Vagueness and Imprecision: Empirical Foundations

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Abstract

Vagueness is a pervasive feature of natural language, which has been studied from a range of perspectives. This review focuses on recent empirical insights into vagueness that have come out of the field of linguistic semantics, as well as the theoretical developments that these have prompted. Topics covered include the distinction between vagueness and imprecision, or what I refer to as Type 1 and Type 2 vagueness; the complex manifestations of vagueness in the adjectival domain; and recent experimental findings regarding "ordinary" speakers' use and interpretation of vague language. Also briefly discussed is the broader question of why language is vague.

1. INTRODUCTION

The simple three-word sentence 1 exhibits a property that has fascinated and puzzled scholars for centuries: vagueness. Simply put, there seems to be no sharp boundary between the individuals who can be called tall and those who cannot; rather, *tall* is characterized by borderline cases, individuals for whom the predicate seems neither clearly to apply nor clearly not to apply.

(1) Anna is tall.

Vagueness, as has often been observed, is pervasive in natural language, occurring in expressions of nearly all grammatical categories. In fact, outside of statements of mathematical truths and the like, it is hard to find an expression (or perhaps a use of an expression) in which it is completely lacking. Vague language has been studied from a variety of perspectives, including philosophical, linguistic, psychological, and computational, and is also relevant to fields including law (Endicott 2001), medicine (Seising 2006), and geography (Varzi 2001). Within formal semantics and pragmatics, vagueness has long been a topic of investigation; classic earlier works include Sapir (1944), Lewis (1970), Lakoff (1973), McConnell-Ginet (1973), Kamp (1975, 1981), Klein (1980), Kamp & Partee (1995), and Pinkal (1995). The topic has witnessed an upsurge of interest in the past 15 or so years, prompted in part by (a) work on comparison and gradability, which has brought into focus questions concerning the vagueness of gradable expressions, and (b) research into the semantics of number and amount, which has likewise drawn attention to their approximate expression. This period has also been characterized by fruitful interaction between linguists and scholars in other disciplines engaged with vagueness, as evidenced by a series of interdisciplinary workshops and their associated proceedings (Cintula et al. 2011, Égré & Klinedinst 2011, Nouwen et al. 2011).

This article is not intended as an overview of theories of vagueness in general; there are already several excellent reviews of this sort (Keefe & Smith 1997, Keefe 2000, Barker 2006, Kennedy 2012, Sorensen 2013, Hyde 2014; for a highly enjoyable popular summary, see van Deemter 2010). Nor is my primary goal to compare and evaluate formal semantic analyses of vagueness, although I briefly discuss some promising new approaches. Rather, the focus is more empirical in nature. Recent work from the perspective of semantics and pragmatics has resulted in a much richer understanding of how vagueness is manifested in natural language. Central to the methods of formal linguistics—including semantics—is the systematic investigation of natural language data, and the use of such data to inform theories of the nature of our linguistic competence. Applied to vague language, the linguistic approach has of late yielded new perspectives on some very old problems. Although we cannot claim to have achieved a completely comprehensive theory of vagueness, what has emerged is a much fuller picture of what such a theory (or theories) must account for. It is this picture that I highlight here.

The first line of research I discuss involves work that in one way or another has contributed to localizing and profiling vagueness in language: Section 2 reviews how vagueness can be recognized, and how it relates to some seemingly similar and sometimes co-occurring phenomena; Section 3 examines the distinction between vagueness and imprecision (or what I refer to as Type 1 and Type 2 vagueness); and Section 4 takes a closer look at vagueness as it is found with gradable adjectives, the aspect of the topic that has received the most attention from semanticists. In Section 5, I turn to some recent and quite surprising findings on how "ordinary" speakers actually use and interpret vague language. It is well known from the psychology literature that in everyday situations, people do not reason like logicians, or at least not like classical logicians (Tversky & Kahneman 1983); here we observe that they do not interpret

language like classical logicians either. Finally, Section 6 concludes with a brief discussion of what might be the most interesting question of all: Why is language vague?

2. DELINEATING VAGUENESS

2.1. Place in Language

Vagueness has been equated with the lack of sharp boundaries and the existence of borderline cases, but the pattern most famously associated with vague expressions, and the one that has been the subject of the most study, is that they give rise to the so-called Sorites paradox, otherwise known as the Paradox of the Heap (from the Greek word *soros* 'heap'). In one of its common forms, the Sorites paradox is exemplified by the sequence in examples 2a-c: The two premises 2a and b strike us as unquestionably true, but the conclusion 2c, which follows from them, is just as clearly false.

- (2a) A man who is 6 feet 8 inches tall is tall.
- (2b) A man who is one-sixteenth of an inch shorter than a tall man is also tall.
- (2c) A man who is 4 feet 3 inches tall is tall.

The crucial property that lies at the heart of this paradox is tolerance (Wright 1975), meaning that the applicability of the predicate is insensitive to small changes in the relevant measure.

Using tolerance and susceptibility to Sorites reasoning as diagnostics allows us to systematically investigate where in language vagueness is found. The first and most obvious observation is that it is seemingly everywhere: adjectives (per above); nouns (if a pile of sand is a <u>heap</u>, it will still be a heap if one grain is removed; but two grains of sand do not make a heap); verbs (a minimally perceptible change to a <u>running</u> motion is still a running motion); quantifiers (if there is <u>much</u> water in a bucket, the evaporation of one molecule does not change that); prepositions (a chair that is <u>behind</u> the table remains so when moved a few millimeters); and even geographical descriptions (a point 1 mm away from a place in <u>London</u> is also in London).

The central place of vagueness in language is also demonstrated by the cross-linguistic prevalence of devices that serve to reduce or add vagueness, notably hedges such as *sort of, literally*, and *loosely/strictly speaking*, as well as precision regulators such as *about* and *exactly*. Vagueness might also underlie other linguistic phenomena, such as the distinction between count and mass nouns (Chierchia 2010).

