

Portrait of Bill Reynolds by Linda Gray-Moin (2007).



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An Appreciation of the Life and Work of William C. Reynolds (1933–2004)

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Annu. Rev. Fluid Mech. 2017. 49:1-21

First published online as a Review in Advance on May 5, 2016

The Annual Review of Fluid Mechanics is online at fluid.annualreviews.org

This article's doi: 10.1146/annurev-fluid-122414-034434

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Keywords

biography, turbulence, boundary layers, flow control, teaching

Abstract

Bill Reynolds was a remarkably creative scientist who combined a natural curiosity with enormous energy to make significant contributions to fluid mechanics research. In this article, we combine our own recollections with those of many others to capture the aspects of Bill's personality and sense of humor that made him the irrepressible person that he was. We discuss his works on turbulent flow and touch on others that illustrate the wide range of his interests. We survey his involvement in education through classroom teaching and mentoring of research students, and his lifelong support of the Division of Fluid Dynamics of the American Physical Society. And we cover his many contributions during his long career at Stanford University, where he spent his entire working life, especially his seminal role with the Center for Turbulence Research.

INTRODUCTION

Bill Reynolds was born on March 16, 1933, and died of a malignant brain tumor on January 3, 2004, at age 70. As the Editors of a special issue of the *International Journal of Heat and Fluid Flow* wrote, "The field of turbulence research and, indeed, the world of Fluid Mechanics thus lost, at a stroke, one of its strongest and most inventive and charismatic leaders" (Kasagi et al. 2004). Bill was a mentor, colleague, and friend to both of us, and we were honored to be invited to write this article.

Bill advised or coadvised over 50 PhD students, and countless MS students and postdoctoral fellows, and he published over 200 papers. He was elected to the National Academy of Engineering (1979), American Academy of Arts and Sciences (1995), American Society of Mechanical Engineers as a Life Fellow (1979), American Physical Society (APS) as a Fellow (1982), Sigma Xi (1958), and Tau Beta Pi. He also received an honorary degree from the University of Manchester (2000). At his death, he was the Donald W. Whittier Professor Emeritus of Mechanical Engineering at Stanford University.

He served as the chair of the mechanical engineering department on two separate occasions, for a total of 15 years, and cofounded both the Institute for Energy Studies (IES) and the Center for Turbulence Research (CTR) at Stanford.

Bill was a gifted educator: He received the G. Edwin Burks Award for Outstanding Teaching from the American Society of Engineering Education (1972), the Tau Beta Pi Outstanding Teaching Award at Stanford (1974), and the Teaching Award from the Society of Women Engineers. The textbooks *Thermodynamics* (Reynolds 1968) and *Engineering Thermodynamics* (Reynolds & Perkins 1977) and Bill's computer program for property estimation, STANJAN, are used at over 100 universities in the United States, and worldwide, including Japan, Russia, Denmark, Germany, Italy, Sweden, Brazil, South Africa, and Australia. The book *Energy: From Nature to Man* (Reynolds 1974) was used in his course about energy and energy technology aimed primarily at nonengineering students. The book is a timely and cogent presentation of quantitative aspects of energy sciences, and the accompanying societal/policy choices.

But these simple statements of facts and honors paint an incomplete picture of Bill.

THE ESSENTIAL BILL REYNOLDS

Bill was a master of all modes of scientific enquiry—experiments, applied mathematics, and numerical simulation—and brought them and his formidable intellect to bear on many problems in fluid mechanics. We review a fraction of his major scientific contributions later in this article. Bill exulted in the joy of tackling new challenges and in finding elegant and fresh solutions to unsolved problems. His research in fluid dynamics, particularly in turbulence, was recognized worldwide, and the knowledge he imparted to several generations of graduate students, postdoctoral scholars, and colleagues has had an influence that persists to this day. As a former student put it, "Hallmarks of Bill's personality are his seemingly endless energy, youthful curiosity and a passionate enthusiasm for any endeavor he embarks upon. His enthusiasm, high standards and appreciation for both fundamental research and practical engineering applications have touched and influenced scores of students and colleagues over the years."

As a scientist, Reynolds was the ultimate independent thinker, a self-starter, do-it-yourselfer, a hands-on problem solver, and, in the most favorable sense of the term, a micromanager. In following his own intuition, Bill might have reinvented the proverbial wheel, but in the process he found novel and exciting ideas and designs that enriched the engineering field and inspired the people around him. He was a true believer in the familiar maxim (which he repeated quite often) that "if you want it done right, you had better do it yourself." He designed his own house and once

told one of us (G.M.H.) who was undertaking a similar venture that "sure, you'll make mistakes, but they will be *your* mistakes." Not a natural delegator, he immersed himself in many diverse projects with boundless energy and indefatigable enthusiasm. In the words of his son Russell, Bill felt that "anything worth doing was worth overdoing."

He took a no-nonsense approach to classroom teaching that combined rigor with a degree of showmanship, which was designed to make concrete the concepts he sought to convey. He was known to play drums in an applied math class to illustrate the effects of wave motion. He could be acrobatic in his demonstrations, famously depicting "blooming jet" vortex rings by a modern dance-like movement reminiscent of Isadora Duncan.

He was one of the first to embrace the computer, and started authoring programs for his own works; many of them were used later by others worldwide in teaching and research. When writing his thermodynamics texts, Bill became frustrated by the inconsistencies in thermodynamic properties and the use of mixed units that pervaded the literature. He spent countless hours in the library, going through original sources, correcting errors and mistabulations, and converting everything to SI units—STANJAN is the result. It was not just the properties and mixed units that frustrated Bill. In the preface to his STANJAN book (Reynolds 1979), he writes,

The thought of hiring a draftsperson to draw all of the charts, and then of my having to check them carefully, was so hideous that I decided to teach our IBM 360 computer and its accessory CALCOMP plotter how to draw the diagrams. Once the program had been perfected, it took only milliseconds to calculate a graph and only a few minutes to draw it, and so I was able to repeat the process several times, if necessary, to get a drawing that was acceptable.

Here Bill is referring to a computer program he wrote that mechanically drew the detailed material property charts; even with all the current technological advances in computer-aided plotting/drafting, very few people today have the talent to make equivalent charts.

When he needed something in his work, he invented it. Stavros Kassinos recounts that he and Bill adapted LaTeX to "crank through some heavy duty tensor algebra" related to structure-based turbulence modeling (Bill referred to this as "doing our i-j-k's"). **Figure 1** is taken from Bill's notebook, showing his analytical ability. Both examples demonstrate Bill's prowess in applied mathematics and the "i-j-k" business.

Bill wrote the computer program for sorting graduate student applications for the mechanical engineering department, which was used until the early 1990s, and he wrote his own word processor to nicely display mathematical equations before LaTeX became the default standard.

When the computer room at the CTR required air conditioning, instead of waiting for the Stanford physical plant staff to draw up and execute plans, he designed and installed the system himself.

After he stepped down as chair of the mechanical engineering department and moved to more modest office space, Reynolds realized that one of the walls was hollow, and managed to remove the sheet rock, expanding his office dimension by one foot. To his delight, during this process, he discovered a water pipe that became a source of water for his own wet bar. He joked that the water was suitable to mix with good scotch, but only this, because the plumbing in the building had not been traced.

EARLY YEARS

Bill was born and raised in the East Bay area of the San Francisco Bay Area. His family moved 13 times between 1933 and 1950, at which point he entered Stanford University as a freshman.

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A page from Bill Reynolds's notes manipulating equations for homogeneous turbulence with mean shear in a time-dependent coordinate system.

Whatever uncertainty was engendered by these moves was replaced by solidity: He obtained all his degrees in Mechanical Engineering from Stanford [BS (1954), MS (1955), and PhD (1957)] and, aside from sabbatical leaves, spent his entire career there.

As a junior in high school and captivated by the rocket science of the day, Bill wrote to Wernher von Braun to ask his advice about how best to join the space program. von Braun wrote back, telling him to go to a fine university and major in engineering. Bill did the first of these, but began his freshman year as a music major. After switching to engineering, his grades suffered as a result of having two jobs in order to work his way through Stanford (difficult even in the 1950s) while in addition playing trumpet and leading his own dance band.

A happy accident led to Bill meeting his wife, Jan. Bill was sharing a house with several other Stanford undergraduates when a delivery intended for them was sent by mistake to the house next door, whose family name was also Reynolds. Tracking down the delivery put Bill and Jan in contact, romance ensued, and they were married in the Stanford Memorial Church between Bill's junior and senior years.

Although it is hard to imagine, Bill's undergraduate record indicated a mediocre student who had no focus and was drifting. That changed as a result of the wisdom and intervention of two Stanford professors—Bill Kays and Lou London. In his junior year, Bill took a course in thermodynamics from Kays and received a B. He was, according to Kays, unremarkable and in the middle of the class. In his senior year, he again took thermodynamics, this time first from London and then again from Kays—receiving the top A in the course! What had changed? London quickly recognized Bill's talent and potential and wanted to convince him to continue in the graduate program, but was shocked when he consulted Bill's transcript, which included an F in a math course! London issued a challenge: "Get straight As in spring quarter and we will admit you to graduate school." Bill never turned down a challenge. The result was foregone: straight As, not only in the spring of his senior year, but the entire first year in graduate school. Kays and Steve Kline intended to hire Bill on their new NASA grant, but there was a twist. Between his first and second years in graduate school, Bill worked at the NASA Ames Laboratory for a summer. He returned to Stanford in the fall, bringing his own research grant with him!

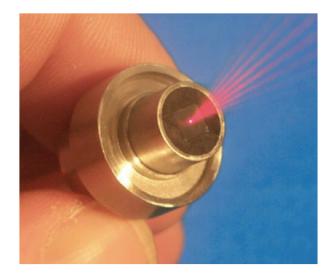
Bill completed his PhD thesis in three years, which was a record at the time, and was appointed to the faculty at Stanford. As mentioned above, he spent his entire career there, officially retiring in 2000.

Bill's first sabbatical (1964–1965), which he spent at the National Physical Laboratory in Teddington, UK, was significant, as it brought him into contact with Trevor Stuart, who became a lifelong friend. In a letter to us, Trevor writes,

At the time John Watson and I, having devised equations for nonlinear waves in plane Poiseuille flow, were keen to solve them numerically, but had made little progress. Bill was intensely interested and enthusiastic, and he found it possible to use a numerical scheme... to produce answers, including the fact that the bifurcation was subcritical, a remarkable result. However the \mathcal{JFM} Editor to whom the paper was sent was less impressed and it was initially rejected... a modified paper was published in 1967 jointly with Merle C. Potter (Reynolds & Potter 1967) but with additional valuable material, the first rational calculation to show a subcritical bifurcation for a hydrodynamic stability problem.

BILL AND FLUID MECHANICS

Bill's PhD dissertation is on convective heat transfer in turbulent boundary layers, which he completed under the supervision of Bill Kays (Reynolds 1957). Following his appointment to the Stanford faculty, he gradually focused his attention on the mechanics, modeling, and control of turbulent flows. Bill was naturally drawn to difficult problems and, fortunately for the field, used his curiosity and intellect to make major advances in them. As mentioned above, he had the breadth and talent to use all three principal approaches to scientific research: theory, experimentation, and computation. He refused to be stymied by lack of tools and techniques, and would invent new ones as he needed them. For example, he and his student Amir Naqwi developed an optical shear stress sensor using a diverging fringe pattern for the measurement of wall stresses in turbulent



Microsensor for measuring wall shear stress. Figure courtesy of Measurement Science Enterprise, Inc.

boundary layers (Naqwi & Reynolds 1987). **Figure 2** shows a miniaturized version of their device. Bill was also quick to recognize emerging fields and to take initiative. Although he did not pursue the field in depth, he liked to point out that he did some of the earliest "microgravity" experiments on drop shapes in zero g by dropping containers from the roof of one of the laboratory buildings at Stanford (Reynolds 1959, 1961)!

In boundary layer experiments, he was an early adopter and developer of microelectromechanical systems (MEMS) for sensing and actuation in active flow control. Reynolds was an avid internal combustion engine enthusiast. He had an experimental project to visualize and measure the flow in a single-cylinder engine with a transparent single-crystal sapphire cylinder (Richman & Reynolds 1984). A special Schlieren system was developed to visualize the flow in the cylinder, which was complemented with a holographic optical element to correct system aberrations. In another automobile-related project, he and student Joseph Pope built a hemi-anechoic room and road-wheel facility to study the noise from automobile tires. They measured the level and spectral characteristics of noise for several tires with different tread patterns (Pope & Reynolds 1976). Perhaps more telling is the motivation for the study. Apparently, the situation that spurred Bill was his annoyance at the traffic noise at his home caused by the newly constructed interstate highway 280, nearby. Typically, Bill was going to study the problem, understand the source of the noise, and get the Interstate Highway System to come fix it!