2.2. Related Phenomena

Vagueness tends to co-occur with several seemingly similar properties that compound it and complicate its analysis, but can nevertheless be diagnosed as separate from it. First of all, vagueness must be distinguished from ambiguity. *Child*, for example, is ambiguous between a meaning of 'immature human' and a meaning of 'first-generation descendent of'; on the second reading, it is not (particularly) vague, whereas on the first it is (at what precise instant in the aging process does an individual cease to be a child?). Vagueness is also distinct from indeterminacy. The truth or falsity of *London is larger than New York* depends on whether *large* is interpreted in terms of land area (true reading) or population size (false reading). As with ambiguity, indeterminacy complicates vagueness, but even when it is resolved, vagueness in the sense of tolerance and Sorites susceptibility remains: A city that is large on the population size reading remains so if one resident moves away.

A phenomenon related to both indeterminacy and ambiguity is multidimensionality (see, e.g., Klein 1980 and especially Sassoon 2007, 2013). In contrast to the single dimension underlying the interpretation of *tall, clever* is based on multiple potentially independent dimensions, such as mathematical ability, ability to manipulate people, and so forth. Similarly, whether one counts as *healthy* is a function of a range of underlying health dimensionality adds a further layer of vagueness, in that the component dimensions and their relative importance are not precisely specified. But again, focusing in on a single dimension does not eliminate vagueness; what counts as *healthy with respect to blood pressure*, for example, itself allows for borderline cases.

Finally, vague expressions are typically subjective, or what has come to be known as judge dependent (Lasersohn 2005, Stephenson 2007, Bylinina 2014). Two speakers who knew Anna's height might nonetheless disagree as to the truth or falsity of example 1, the point of dissent being how tall one must be to be considered tall. Vagueness, however, persists even when we consider the judgments of a single speaker.

2.3. Thresholds and Prototypes

The vagueness of example 1 might be called threshold related, as it reduces to the question of the minimum height required for an individual to qualify as tall in the given context. A somewhat different variety of vagueness involves similarity to a category prototype (classic works on prototype theory include Rosch 1973, 1975; discussions of the connection between prototypes and linguistic vagueness can be found in Kamp & Partee 1995, Hampton 2007, Sassoon 2007). Whether a shade can be called red depends on its nearness in the color space to the focal shade of true red; whether an object is a chair is a function of its similarity to prototypical chairs. In both instances there are borderline cases, and Sorites series can be constructed.

The relationship between prototypicality and vagueness is complex (Kamp & Partee 1995). A category need not have a prototype to have fuzzy boundaries, as shown by example 1: We do not seem to have a prototype for tall things. Conversely, the existence of a prototype does not entail vagueness, because prototypes do not necessarily determine predicate extensions. It is easy to imagine a prototypical grandmother, for instance—perhaps characterized by knitting and gray hair—but what determines membership in the class of grandmothers is not resemblance to this image but simply gender and the property of having offspring who have offspring, criteria that leave little room for vagueness. When prototype resemblance does determine category membership, however, as in *red* and *chair*, vagueness seems to be an inescapable consequence.

The prevalence of prototype-related vagueness is connected to the observation that many of the concepts denoted by natural language predicates cannot be defined in terms of necessary and sufficient conditions, but are instead characterized by patterns of family resemblances (Wittgenstein 1953). Prototype theory captures this insight by modeling prototypicality in terms of distances from the category prototype in some multidimensional space. Douven et al. (2013), building on Gärdenfors (2004), develop an analysis of prototype-related vagueness based on this sort of distance measure.

3. VAGUENESS VERSUS IMPRECISION

3.1. Empirical Findings

A distinction that has become quite central in the literature is that between vagueness and imprecision (for the latter, see especially Lakoff 1973; Sadock 1977; Pinkal 1995; Krifka 2002, 2007; Sauerland & Stateva 2007). This difference is illustrated by the contrast between the sentences in examples 3a-e and 4a-c. All of the underlined expressions in examples 3a-e can be used imprecisely or approximately: Example 3a could in some contexts describe a rope whose length fell roughly between 45 and 55 meters; example 3b an arrival time a few minutes before or after 3:00; example 3c an attendance number close to 100; and so forth. But each of these has an underlying precise meaning: the point 50 m, the exact time 3:00, the mathematical center of the square, and so on. Examples 4a-c, by contrast, like the original example 1, have no corresponding precise concepts.

- (3a) The rope is <u>fifty meters</u> long.
- (3b) Sue arrived at three o'clock.
- (3c) There were <u>one hundred</u> people at the rally.
- (3d) I wrote the paper in <u>twenty-four</u> hours.
- (3e) The circle is in the <u>center</u> of the square.
- (4a) The rope is <u>long</u>.
- (4b) The shirt is expensive.
- (4c) The sand on the tray forms a <u>heap</u>.

Importantly, not all measure expressions are equal with respect to their potential for imprecise usage (see especially Krifka 2007). Whereas round numbers such as those in examples 3a-d allow or even favor an approximate interpretation, the corresponding nonround numbers in examples 5a-d must be interpreted (more) precisely.

- (5a) The rope is <u>fifty-one meters</u> long.
- (5b) Sue arrived at three-oh-one.
- (5c) There were <u>ninety-nine</u> people at the rally.
- (5d) I wrote the paper in twenty-three hours.

In decimal languages such as English, "round" for these purposes corresponds roughly to being a multiple of 10^n or 5×10^n (for a more nuanced view, see Jansen & Pollmann 2001); but a different characterization may be required in languages with other (e.g., vigesimal) number systems. In particular domains, it is values corresponding to higher-level units of measurement (e.g., 24 h in example 3*d*) that behave as round in this respect.

The use of round numbers to express approximations is common cross-linguistically (Dehaene & Mehler 1992). Furthermore, speakers often round even when they have more precise information available, for example reporting the time as *three-fifteen* when one's watch reads 3:13 (Van der Henst et al. 2002).