Bill developed computer programs that were suitable not only for his own use, but also for the general user. For example, his ORRSOM program (Reynolds 1969) was one of the early codes for the computation of the Orr-Sommerfeld modes. The code is user-friendly and robust, reads input parameters (the Reynolds number, a user-defined arbitrary mean velocity profile, and the wave number of the desired mode), and returns the corresponding eigenfunction, growth rate, and phase speed.

He was the ultimate do-it-yourselfer in conducting research as well as in life. When he needed a numerical solver, he found it more desirable to write it himself from scratch than to search for it elsewhere. As mentioned above, this sometimes resulted in the reinvention of the wheel, but the process often led to a fresh look at old concepts and ideas for improvements. The Stanford Engine Simulation Program (ESP; available at http://esp.stanford.edu) is such an example of Reynolds's creativity and out-of-the box approach to problems. Reynolds wrote ESP in 1987 for instructional and engine design purposes. It is a fast, user-friendly interactive program for simulating the thermodynamic performance of homogeneous charge spark ignition engines, including a simplified dynamic model of the intake and exhaust manifolds. An equation for the time rate of change of turbulent kinetic energy is solved to account for the effects of turbulence on combustion and heat transfer (Lumley 1999).

Although these vignettes indicate the breadth of Reynolds's intellect and interests, it is undeniable that his major scientific contributions were in the study of turbulence. His strong reputation among the community is exemplified by the symposium on turbulence held near Monterey, California, in March 1993 to honor Bill on his sixtieth birthday. A special issue of the *Physics of Fluids* dedicated to Bill (volume 6, issue 2) contains over 30 articles on various aspects of turbulence authored by the leading researchers of the day (see **Figure 3**).

Below we review four turbulence research projects that occupied Bill and several students for some years.

DISCOVERY AND THE MECHANICS OF COHERENT STRUCTURES IN TURBULENT BOUNDARY LAYERS

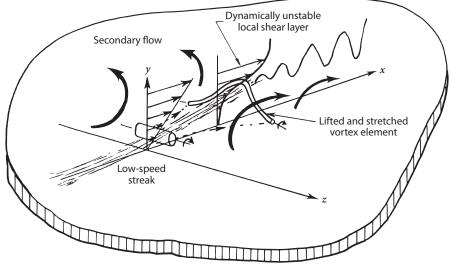
Reynolds's collaboration with Steve Kline throughout the 1960s resulted in landmark studies of the structure of turbulent boundary layers. In the late 1950s, Kline and his student P.W. Runstadler had observed alternating low- and high-speed flow regions, which they called wall layer streaks. Although these earlier studies were based largely on flow visualizations, subsequent publications in the next decade included substantial quantitative information and key conceptualizations of the mechanics of turbulence production in boundary layers. Here we find Reynolds to be highly influential in asking key questions and seeking answers grounded in quantitative data and theory. His contributions led to a deep understanding of the structure of wall-bounded flows and brought a rigor to the work that was heretofore lacking. The results appeared in two classic articles in the *Journal of Fluid Mechanics*. In the first article, Kline et al. (1967) firmly established the presence of well-organized spatially and temporally dependent motions in the immediate vicinity of the wall. Their data indicated that the mean spanwise spacing of the high- and low-speed streaks scaled with wall region parameters and measured at approximately 100 wall units, a figure that has endured the test of time and has been validated by numerous later measurements. This study also put to rest the notion that the flow in the vicinity of the wall is strictly laminar: The so-called laminar sublayer is a misnomer and is now referred to as the viscous sublayer in the modern literature.

Another important legacy of this study was the observation of streak breakup and ejection from the wall. They described in detail the events leading to the so-called bursting event and measured its frequency, which when expressed in wall units exhibited an approximate scaling. They conjectured that the bursting event is the dominant source of turbulence production in boundary layers. Bill provided a mechanistic explanation for the bursting process in terms of vortex dynamics and hydrodynamic stability theory. The key physical processes in bursting are depicted in **Figure 4** (a reproduction of figure 19 from Kline et al. 1967), which is believed to be drawn in Bill's own hand.

In the second, related, and definitive study, Bill and his student T. Uzkan conceived and built a novel experimental facility to isolate the two main effects of the wall on the flow: blockage and mean shear (Uzkan & Reynolds 1967). In it, grid turbulence was passed over a moving wall at the mean flow speed. The resulting turbulent boundary layer is without mean shear and hence devoid of production of turbulence. Visualizations showed no evidence of streaks, providing indirect



Group photo taken at the symposium on turbulence in honor of Bill Reynolds's sixtieth birthday, Asilomar Conference Center, near Monterey, California, March 22–23, 1993.



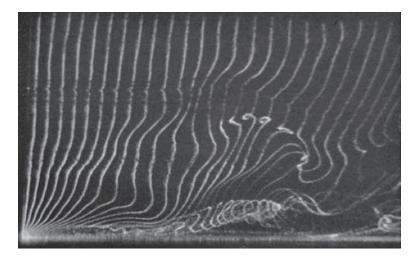
Schematic of the mechanics of streak breakup. Figure reproduced with permission from Kline et al. (1967).

confirmation of the connection between streaks and turbulence production. The results of this study have informed turbulence modeling, in particular, in the pressure–strain correlations in which separate terms due to mean shear and turbulence fluctuations are modeled (Launder et al. 1975).

Reynolds's in-depth study of turbulent boundary layer structure culminated in the paper by Kim et al. (1971). Both pulsed hydrogen bubble visualizations and hot wire measurements, most notably in planes normal to the wall, provided quantitative support for their 1967 conjectures. Instantaneous velocity profiles and Reynolds shear stress measurements were used to substantiate their claim that essentially all of turbulence production occurs during the bursting events. The hydrogen bubble flow visualizations, an example of which is shown in **Figure 5**, are striking and clearly depict instantaneous velocity profiles with inflection points that are qualitatively distinct from the mean velocity profile. An almost decade-later comparison to these profiles, in a film first presented at the APS Division of Fluid Dynamics (DFD) meeting at Cornell, became the basis for establishing the credibility and physical realism of the large eddy simulation (LES) technique (Kim & Moin 1980).

TURBULENCE SUBJECT TO IMPOSED PERIODIC DISTURBANCES

Reynolds's research in turbulence in the 1960s included experimental and theoretical studies of the effect of waves and imposed unsteadiness on turbulent boundary layers. The main objective was to explore the possibility of representing turbulent flows as a random superposition of characteristic waves, an idea that was advanced by Landahl (1967). Landahl's representation included waves that were born at some location and decayed as they traveled downstream. With his student Fazle Hussain, Reynolds artificially introduced periodic waves in a turbulent channel flow and tracked their downstream evolution (Hussain & Reynolds 1970, 1972). In terms of analysis, they advanced a celebrated triple decomposition of turbulent flows into mean, phase-averaged, and fluctuating components (Reynolds & Hussain 1972), and derived the dynamical equations governing



Hydrogen bubble visualizations in the vertical plane showing time lines and the evolution of a downstream vortex. Figure reproduced with permission from Kim et al. (1971).

small-amplitude wave disturbances in turbulent shear flows. Their data did not support the notion that in this turbulent flow the waves evolved according to the Orr-Sommerfeld theory. Through his calculations, Reynolds argued that separate modeling considerations should be used for the stresses due to the imposed deterministic waves and the remaining "random turbulence," and that the modeling of the fine-scale turbulence stresses should take into account the stresses due to wave motions. The triple decomposition of Reynolds and Hussain has been adopted in representing turbulent flows by mean, coherent modes, and random fluctuations in several later studies (see Sayadi et al. 2014). Reynolds continued his experimental studies of imposed unsteady fluctuations in turbulent shear flows for several years (Acharya & Reynolds 1975, Norris & Reynolds 1975, Brereton et al. 1990).

Bill was keenly aware of Lumley's (1967) contrasting viewpoint that turbulence consists of characteristic energetic eddies of compact support as opposed to traveling waves. Both views, of course, do not avoid the problem of turbulence closure, but which representation is the more useful in statistical turbulence modeling was very much on Reynolds's mind. Years later Reynolds encouraged one of us (P.M.) to explore Lumley's decomposition to analyze turbulent shear flows using newly available data from LES of channel flow (Moin 1984, Moin & Moser 1989).

BIFURCATING AND BLOOMING JETS

Bill had strong interests and an experimental program in turbulent flow control. This included the design and fabrication of small sensors and actuators to delay transition or reduce skin friction in boundary layers (Jacobson & Reynolds 1998). From the early 1980s to the mid-1990s, he and three students focused on the active control of turbulent jets (see Reynolds et al. 2003 for a review of their work). Significant flow control was achieved through a combination of axial and circumferential excitations at the nozzle exit. With one set of excitations, a jet was forced to divide into two separate jets, referred to as bifurcating jets (see **Figure 6**). With another set of excitations, a blooming jet resulted, in which the jet explodes into an amazing shower of vortex



Two orthogonal photos of a bifurcating water jet taken by Bill Reynolds in the smaller water flow apparatus that he built during his 1984 sabbatical leave at Caltech. Figure reproduced with permission from Reynolds et al. (2003).

rings (see **Figure 7**). Both jets produce a significant increase in turbulent mixing with promising engineering applications.

The enthusiasm with which Bill pursued these studies was infectious, and was emblematic of his joy of discovery—in his view, research should be *fun*. One of us (G.M.H.) recalls how Bill nearly forced his way onto the Tuesday afternoon Fluid Mechanics Seminar schedule at Stanford so that

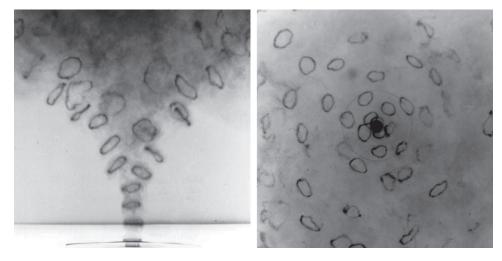


Figure 7

Side and axial views of a blooming jet in a mechanically oscillated water flow. Figure reproduced with permission from Reynolds et al. (2003).

he could share this fascinating phenomena with the community. In 1984, during his sabbatical leave at the California Institute of Technology, Reynolds built a smaller water flow facility to replicate his now favorite jet control experiments. He was eager to show them off and was particularly proud that they impressed the renowned Caltech physicist Richard Feynman.

These flow control studies eventually led to a demonstration of an active flow control on an aircraft engine (Kibens et al. 1999). As Reynolds pointed out, these and similar fundamental studies at other institutions were excellent examples of how long-term investment in fundamental research in fluid mechanics led to important new applications. He was particularly appreciative of the sustained support from the Air Force Office of Scientific Research that enabled him to carry out his flow control experiments.

LARGE EDDY SIMULATION AND TURBULENCE MODELING

Reynolds's reputation in the 1960s in turbulence research was well established as a result of his highly regarded and well-conceived landmark experimental investigations of the mechanics of turbulent shear flows. In 1970, Hans Mark, director of the NASA Ames Research Center, contacted Joel Ferziger, who was Reynolds's colleague at Stanford, and offered to fund a program at Stanford in high-fidelity numerical simulations of turbulent flows. This was part of the grand vision of the NASA Ames leadership to promote computational fluid dynamics for aircraft design and to replace wind tunnel tests with virtual ones to the extent possible (Chapman et al. 1975). Of course, turbulence posed the main obstacle to obtaining accurate aerodynamic calculations. Mark was a nuclear engineer and knew Ferziger's work through his books in this area. In a very astute move, Mark stipulated that Reynolds should be part of the new computational turbulence program at Stanford to give it credibility in the turbulence research community (which at the time was dominated by experimentalists). Thus began a long-term cooperative research program on numerical simulation of turbulent flows between Stanford and NASA Ames. A series of technical reports, numbered as TF-X (mainly reprints of PhD theses), was initiated, and these were widely distributed.

At that time, three-dimensional time-dependent numerical simulations of turbulent flows using subgrid-scale models to represent small-scale motions were being carried out primarily at the National Center for Atmospheric Research. In 1970, Jim Deardorff's landmark paper on simulation of infinite–Reynolds number channel flow, which was carried out on a CDC 6600 computer, appeared in the *Journal of Fluid Mechanics* (Deardorff 1970). However, it appears that Stanford's program was the first to embark on turbulent flow simulations for engineering analysis.¹ Reynolds referred to this method of calculating turbulent flows as large eddy simulation (LES), a terminology that has been universally adopted.