Two sorts of linguistic tests have been developed to differentiate vagueness from imprecision. First, imprecisely used expressions allow for contextual "precisification" that reduces or eliminates the fuzzy boundary region, whereas vague ones do not (Pinkal 1995, Kennedy 2007). We could create a context (e.g., building a space station) in which *ten meters long* distinguished between objects on the basis of arbitrarily small differences in length, such that an object a few centimeters longer or shorter than exactly 10 m could not be described as such. But it seems impossible to establish a corresponding precise lower bound for the bare *long* (such that it would mean something like 'longer than exactly 10 m'). Second, expressions that are imprecise rather

than vague can compose with approximators, words that regulate the level of precision at which they are interpreted (Sauerland & Stateva 2007): <u>exactly</u> ten meters; <u>about</u> three o'clock; <u>approximately</u> one hundred; <u>roughly</u> in the center. These are infelicitous in combination with vague expressions (*exactly/about tall/expensive/long/a heap).

Imprecision is superficially a simpler phenomenon than vagueness: The former merely involves looseness in use around some definitive, unique value, whereas the latter is characterized by the puzzling lack of any nonarbitrary threshold value whatsoever. At the same time, it is important to recognize that imprecisely used expressions exhibit some of the same interpretive patterns associated with vagueness. In particular, they give rise to the Sorites paradox (Sauerland & Stateva 2007, van Rooij 2011b). For example, if a time *t* distinct from 3:00 counts in the context as *three o'clock*, then so too must t + 1 ms; again, we derive nonsensical conclusions. As such, imprecision can be considered a type of vagueness. Sauerland & Stateva (2007) propose in place of vagueness and imprecision the terms epistemic vagueness and scalar vagueness, nomenclature that embodies a particular view on the nature of the vagueness (i.e., "classical" vagueness) and Type 2 vagueness (imprecision). In any case, a comprehensive theory of vagueness needs to accommodate the latter variety as well.

3.2. Theories of Imprecision

Some analyses of imprecision—or what I refer to as Type 2 vagueness—have applied the same mechanism to it and to Type 1 vagueness (notably Lakoff 1973). Other authors have used differences in linguistic behavior of the sort described above to support the need for separate mechanisms (e.g., Pinkal 1995). In the recent literature, two promising analyses focused in particular on imprecision have been developed.

The first, due to Lasersohn (1999), is based on the notion of pragmatic halos. In addition to its denotation, each expression of language is associated with a set of values—its halo—that differ from the denotation in only pragmatically ignorable ways. For example, *three o'clock* denotes the point of time 3:00; its halo is some set of times that are close enough to 3:00 for the purposes at hand. Expressions that regulate (im)precision operate on halos, reducing their size (e.g., *exactly*) or expanding the denotation to include the halo (e.g., *roughly* and *loosely speaking*). The mechanism of halos has also been applied to other phenomena, for example, metalinguistic comparison (Morzycki 2011).

The second approach, developed by Krifka (2007), analyzes imprecision as deriving from the granularity of measurement scales (Hobbs 1985). In this view, the results of measurement can be reported with respect to scales that differ in their level of granularity, conceptualized as density of scale points. Distances might, for instance, be measured relative to scales with basic units of 1 cm, 1 m, 5 m, and so forth. Relative to the 1-m granularity level, *ten meters* denotes a 1-m interval around the point 10 m; relative to the 1-cm level, it picks out a narrower 1-cm interval, yielding a more precise interpretation. Approximators can then be analyzed as determining granularity level (Sauerland & Stateva 2007).

A potential advantage of the granularity-based approach is that it accounts for the above observation that not all numerical expressions allow equally approximate interpretations. Available granularity levels are typically based on powers of 10 and the result of halving and doubling these, or on domain-specific measurement conventions. Rounder values occur on coarser scales and, thus, can be interpreted more approximately. This pattern is less easy to account for via pragmatic halos: "different in no more than pragmatically ignorable ways" would seem to be a symmetric relation, such that if *a* is in the halo of *b*, *b* should likewise be in

the halo of *a*. One might, however, attempt to overcome this objection by modifying the halo approach to incorporate findings from psychological research regarding asymmetry in judgments of similarity (Tversky 1977).

The two frameworks outlined here differ not only in the mechanisms they apply to imprecise interpretations, but more fundamentally in the basic view they take of the phenomenon. For Lasersohn (1999), imprecision is pragmatic: To say *The rope is ten meters long* in the case that it is in fact somewhat longer or shorter is to say something strictly false but "close enough" to true for the purposes at hand. In the granularity view of Krifka and others, imprecision is—or at least can be—semantic in nature; that is, a sentence such as this has an interpretation on which it is true in the case that the actual measure deviates slightly from the point denotation of the measure expression. It is not obvious that the choice between these two views can be made on empirical grounds (but see Lasersohn 1999 and Lauer 2012 for discussion). Rather, the question seems to be one of philosophy or preference: Would we rather have a system in which number words are ambiguous (per the scale granularity analysis) or one in which much of what we say with number words must be analyzed as literally false (a consequence of the halo-based approach)?

4. ADJECTIVAL VAGUENESS: A CLOSER LOOK

4.1. Empirical Findings

Much of the recent work on the semantics of vagueness (or, more specifically, Type 1 vagueness) has focused on adjectives such as *tall*. In this section I summarize some of the most important insights that have emerged.

4.1.1. Context sensitivity and gradability. Adjectival vagueness tends to co-occur with two other phenomena that are relevant to its proper analysis, namely context sensitivity and gradability (Kennedy 2007 and many others). *Tall* in sentences such as example 1 exhibits all three. By gradability, we mean the ability to form comparatives (*Anna is taller than Zoe*) and to compose with degree modifiers (*Anna is very/too/that/as/so tall*). As an illustration of context sensitivity, even if Anna's height did not change, judgments as to the truth or falsity of example 1 might vary depending on whether she were considered in the context of, say, gymnasts or women basketball players. Even further sensitivity to context emerges when we consider the minimum height required to call a 5-year-old child—or a building—tall.