Reynolds influenced this burgeoning LES program in several important ways. In particular, he insisted on a rigorous derivation of the constitutive equations through explicit filtering of the Navier-Stokes equations, and a clear separation between numerical discretization errors and those due to a subgrid-scale model. Tony Leonard, who was very much involved in the Stanford program from its inception, derived the LES equations using explicit filtering kernels, which became the first TF-1 report (Leonard 1973, 1974). Leonard's derivation exposed additional turbulence stresses

¹At about the same time, Ulrich Schumann at the University of Karlsruhe was working on his doctoral thesis, which appeared in 1973 and was later summarized in the 1975 *Journal of Computational Physics* article entitled "Subgrid Scale Model for Finite Difference Simulations of Turbulent Flows in Plane Channels and Annuli" (Schumann 1975).

(commonly referred to as the Leonard stresses) that could be computed explicitly. Because the multiplicative factor of the Leonard stress (as well as that for Smagorinsky's subgrid-scale model) included the square of the filter width, Reynolds insisted on using fourth- or higher-order finite difference schemes in LES in order not to overwhelm the Leonard and subgrid-scale stress terms with the numerical truncation errors.

The Stanford program pioneered the use of simulation databases, with their presumed time and space resolution and ability to be analyzed in a variety of ways, as a complement and supplement to experimental data in turbulence research. With several students, Bill oversaw simulations of canonical flows to gain insight into turbulence physics and turbulence modeling concepts for engineering computations. For example, in their pioneering study, Clark et al. (1977, 1979) used data from direct numerical simulation (DNS) of isotropic turbulence to "exactly" compute the subgrid-scale modeled terms used in LES. They realized the potential of DNS for a priori testing and evaluation of reduced-order models of turbulence, including subgrid-scale models for LES.

Several numerical simulations of canonical flows were conducted to study the effect of external forcing on turbulence. These included isotropic turbulence subjected to plane and axisymmetric strain and its subsequent return to isotropy, homogeneous sheared turbulence, homogeneous turbulence subject to system rotation, compressibility effects, and scalar transport. Reynolds's motivation in studying these canonical flows was often rooted in his interest in developing engineering turbulence models, and his belief that properly constructed turbulence models should behave correctly in these limiting cases. For example, his interest in simulation data from rapidly strained turbulence and return to isotropy was motivated by the need to test pressure–strain correlation models for the Reynolds stress equations. Additionally, it was known that system rotation inhibits the rate of dissipation of turbulence; therefore, he demanded that dissipation models should be consistent with this fact.

Although Reynolds was among the first to recognize the potential of high-fidelity simulations in turbulence research, he always harbored a deep interest in engineering turbulence modeling using the Reynolds-averaged Navier-Stokes (RANS) equations. Arguably, the motivation for much of his fundamental turbulence research was to gain the insight required to improve predictive statistical turbulence models. His article in the *Annual Review of Fluid Mechanics* (Reynolds 1976) is an insightful review of the subject. He devoted the last section of that article to LES and introduced it as a promising approach for the prediction of turbulent flows. In fact, engineering turbulence modeling became his main research focus during the last years of his life.

In the 1980s, research and the discovery of coherent structures in turbulent flows brought renewed criticism of the viability of the RANS approach for the prediction of turbulent flows. As Lumley (1990) had pointed out, the statistical modeling of turbulent flows and the existence of coherent structures need not be mutually exclusive. Reynolds (1992) addressed this issue by introducing his structure-based turbulence modeling. In this creative departure from the traditional approaches to turbulence modeling, a new one-point quantity, the eddy structure tensor, was introduced, which carries information about the two-point structure of turbulence. Up to this point, the conventional turbulence models were based only on turbulence stresses. Reynolds's original formulation for homogeneous turbulent flows involved 13 variables tracked in time through differential equations. This included six equations for the components of the eddy orientation tensor.

The structure-based modeling concept was significantly refined and expanded with his student Stavros Kassinos (Kassinos & Reynolds 1994, Kassinos et al. 2001). They introduced five new tensors, depicted schematically in **Figure 8** and labeled componentality, dimensionality, circulicity, inhomogeneity, and stropholysis, as descriptors of turbulence structures. A systematic framework

$$M_{ijpq}^{|\mathbf{a}|} = \widetilde{V}^{2}\widetilde{\zeta} \frac{1}{8} \epsilon_{irr} \epsilon_{jzs} [(\delta_{rz} \delta_{pq} + \delta_{rp} \delta_{zq} + \delta_{rq} \delta_{zp}) - \delta_{rz} a_{p} a_{q}] a_{t} a_{s}$$

$$Vortical eddy$$

$$(2D-2C)$$

$$+ \widetilde{\gamma} \widetilde{V}^{2} \frac{\Omega}{\Omega} (\epsilon_{irr} a_{t} a_{j} + \epsilon_{jrr} a_{t} a_{i}) [\frac{1}{8} (\delta_{rk} \delta_{pq} + \delta_{rp} \delta_{kq} + \delta_{rq} \delta_{kp})$$

$$- \frac{1}{8} (\delta_{rk} a_{p} a_{q} + \delta_{rp} a_{k} a_{q} + \delta_{rq} a_{k} a_{p})]$$

$$+ \frac{1}{2} \widetilde{V}^{2} (1 - \widetilde{\zeta}) [\delta_{pq} a_{i} a_{j} - a_{i} a_{j} a_{p} a_{q}]$$

$$Jettal eddy$$

$$(2D-1C)$$

Schematic illustration of basic turbulent eddies in structure-based modeling, given in terms of vortical, helical, and jettal eddy structures. Moments conditioned on a fixed eddy axis vector **a**, such as the fourth-rank tensor $M_{ijpq}^{|a}$ shown, are constructed to model a 3D-3C turbulence field as a superposition of these building blocks. D and C denote dimensionality and componentality of the structures shown. Arrows signify which fluctuating velocity components are strong: The three eddies have different componentalities. Elongated shapes represent the strong correlation in fluctuating velocity along and around the eddy axis (dimensionality). Figure adapted with permission from Kassinos & Reynolds (1994).

was developed for exploring the role of turbulence structures in the evolution of one-point turbulence statistics with flow undergoing diverse modes of mean deformation. These additional tensors lead inevitably to a very complex set of equations, examples of which are given in **Figure 8**. In yet another characteristic creative feat, in order to carry out the required extensive algebraic manipulations involving proper tensor expansions, Kassinos and Reynolds developed their own symbolic tensor manipulation software. This procedure minimized errors in the derivation of the equations as well as their transcription into computer readable form suitable for both computations and presentation in publications.