It is common to analyze context sensitivity of this sort as deriving from the choice of a comparison class, a set of individuals that provides a frame of reference or standard of comparison (Bartsch & Vennemann 1973, Cresswell 1977, Klein 1980, von Stechow 1984, Ludlow 1989, Fults 2006, van Rooij 2011a, Bylinina 2014). This analysis is supported by the observation that a comparison class can seemingly be made overt via a *for*-phrase.

(6) Anna is tall for a gymnast/a basketball player/a 5-year-old child.

Recent experimental work has shown that speakers are highly sensitive to the statistical properties of comparison classes, changing their thresholds for the application of predicates such as *tall* systematically in response to variation in the experimentally provided context (Barner & Snedeker 2008, Schmidt et al. 2009, Tribushinina 2011, McNabb 2012, Solt & Gotzner 2012).

The contextual flexibility of adjectives of this sort is further illustrated by their interaction with definiteness. By way of example, a farmer looking at two quite corpulent pigs might use *the fat pig* to refer to the fatter of the two (Kyburg & Morreau 2000). Thus we are free to shift the threshold

for *fat* as needed to satisfy the uniqueness presupposition of the definite article. This relationship between vagueness and context also has the consequence that vague adjectives can be used to dynamically update the context: Example 1 might be used to report a fact about Anna's height; but if both speaker and hearer were aware of her height, it could also be used to convey something about the standard for *tall* in the context of utterance, namely that it is lower than Anna's height (the metalinguistic use discussed in Barker 2002).

That these three properties tend to cluster together has prompted semantic analyses that establish a causal link between them. A prime example is the delineation framework of Klein (1980), which derives gradability from variation in the extension of an adjectival predicate across contexts. I return briefly to these connections below; but first it is necessary to point out that vagueness, gradability, and context sensitivity do not always co-occur, indicating that neither of the latter two is a necessary or sufficient condition for vagueness. As simple examples, *left* and *right* are context sensitive without being particularly vague, whereas nouns such as *bush* and *tree* are vague without being particularly context sensitive (Kamp & Partee 1995). More significantly, there is a class of adjectives that are gradable without being either context sensitive or vague in the way observed with adjectives of the *tall* sort. I turn to these next.

4.1.2. The absolute/relative distinction. So-called absolute gradable adjectives such as *bumpy*, *flat*, *dirty*, *clean*, *full*, and *empty* pattern differently in a number of respects from relative gradable adjectives such as *tall*, *long*, and *expensive* (Kennedy & McNally 2005, Kennedy 2007). Like the latter class, they form comparatives (*flatter*, *bumpier*, *dirtier*, *emptier*, etc.) and compose with degree modifiers (*too flat*, *very bumpy*, *so empty*, etc.). But they do not exhibit the same context sensitivity as relative gradable adjectives. They are at best marginally felicitous with *for*-phrases (*?dirty for a table*; *?empty for a gas tank*), suggesting that their interpretation does not depend in the same way on a comparison class. Even more clearly, they interact differently with definiteness, an effect that has been substantiated experimentally (Syrett et al. 2010): When presented with two small objects, subjects identify *the big one* as the bigger of the two; but when presented with two partially full containers, *the full one* is not interpreted as referring to the fuller one.

Absolute gradable adjectives also behave differently with respect to diagnostics for vagueness; in particular, they are less susceptible to Sorites reasoning. For example, the paradigm 7 lacks the paradoxical force of that in example 2, in that a moment's reflection tells us that the inductive premise 7b is false: There is a point at which removal of a speck of dirt will result in a table that is completely clean.

- (7a) A table that is covered with dirt is dirty.
- (7b) A table with one speck of dirt less than a dirty table is dirty.
- (7c) A completely clean table is dirty.

Yet although adjectives such as *dirty* and *empty* are not vague or context sensitive in the same way as *tall*, it would be incorrect to conclude that they lack these properties entirely. What counts as clean in the case of a kitchen knife is different from what counts as clean in the context of surgical instruments. Furthermore, once we allow that *clean* can be applied to objects that are not completely free of dirt, the issues of tolerance and borderline cases rear their heads again: If a kitchen knife with a microscopic amount of dirt on it can be called clean, adding just one further imperceptible speck does not make it not so.

Burnett (2014) observes that the difference between absolute and relative gradable adjectives can be characterized as follows: For members of the absolute class, there are entities for which the adjective always or never applies. Imagine a knife that is entirely free of dirt (something that is possible in principle if not in practice). No matter the context, such an object must always count as clean, and never as not clean; conversely, it can never be called dirty, but always not dirty. Relative gradable adjectives behave differently; for example, anything that can be called *tall* might in a different context be called *short*, and vice versa.

Put somewhat differently, the dimensions encoded by absolute gradable adjectives are those that feature natural transitions on the basis of which two individuals can be distinguished, such as the transition between completely clean things and those with a nonzero amount of dirt on them. These provide the basis for absolute thresholds of application for an adjective, anchoring its interpretation and limiting contextual flexibility. Relative gradable adjectives lack any such absolute or nonarbitrary transitions.

The reference to nonarbitrary thresholds for absolute gradable adjectives allows a connection to be drawn to the vagueness observed with measure expressions such as those in examples 3*a*-*d*. Indeed, as pointed out by Kennedy (2007) and others, the above-discussed diagnostics allow the loose use of absolute gradable adjectives to be classified as Type 2 vagueness (imprecision). Precisification is possible: We can create a context (e.g., surgery) in which the standard for what counts as clean takes into consideration arbitrarily small amounts of dirt. Furthermore, just as in the case of measure expressions, there is a class of modifiers that regulate the level of precision at which absolute gradable adjectives are interpreted; these include low-degree modifiers such as *slightly* and *a bit* (e.g., *slightly wet/bumpy/dirty*) and maximum-degree modifiers such as *perfectly* and *absolutely* (e.g., *perfectly dry/flat/clean/full/ empty*). Both sorts are typically infelicitous with relative gradable adjectives (**slightly/ perfectly tall/long*).

4.1.3. Compositional regulation. Examples 8*a* and *b* demonstrate that the Type 1 vagueness observed with members of the relative class does not manifest itself across all forms of an adjective, indicating that it is not an inherent lexical property of the adjective itself. In contrast to the original example 1, those below are characterized by sharp rather than fuzzy boundaries, namely Zoe's height and the maximum allowable height for the rides.

- (8a) Anna is taller than Zoe.
- (8b) Anna is too tall to go on the kiddie rides.

To be sure, both of these sentences exhibit Type 2 vagueness. For instance, is a difference in height of half an inch sufficient to judge example 8a true? A quarter of an inch? An eighth of an inch? But they allow for precisification, just as do measure expressions such as those in examples 3a-d. These forms also allow the same low-degree modifiers found with absolute gradable adjectives (e.g., *slightly taller than; slightly too tall*). It is the unmodified, "positive" form of relative gradable adjectives that behaves differently.

A difference between the comparative and excessive in examples 8a and b and the positive in example 1 is that in the former case but not the latter, the standard of comparison is introduced explicitly via linguistic means. But interestingly, providing an explicit standard is not enough to eliminate the Type 1 vagueness of the positive form, as demonstrated by sentences such as example 9, which have come to be known as implicit comparatives (Kennedy 2011, van Rooij 2011a).

(9) Anna is tall compared to Zoe.

Like the explicit comparative in example 8*a*, example 9 entails that Anna's height exceeds Zoe's. But in contrast to the former, it cannot be used to express a crisp comparison: If Anna is only very slightly taller than Zoe, example 8*a* is potentially felicitous, whereas example 9 is not. Thus, it is apparently impossible to set the threshold for bare *tall* in such a precise way that a very small difference in height is sufficient for two individuals to fall on either side of it. That is, vagueness of this sort appears to be an inherent aspect of the interpretation of the positive form of members of the relative class.

4.1.4. Standards and vagueness. There is, however, a final twist in the story, namely that the positive form of relative gradable adjectives can take on at least two distinct contextually determined interpretations, which pattern differently with respect to diagnostics for vagueness (Kagan & Alexeyenko 2011; Bylinina 2012, 2014; Solt 2012). This difference is brought out in constructions involving low-degree modifiers. As noted above, these are compatible with certain absolute gradable adjectives and with the comparative and excessive forms of the relative class, but typically not with their positive forms. But there is an exception, namely when the adjective has an interpretation that is close to that of an excessive, as in the following.

- (10a) The actress is slightly tall to play the part.
- (10b) The jacket sleeves are a bit long.

The standard involved in such examples has been called a functional standard, in that it corresponds to the maximum degree that is suitable for a given function or purpose (for example 10*b*, this might be the purpose of wearing the jacket). This standard contrasts with the so-called distributional standard—that is, a standard set relative to a comparison class—that is involved in the usual reading of sentences such as example 1.

Compatibility with low-degree modifiers might be considered a diagnostic for the presence of a potentially precise standard, as is the case with those invoked by the absolute class and modified forms such as the comparative. Their felicity with relative gradable adjectives on their functional reading suggests that these standards too are potentially precise (although open to looseness in use); their unavailability on the distributional reading again points to the conclusion that distributional standards are somehow different, in that they resist any attempt to reduce them to a single nonarbitrary value.

4.1.5. Summary: context sensitivity after all? We are now in a position to characterize the distribution of Type 1 adjectival vagueness: It is a property of relative gradable adjectives, particularly of their positive forms, and these specifically on their distributional interpretation. A different sort of vagueness—which may be aligned to the imprecision or Type 2 vagueness observed with measure expressions—is found with absolute gradable adjectives, modified forms of the relative class, and functional readings of the positive.

Because Type 1 vagueness appears closely related to comparison classes, we might ask whether this pattern—or more specifically the difference between it and Type 2 vagueness—can be reduced to context sensitivity, particularly to the choice of comparison class. Some semantic analyses of gradable adjectives do entail that fixing the comparison class fully determines the corresponding standard (Bartsch & Vennemann 1973), but this type of approach is problematic (Kennedy 2007). By way of illustration, in examples with *for*-phrases such as example 6, the comparison class is specified, but vagueness remains in the form of borderline cases, and

the absence of a clear boundary between what does and does not count as tall in the specified context. A further issue is illustrated by example 11, which suggests that the standard provided by the comparison class cannot be the average over that class—but if not an average, what value would it be?

(11) Anna is taller than the average gymnast, but she isn't tall for a gymnast.

It has also been argued that the standard of application for such adjectives depends on other contextual factors beyond a comparison class, such as the interests of discourse participants (Fara 2000).

However, it may be possible to overcome these objections by, for instance, invoking the impossibility of uniquely determining the comparison class and knowing all of the relevant facts about it (Williamson 1994); by moving from extensional to intensional comparison classes (Bylinina 2014); or by taking the standard calculated from the comparison class to be not a single point but a range (von Stechow 2009, Solt 2011). Thus the relation between Type 1 vagueness and comparison classes remains to be better understood.

4.2. Theories of Adjectival Vagueness

In this section, we examine some recent theoretical proposals that have been put forward to account for the patterns discussed above.

4.2.1. Vagueness and scale structure. The most influential recent semantic analysis of adjectival vagueness is Kennedy (2007). Kennedy approaches the topic from the perspective of a degreebased semantic framework in which gradable adjectives relate individuals to degrees on scales, and the standard of comparison or threshold for the positive form is introduced by a phonologically null degree morpheme pos (Cresswell 1977, von Stechow 1984, Heim 2000, among others). On Kennedy's version of this approach, the standard is set in such a way that items for which the positive form is true "stand out" in the context of utterance relative to the dimension introduced by the adjective. Crucially, Kennedy proposes that the interpretation of the "stands out" relation interacts with the structure of the underlying measurement scale, a line of analysis that builds on Rotstein & Winter (2004) and Kennedy & McNally (2005). Scales differ as to whether or not they have maximum and/or minimum points. When such endpoints are present, what it means to stand out is determined in relation to them, resulting in an absolute interpretation: To be clean, for example, is to have the maximum degree of cleanness, whereas to be dirty is to have more than the minimum degree of dirtiness. The preference for endpoint-based standards, if they are available, is proposed to derive from a principle of Interpretive Economy, which requires that truth conditions be calculated whenever possible on the basis of conventional semantic elements. It is only when the scale lexicalized by the adjective is open at both ends that the standard is contextually determined, vielding a relative interpretation.