Reynolds was very much aware that the complexity of the structure-based equations would deter their eventual use in engineering computations. With his last two students, Scott Haire and Carlos Langer, he worked on the development of algebraic structure-based models for engineering computations. Reynolds's last technical act was to ensure that these two PhD students had successfully defended their dissertations before he was gone.

BILL AND STANFORD

Reynolds was a Stanford stalwart who left a deep mark on the institution and on its students, faculty, and staff. As one Stanford Dean of Engineering commented, "Every organization . . . has within it individuals about whom one would say that they vibrate at an especially high frequency. Bill Reynolds is such a person." Bill won admiration from his colleagues for his unselfish administrative skills and commitment to excellence. Everything he did, or was asked to express an opinion on, had to measure up to what he referred to as "Stanford quality." On the occasion of his appointment as the Donald W. Whittier Professor, his chosen quote was, "What I enjoy most about Stanford is that anything is possible; good ideas are always encouraged and supported. This makes it an exciting place for everyone."

He had no time for recrimination or complaints—he was moving too fast. One of us (G.M.H.) was corralled into helping Bill write a Department of Energy proposal for graduate fellowships

in energy studies that would go to three Stanford departments. He opined that he "hoped that the [other] department appreciates the work we're putting in on this." As it turned out, the other departments were awarded funding while the mechanical engineering department was not, and the hoped-for thanks were not forthcoming. He never complained or commented on it again: He was on to the "next great thing."

Bill was also deeply admired as one of Stanford's very best classroom teachers, and his commitment to student learning was remarkable. One of us (G.M.H.) was present at an award ceremony during which members of the Society of Women Engineers recounted an occasion when Bill drove from his home in Los Altos to the dormitory at 10 pm to help some students work on that week's problem set. Needless to say, the students were both astonished at and appreciative of this level of commitment from a Full Professor.

Bill's enthusiasm and creativity extended to another of his passions—Stanford football. When the gunpowder cannon used after every Stanford touchdown misfired in 1970 and was declared unsafe to use, he and graduate student Chris Flegal designed and built an impulse cannon essentially a shock tube with (of course) a trumpet-shaped bell—that mimicked the original cannon and was in use for many years. The bell shape was neither an homage to his avocation as a musician nor a design flourish: It was necessary to disperse the acoustic energy sufficiently to avoid serious hearing damage to the operator.

Bill's quick and agile mind lent itself to opportunism. When the energy crisis of the 1970s was in full swing, he gathered a group of faculty together for an evening's brainstorming about energy research at Stanford. He quickly took the group's scribbled notes and half-baked ideas and molded them into a successful proposal to found the IES, which he directed from 1974 to 1981. When the IES needed space and Stanford phased out their telephone switchboard, Bill, knowing both that vacant space does not last long and that possession is nine-tenths of the law, quickly commandeered the building and claimed "squatter's rights." That building currently houses the CTR.

Bill, together with one of us (P.M.) and John Kim, was one of the cofounders of the CTR. In the decade of 1975–1985, several new developments had taken place that justified a concerted new assault on the turbulence problem. These included the advent of supercomputers, which made possible the computation of turbulence from first principles using direct simulations. The databases generated from these simulations had begun to make a large impact on turbulence research. Effective utilization of these new developments required a critical mass of experimental, theoretical, and computational expertise in an environment conducive to synergetic, high-quality research—an environment capable of attracting the best, most experienced people in the field and the brightest new young researchers.

At the time, Stanford and the nearby NASA Ames Research Center had research activities in many areas of aerodynamics, and in particular, both had interest in understanding the fundamental physics of turbulence, developing turbulence models, and utilizing them for turbulent flows of engineering interest. The capabilities possessed by each were complementary, and it was clear that a cooperative activity to develop a program for turbulence research would be mutually attractive, synergistic, and effective. On February 5, 1987, Stanford and NASA Ames entered into an agreement to establish the CTR.

To ensure that the CTR was truly a joint effort rather than a more conventional center of excellence funded by a government agency, Bill also agreed to accept a part-time civil service appointment at NASA. This arrangement between a great university and a distinguished national laboratory was unique and provided for a relationship based on scholarship (as opposed to that of a grantor and grantee) between the NASA Ames scientists and the cognizant Stanford faculty and researchers. The launching of the biennial CTR Summer Programs was Bill's idea for reaching out to the turbulence research community. He felt that while having access to NASA's supercomputers,



Group photo of the first Center for Turbulence Research Summer Program participants, taken in 1987. Bill Reynolds stands confidently second from the right (wearing a tie).

CTR researchers had generated large turbulence databases that should be made available to the summer program participants to test their own hypotheses and models. These biennial CTR Summer Programs, the first of which was in 1987 (see **Figure 9**), have proven to be a highly successful venue for international turbulence research.

Bill's memorial service was held in Stanford's Memorial Church on January 20, 2004. Memorial Church was overflowing, with many people standing far in the back, demonstrating the respect and affection with which he was held both by the Stanford community and by many outside it. At the conclusion of the memorial service, Stanford students fired his football cannon three times, and in a most moving moment, a student from the Stanford marching band played taps on her trumpet.

BILL AND THE APS DIVISION OF FLUID DYNAMICS

The DFD of the APS was the beneficiary of Bill's enthusiasm and creativity. He played a major role in the DFD's governance and, in the early years of Andy Acrivos's tenure as Editor of the *Physics of Fluids*, was instrumental in strengthening the connection between the *Physics of Fluids* and the DFD. He was also a constant, visible, and vocal (!) attendee at every annual meeting. However, Bill's love for the DFD and his equally strong love of Stanford football created an annual head-on collision. The Saturday big game between Cal Berkeley and Stanford overlapped with the Sunday start of the DFD meeting! Bill solved this problem in typical fashion: He would attend the game and then take a red-eye flight to the meeting, arriving sleep-deprived but going full steam. **Figure 10**, taken from his 1999 pocket calendar, shows not only an "average" week for Bill, but also the lengths to which he would go to attend both events: leaving the San Francisco International Airport at midnight, changing planes, landing in New Orleans at 10:48 am (7:48 am

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Bill Reynolds's pocket calendar for the week of November 15, 1999. The small square boxed item in the lower right gives his flights and connections for the red-eye flight to New Orleans for the 1999 APS Division of Fluid Dynamics meeting.