According to this analysis, therefore, the locus of the Type 1 vagueness characterizing relative gradable adjectives is the calculation of the minimum degree required to stand out in the context, something that may involve multiple and perhaps unknowable factors. Comparison classes are not semantically represented, although a property of this sort may contribute to standard setting by restricting the domain of the adjective.

Kennedy's (2007) analysis can be supplemented with ancillary mechanisms to account for other aspects of the data discussed above. Kennedy suggests that the Type 2 vagueness found with absolute gradable adjectives might be analyzed via either of the two above-described

approaches to imprecision (Section 3.2); subsequent work within this framework has in fact applied granularity-based analyses to these phenomena (Sauerland & Stateva 2007, Sassoon 2012, Sassoon & Zevakhina 2012). Functional readings might then be analyzed as involving coercion to a lower-closed scale (Bogal-Allbritten 2012). However, other patterns are less easy to account for. Particularly problematic is that adjectives with closed scales do sometimes seem to take on contextual rather than endpoint-based standards, suggesting that the principle of Interpretive Economy, in the form stated above, is too strong. The scale of fullness, for example, has minimum and maximum points, but a wineglass is conventionally considered full when it is filled to something like the halfway point. Objections of this sort have led to several alternative analyses.

4.2.2. Alternative analyses. McNally (2011) proposes that the absolute versus relative distinction corresponds to that between classification by rule and classification by similarity, concepts originating in the field of cognitive psychology (Hahn & Chater 1998). What characterizes absolute gradable adjectives is that the decision about whether they apply to a particular entity can be formulated in terms of a simple rule. Such a rule might be stated relative to a scalar endpoint, but also relative to some other conventional standard point, as in the wineglass example. In neither case is it necessary to consider any other individuals in the domain. For relative gradable adjectives, by contrast, no such simple rule can be formulated. Instead, ascription of the property encoded by the adjective depends on comparison to other individuals in the domain. For example, once tall and short individuals have been identified, a third individual can be judged tall if she is more similar in height to the tall one than to the short one. On this account we have an explanation for the intuition that relative gradable adjectives (but not absolute ones) are necessarily interpreted relative to a comparison class. The vagueness of such adjectives then derives from properties of similarity-based classification.

Somewhat relatedly, Toledo & Sassoon (2011) propose that all gradable adjectives are interpreted with reference to comparison classes, the difference between the two subtypes reducing to the <u>type</u> of comparison class invoked. Relative gradable adjectives select extensional comparison classes, sets of which the subject of predication is a member; absolute gradable adjectives select counterpart comparison classes, essentially alternate possible stages of the same individual. The difference between extensional and counterpart comparison classes is related in turn to that between individual-level and stage-level properties (Carlson 1977). In this way, the authors seek to ground the relative/absolute distinction in a more fundamental aspect of adjective meaning.

Lassiter & Goodman (2013) develop an alternate probabilistic account (see Edgington 1997 for a previous probabilistic analysis of vagueness). The starting point is a simple semantics in which the positive form contains a free variable over thresholds of application. Uncertainty in the value of this variable is represented as a probability distribution, which is derived via a process of coordination between speaker and listener, modeled using methods developed in Bayesian pragmatics (Frank & Goodman 2012). The type of interpretation that is inferred in this way depends on prior assumptions about the statistical properties of the reference class. A vague relative reading (roughly "significantly greater than average") arises when a normal-like distribution of the relevant property is assumed, whereas the assumption of a flat or end-skewed distribution yields a less vague absolute interpretation. The first of these proposed priors seems quite reasonable (consider, for instance, the distribution of heights of adult women); whether the second can be supported is a question that merits further study.

Finally, Burnett (2014) combines Klein's (1980) delineation framework with a tolerance-based logic (see Section 5.2) to yield a system in which the absolute/relative distinction is related to

different constraints on how an adjective's extension may vary across comparison classes. Which of these sorts of approaches will ultimately prove to offer the best account of adjectival vagueness remains to be determined.

5. VAGUENESS AND LOGIC

A wide variety of logical frameworks have been applied to vagueness: multivalued and fuzzy logics (Zadeh 1975, Tye 1994), supervaluations (Fine 1975), subvaluations (Hyde 1997), epistemicism (Williamson 1994), and contextualism (Raffman 1996, Fara 2000); see Keefe & Smith (1997) for an in-depth review. Over the years, linguists have contributed to several of these: Lakoff (1973) was an early proponent of fuzzy logic; Kamp (1975) is one of the foundational works in supervaluation theory; and important contributions to contextualism include Klein (1980), Kamp (1981), Bosch (1983), and Kennedy (2007). Recently, the trend has been toward empirical research, the results of which have, in turn, prompted developments in the logical analysis of vague language.

5.1. Experimental Findings

There is now a small but growing body of experimental research probing "ordinary" speakers' intuitions about the interpretation of vague expressions (see the sidebar, Experimenting with Vagueness). The overall picture that has emerged is that the principles of classical logic seem not to adequately characterize speakers' understanding of sentences containing vague terms.

In one of the first important examples of such work, namely Bonini et al. (1999), subjects were asked questions of the following form.

(12) When is it true [false] to say that a man is 'tall'? . . .It is true [false] to say that a man is tall if his height is greater [less] than or equal to _____ centimeters.

Across different vague predicates, a gap was consistently found between the two values elicited in this way: The average <u>minimum</u> value at which the predicate was judged true was greater than the average <u>maximum</u> value at which it was judged <u>false</u>. Thus, participants appeared not to be

EXPERIMENTING WITH VAGUENESS

Experimental approaches have come to play an increasing role in formal semantics, but the design and interpretation of such experiments are far from simple matters, because subjects' performance on experimental tasks is influenced by a range of factors beyond the underlying semantic representations of the linguistic expressions under investigation. In the case of vagueness in particular, potential confounds might include pragmatic principles governing the felicitous <u>use</u> of language; nonlinguistic biases in response orientation, such as a preference for errors of omission over errors of commission (Bonini et al. 1999); subjects' own "naïve" theories of logic (Ripley 2011); and perceived expectations of the experimental task. Even question phrasing may play a role: Eliciting judgments of truth or agreement on a multipoint scale (as in Ripley 2011, Sauerland 2011) suggests a graded notion of truth, whereas using 'true/false/can't tell' as response options (per Alxatib & Pelletier 2011) might instead imply an epistemic view. Further work, employing a diversity of methodologies, would be beneficial in understanding the interplay among semantics, pragmatics, and experimental factors underlying the judgments reported here. committed to the principle of bivalence, in that there are heights for which *The man is tall* would be judged neither true nor false.

Although there is some evidence that the finding of a gap is sensitive to the precise phrasing of the questions asked (Serchuk et al. 2011), similar "gappy" results have also been obtained via different experimental methodologies (Alxatib 2010, Solt & Gotzner 2010), suggesting that this is a robust pattern that requires explanation. Perhaps a more substantive objection is that these results are not completely surprising. Rather, they essentially mirror the original intuition that words such as *tall* are characterized by borderline cases where we hesitate to apply either the predicate or its negation, and are thus amenable to as many potential formal explanations as that observation itself. Bonini et al. (1999) propose an epistemic account, supporting this with an additional experiment that demonstrated a similar gap around a definite but unknown value, such as the average height of adult Italian men; but alternate explanations, such as one based on fuzzy logic, are also possible.

Other recent research has yielded results that do more to constrain possible logical theories of vagueness. A significant paper in this area is Alxatib & Pelletier (2011), which again derives theoretically significant insights from a simple experimental methodology. As stimuli, subjects saw a picture of a mock police line-up including five men whose heights ranged from 5 feet 4 inches to 6 feet 6 inches. For each "suspect," participants were asked to judge the four sentences 13a-d using the response options 'true,' 'false,' and 'can't tell':

- (13a) X is tall.
- (13b) X is not tall.
- (13c) X is tall and not tall.
- (13d) X is neither tall nor not tall.

The crucial judgments are those for the middle suspect, who at 5 feet 11 inches represented a borderline case of *tall*. Considering first examples 13*a* and *b*, the authors found a preference for false versus true responses: Subjects were more likely to judge it false that the borderline individual was tall than true that he was not tall and, similarly, more likely to judge it false that he was not tall than true that he was tall. These judgments constitute a divergence from classical logic, according to which a proposition is false if and only if its negation is true. They could, however, be accounted for under certain nonclassical logics, such as three-valued logic with strong negation.

The findings relating to the conjunctions in examples 13c and d were even more surprising, and problematic for both classical and nonclassical logical approaches to vagueness. Classically, conjunctions of this form are contradictions, and even authors who have argued for systems having intermediate or undetermined truth values in addition to the standard true/false (e.g., Kamp 1975) have typically assumed that propositions of this form are necessarily false. Alxatib & Pelletier (2011) found, however, that in the case of the borderline-tall suspect (but not the clear cases of tall and not tall), roughly half of subjects evaluated these as true. Furthermore, 'tall and not tall' was judged true by some subjects who judged both individual conjuncts false (and similarly, *mutatis mutandi*, for the negative conjunction). This is impossible even in nonclassical logics (e.g. Priest's 1979 logic of paradox) that do allow contradictions to have truth values other than false. The relative acceptability of these so-called borderline contradictions—that is, classical contradictions involving borderline cases of vague predicates—has been further substantiated in additional experiments by Ripley (2011), Sauerland (2011), and Serchuk et al. (2011), implying that again we are dealing with a real result.

A final strand of experimental research—although one whose theoretical implications are not yet clear—has involved eliciting judgments on dynamic versions of Sorites series, such as a series of color chips ranging from clear cases of blue to clear cases of green (Égré et al. 2013, Raffman 2014). The main finding is that the location of the boundary drawn between the extensions of two adjacent color words depends on the order of presentation.

5.2. New Logical Approaches

Across all of the studies discussed here, subjects' performance is systematic, suggesting they are in some way behaving "logically," but the logic involved seems to be a different one from those typically invoked in the analysis of vague language. Below, I briefly summarize some of the alternatives that have been put forward recently to capture these facts.

5.2.1. Super- and **subvaluations**. To account for their pattern of experimental results, Alxatib & Pelletier (2011) propose that vague predicates are ambiguous between a superinterpretation on which they are <u>neither</u> true nor false for borderline cases and a subinterpretation on which they are <u>both</u> true and false for such individuals. Pragmatic principles call for the selection of the strongest interpretation on which the resulting sentence could be true. For simple assertions such as 'X is tall,' this is the superinterpretation, resulting in the occurrence of gaps; for contradictions, the superinterpretation is trivially false, allowing the emergence of the subinterpretation, on which borderline contradictions are true. This composite approach thus achieves better empirical coverage than either a supervaluation theory or a subvaluation theory alone.

5.2.2. Tolerant semantics. Another class of approaches that has recently gained ground involves in one way or another building tolerance into the formal semantic apparatus. Whereas standard analyses of gradability and comparison assume that the ordering relation underlying the interpretation of gradable adjectives such as *tall* corresponds at least to a strict weak order (a relation that is irreflexive, asymmetric, transitive, and transitive with respect to incomparability), tolerance-based systems relax this requirement. This modification is argued to provide an account for both classical puzzles of vagueness such as borderline cases and Sorites susceptibility, as well as some of the newer observations discussed above, in particular the difference between explicit and implicit comparatives, and the surprising felicity of borderline contradictions.