West Coast time), renting a car, and arriving just in time to quiz some of the first speakers of the morning sessions. Indeed, it was his absence at a subsequent meeting that caused many to think that something was wrong—a premonition that was unfortunately on the mark.

BILL REYNOLDSISMS

Here we recount a few stories and vignettes contributed by several colleagues that illustrate Bill's sharp wit and sense of humor. For instance, several of Bill's former students have commented on his ability to concentrate sharply and without distraction on a technical project. With characteristic tongue-in-cheek drama, he was known to employ the plastic yellow caution tape used on construction sites to form barriers, strapping it boldly across his own office doorway as a sure indication of "no trespassing." Additionally, when teaching perturbation theory one quarter, he posted a sign on his office door stating "Do not perturb."

Once when caught in an impenetrable traffic jam after a Rose Bowl game in Pasadena, Bill jumped out of the car, and acted as a traffic cop, giving favor to the lane in which his family car was crawling. When the coast was clear, he jumped in and drove off.

Once when teaching applied math, he inadvertently (and uncharacteristically) made an error in his presentation. When a student pointed this out, Bill dropped the chalk and exited the lecture hall to the right. He then circled around the building, entered from the left, and remarked to the effect that "You know I have a twin brother who doesn't know beans about mathematics."



(*a*) Bill Reynolds on the California coast wearing a Stanford football letterman jacket (Half Moon Bay, March 29, 1953). (*b*) Bill in 2003 after his brain tumor diagnosis. (*c*) Bill playing in his high school band for the Legion Hall Veteran's Dance, July 1948. (*d*) Bill speaking about his "drop tower" experiments at the symposium on Physical and Biological Phenomena Under Zero g Conditions sponsored by the American Astronautical Society, July 1, 1960 (see Reynolds 1959, 1961). (*e*) Bill and his wife Janice on the occasion of receiving an honorary degree from the University of Manchester in 2000. (*f*) Stanford news photo from approximately 1974 covering the founding of the Institute for Energy Studies.

Bill loved inside jokes, even when it involved gallows humor. After his diagnosis and surgery for a brain tumor, he responded to a particularly annoying bureaucratic request for a research report by pointing out that the report should come from the primary investigator, not him, and made an apology for his "half-brained response."

Tony Leonard recounts how he and Bill were giving a joint talk at a symposium at Caltech honoring Hans Liepmann's seventieth birthday when time was short (Leonard & Reynolds 1988).

Bill quipped, "We are running out of time, so we will have to go to parallel sessions" at which point they finished their talk by alternating sentences. The transcript reads as follows:

AL: Moser did establish that a full simulation would give the right scales in the wall region, and Spalart wanted to see if it would in the boundary layer, too.

WCR: Here are the mean profiles ...

AL: ... and here are the turbulent profiles

WCR: Note that the mean profiles are in excellent agreement with experiments ...

AL: ... as are the turbulence profiles.

We leave it to the reader to guess whose was the quicker delivery.

As Department Chair in 1991, he wrote and signed a "Dear Bill" letter to himself, thanking himself for taking a zero raise that year in order to enable raises for the junior faculty, and for refurbishing his office at his own cost. After agreeing to reimburse himself some of the costs, he ended by saying that "Sadly, the hours that you spent doing it will have to remain a donation."

CONCLUDING REMARKS

Bill Reynolds was a remarkable individual whose dynamism, optimism, and accomplishments are an inspiration to all. Despite an official retirement in 2000, Bill never ceased working. His contributions were still forthcoming, new projects were incubating, and there were PhD students whose thesis work needed his input. One can only guess what magic he would next perform, or which envelope he would next seek to push. His work was cut short early, but he had a glorious career and accomplished much in both science and technology (Figure 11).

We can do no better than to end with two quotations.

This first is from Peter Bradshaw's obituary in *Physics Today* (Bradshaw 2005): "An unknown hand has inscribed Reynolds's initials on wet concrete at the entrance to the building where he worked for many years. His scientific reputation will doubtless outlast the concrete."

And the second is from Bill himself: "What is the meaning of a few years more or a few years less in the measure of the infinite? Just make sure you do the most with the time you were given."

Bill certainly did.

DISCLOSURE STATEMENT

The authors are not aware of any biases that might be perceived as affecting the objectivity of this review.

ACKNOWLEDGMENTS

Many of Bill's former students and colleagues contributed reminiscences that enlivened this article in unique ways. Thanks are due to Stavros Kassinos, Brian Launder, Anthony Leonard, Godfrey Mungal, and Trevor Stuart for sharing these with us. We are especially grateful to Jan Reynolds for access to many family photos and records. The material on Bill Reynolds's student days was excerpted from Bill Kays's comments at the memorial service. Curtis Hamman provided invaluable help in the preparation of this article.