The tolerance-based approach has been implemented in a variety of ways. van Rooij (2011a,b) makes use of semiorders, ordering relations in which the "greater than" relation > is transitive but the incomparability relation \sim is not, such that we might have three individuals *a*, *b*, and *c* for which $a \sim b \sim c$ but a > c. Semiorders were first introduced by Luce (1956) to account for intransitivity of indifference in preference; but we can imagine a similar relation underlying the interpretation of dimensional predicates such as *tall*, if the > relation is interpreted as "perceptibly," "relevantly," or "significantly" taller than.

Along similar lines, Cobreros et al. (2012) develop a logical framework featuring three related concepts of truth: classical truth and two additional notions derived from it on the basis of a potentially intransitive similarity relation, strict and tolerant truth. As in Alxatib & Pelletier's (2011) approach, an independently motivated pragmatic principle—the strongest meaning hypothesis—specifies that a sentence be interpreted in the strongest way in which it can be true. For classical contradictions this is the tolerant interpretation, which is tolerantly true for borderline individuals. As discussed earlier, Burnett (2014) extends the tolerant/classical/ strict framework to arrive at a theory that accounts for the differences between absolute and relative gradable adjectives.

Finally, Fults (2011) and Solt (forthcoming) develop a similar insight in a degree-based semantic framework by building tolerance into the structure of measurement scales. Both of their accounts draw on psychological models of humans' approximate numerical abilities (Dehaene 1997).

The sometimes unspoken assumption underlying all of these divergent implementations is that tolerance is a cognitive or perceptual primitive. Thus, an adequate formal account of vague language requires incorporating this directly into formal models.

5.2.3. Fuzzy logic revisited. A different approach applied recently to borderline contradictions is based on a variant of fuzzy logic, that is, a logic in which the set of truth values is extended from the traditional {0, 1} (false/true) to the entire real interval [0, 1]. Linguists have largely rejected fuzzy logic as a model for vague language since the classic demonstration by Kamp (1975) that in the case of a proposition ϕ with a truth value of 0.5, standard fuzzy logics derive a nonzero truth value for the conjunction $\phi \wedge \neg \phi$ and, furthermore, the same truth value for $\phi \wedge \neg \phi$ (which Kamp assumes must always be 0) and $\phi \wedge \phi$ (which should have the same truth value as ϕ itself). However, the above finding regarding the acceptability of borderline contradictions has prompted a new look at this type of approach. Alxatib et al. (2013) develop a fuzzy logic-based system in which conjunction and disjunction are defined intensionally, rescaling the truth value derived via the ordinary operators \wedge and \vee to span the full range of truth values [0, 1] by considering the truth value of a conjunction or disjunction in the context of the range of truth values it could have had. A borderline conjunction formed from a proposition with a truth value of 0.5 receives a rescaled truth value of 1. By contrast, the conjunction of two independent propositions, or of a proposition with itself, is not affected by rescaling. With this modification, fuzzy logic may in fact be suitable to model vague language, although at the cost of abandoning the semanticist's typical conception of truth and falsity as absolute notions.

6. CONCLUDING REMARKS: WHY SO VAGUE?

A central theme of this review has been that vagueness of various sorts is a ubiquitous phenomenon in language. This observation leads us to ask <u>why</u> language should work this way, rather than employing lexical items with sharply defined extensions. Surely a precise language would be a better tool for communication, an advantage seemingly confirmed in some game-theoretical modeling (Lipman 2009). A variety of possible answers have been suggested (for discussion, see Lipman 2009, as well as van Deemter 2009, van Rooij 2011b).

Vagueness may sometimes be necessary due to limitations in knowledge or memory: Even if I do not know Anna's height, I might be able to assert the sentence in example 1 if I know that she is taller than the relevant contextual norm. Alternately, vagueness in language might arise as a consequence of properties of our perceptual systems (Égré 2011, among others), or simply from the need to describe continuous reality via discrete linguistic categories.

Putting aside issues of incomplete knowledge or perceptual limitations, many vague words have the benefit of flexibility: We can apply *tall* to people, grass, trees, and buildings, all with the same core meaning, which might confer an advantage in learnability. Such words also let us express value judgments: To say that a shirt is expensive is to say something not merely about its price, but also about what I consider a reasonable price for shirts (cf. Barker's 2002 metalinguistic usage).

It has further been proposed that the use of vague or imprecise expressions can lower processing costs: *tall* is briefer and easier to produce than *taller than five foot ten*. Conversely, *three-fifteen* might be easier for a hearer to process than *three-thirteen*, with little reduction in cognitive benefit (Krifka 2002, Van der Henst et al. 2002).

From a practical perspective, vagueness reduces speaker commitment, a benefit when the interests of speaker and hearer do not coincide; as an example, imagine a politician promising a "large" reduction in taxes. It furthermore leaves room for discretion and adaptation to future contingencies, potentially advantageous in contexts like legal statutes (Poscher 2012). Vagueness might even improve communication in cases of mismatch between speakers' and hearers' interpretations, or their preferences (Parikh 1994, de Jaegher 2003).

A moment's reflection will show, however, that not all of these proposed benefits accrue to vagueness itself, in the sense of tolerance and borderline cases. Flexibility derives from context sensitivity, which is not fully coextensive with vagueness. Similarly, value judgments need not involve fuzzy boundaries. Likewise, apparent advantages relating to knowledge, memory, or processing load might in fact relate to the use of coarser-grained categories (e.g., tall/medium/ short instead of height in inches) or perhaps of verbal versus numerical form, rather than vagueness per se.

This discussion brings us full circle to the first part of this review, where we saw how closely vagueness is intertwined with other interpretive phenomena. We can thus restate the question as follows: Is vagueness—interpreted as having fuzzy boundaries—itself necessary or beneficial in language? Or is it rather that there is some other property or properties that benefit speakers and hearers in communication, and these bring along vagueness as a sort of side effect? These questions represent an important area for future research.

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