LITERATURE CITED

- Acharya M, Reynolds WC. 1975. Measurements and predictions of a fully developed channel flow with imposed controlled oscillations. Rep. TF-8, Dep. Mech. Eng., Stanford Univ., Stanford, CA
- Bradshaw P. 2005. William Craig Reynolds. Phys. Today 58(4):85-86
- Brereton GJ, Reynolds WC, Jayaraman R. 1990. Response of a turbulent boundary layer to sinusoidal freestream unsteadiness. *J. Fluid Mecb.* 221:131–59
- Chapman D, Mark H, Pirtle MW. 1975. Computers vs. wind tunnels for aerodynamic flow simulations. Astronaut. Aeronaut. 13(4):22–35
- Clark RA, Ferziger JH, Reynolds WC. 1977. Evaluation of subgrid-scale models using a fully simulated turbulent flow. Rep. TF-9, Dep. Mech. Eng., Stanford Univ., Stanford, CA
- Clark RA, Ferziger JH, Reynolds WC. 1979. Evaluation of subgrid-scale models using an accurately simulated turbulent flow. J. Fluid Mecb. 91(1):1–16
- Deardorff JW. 1970. A numerical study of three-dimensional turbulent channel flow at large Reynolds numbers. *J. Fluid Mech.* 41(2):453–80
- Hussain AKMF, Reynolds WC. 1970. The mechanics of an organized wave in turbulent shear flow. J. Fluid Mech. 41(2):241–58
- Hussain AKMF, Reynolds WC. 1972. The mechanics of an organized wave in turbulent shear flow. Part 2. Experimental results. J. Fluid Mech. 54(2):241–61
- Jacobson SA, Reynolds WC. 1998. Active control of streamwise vortices and streaks in boundary layers. *7. Fluid Mecb.* 360:179–211
- Kasagi N, Launder BE, Schmidt FW. 2004. In memoriam—William C. Reynolds (1933–2004). Int. J. Heat Fluid Flow 25(3):330
- Kassinos SC, Reynolds WC. 1994. A structure-based model for the rapid distortion of bomogeneous turbulence. Rep. TF-61, Dep. Mech. Eng., Stanford Univ., Stanford, CA
- Kassinos SC, Reynolds WC, Rogers MM. 2001. One-point turbulence structure tensors. J. Fluid Mech. 428:213-48
- Kibens V, Doris J III, Smith D, Mossman M. 1999. Active flow control technology transition—the Boeing ACE program. Presented at AIAA Fluid Dyn. Conf., 30th, Norfolk, VA, AIAA Pap. 1999-3507
- Kim HT, Kline SJ, Reynolds WC. 1971. The production of turbulence near a smooth wall in a turbulent boundary layer. J. Fluid Mech. 50(1):133–60
- Kim J, Moin P. 1980. Investigation of flow structure in the wall region of turbulent boundary layers via a numerical simulation. Bull. Am. Phys. Soc. 25:1103 (Abstr.)
- Kline SJ, Reynolds WC, Schraub FA, Runstadler PW. 1967. The structure of turbulent boundary layers. J. Fluid Mecb. 30(4):741–73
- Landahl MT. 1967. A wave-guide model for turbulent shear flow. J. Fluid Mech. 29(3):441-59
- Launder BE, Reece GJ, Rodi W. 1975. Progress in the development of a Reynolds-stress turbulence closure. *J. Fluid Mecb.* 68(3):537–66
- Leonard A. 1973. On the energy cascade in large-eddy simulations of turbulent fluid flows. Rep. TF-1, Dep. Mech. Eng., Stanford Univ., Stanford, CA
- Leonard A. 1974. Energy cascade in large-eddy simulations of turbulent fluid flows. Adv. Geophys. 18A:237-48
- Leonard A, Reynolds WC. 1988. Turbulence research by numerical simulation. In Perspectives in Fluid Mechanics, ed. D Coles, pp. 113–42. New York: Springer
- Lumley JL. 1967. The structure of inhomogeneous turbulent flows. In *Atmospheric Turbulence and Radio Wave Propagation*, ed. AM Yaglom, VI Tatarski, pp. 166–78. Moscow: Nauka
- Lumley JL. 1990. The utility and drawbacks of traditional approaches: comment 1. In Whither Turbulence? Turbulence at the Crossroads, ed. JL Lumley, pp. 49–58. New York: Springer

- Moin P. 1984. Probing turbulence via large eddy simulation. Presented at Aerosp. Sci. Meet., 22nd, Reno, NV, AIAA Pap. 1984-0174
- Moin P, Moser RD. 1989. Characteristic-eddy decomposition of turbulence in a channel. J. Fluid Mech. 200:471-509

Lumley JL. 1999. Engines: An Introduction. Cambridge, UK: Cambridge Univ. Press

- Naqwi AA, Reynolds WC. 1987. Dual cylindrical wave laser-Doppler method for measurement of skin friction in fluid flow. Rep. TF-28, Dep. Mech. Eng., Stanford Univ., Stanford, CA
- Norris HL, Reynolds WC. 1975. *Turbulent channel flow with a moving wavy boundary*. Rep. TF-7, Dep. Mech. Eng., Stanford Univ., Stanford, CA
- Pope J, Reynolds WC. 1976. *Tire noise generation: the role of tire and road*. SAE Tech. Pap. 762023, SAE Int., Warrendale, PA
- Reynolds WC. 1957. Heat transfer in the turbulent incompressible boundary layer with constant and variable wall temperature. PhD Diss., Stanford Univ., Stanford, CA
- Reynolds WC. 1959. Behavior of liquids in free fall. J. Aerosp. Sci. 26(12):847-48
- Reynolds WC. 1961. Behavior of liquids in free fall. In *Weightlessness: Physical Phenomena and Biological Effects*, ed. ET Benedikt, pp. 53–55. New York: Springer
- Reynolds WC. 1968. Thermodynamics. New York: McGraw-Hill. 2nd ed.
- Reynolds WC. 1969. ORRSOM: a Fortran IV program for solution of the Orr-Sommerfeld equation. Rep. FM-4, Dep. Mech. Eng., Stanford Univ., Stanford, CA
- Reynolds WC. 1974. Energy: From Nature to Man. New York: McGraw-Hill
- Reynolds WC. 1976. Computation of turbulent flows. Annu. Rev. Fluid Mech. 8(1):183-208
- Reynolds WC. 1979. Thermodynamic Properties in SI: Graphs, Tables, and Computational Equations for Forty Substances. Stanford, CA: Stanford Univ. Dep. Mech. Eng.
- Reynolds WC. 1992. Towards a structure-based turbulence model. In *Studies in Turbulence*, ed. TB Gatski, CG Speziale, S Sarkar, pp. 76–80. New York: Springer
- Reynolds WC, Hussain AKMF. 1972. The mechanics of an organized wave in turbulent shear flow. Part 3. Theoretical models and comparisons with experiments. *J. Fluid Mecb.* 54(2):263–88
- Reynolds WC, Parekh DE, Juvet PJD, Lee MJD. 2003. Bifurcating and blooming jets. *Annu. Rev. Fluid Mecb.* 35:295–315
- Reynolds WC, Perkins HC. 1977. Engineering Thermodynamics. New York: McGraw-Hill
- Reynolds WC, Potter MC. 1967. Finite-amplitude instability of parallel shear flows. J. Fluid Mech. 27(3):465– 92
- Richman RM, Reynolds WC. 1984. The development of a transparent cylinder engine for piston engine fluid mechanics research. SAE Tech. Pap. 840379, SAE Int., Warrendale, PA
- Sayadi T, Schmid PJ, Nichols JW, Moin P. 2014. Reduced-order representation of near-wall structures in the late transitional boundary layer. J. Fluid Mech. 748:278–301
- Schumann U. 1975. Subgrid scale model for finite difference simulations of turbulent flows in plane channels and annuli. 7. Comput. Phys. 18(4):376–404
- Uzkan T, Reynolds WC. 1967. A shear-free turbulent boundary layer. 7. Fluid Mech. 28(4):803-21