# Neutrino Mass Models

André de Gouvêa

Physics and Astronomy Department, Northwestern University, Evanston, Illinois 60208

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## Abstract

The discovery of nonzero neutrino masses is among the most important particle physics results of the last two decades: It indicates that the Standard Model of particle physics is incomplete. After 20 years of intense experimental and theoretical research, we still do not know the physics that leads to nonzero neutrino masses. The purpose of this review is to discuss the different models for nonzero neutrino masses. In doing so, I describe the differences between the models, how they connect to other aspects of particle physics, and whether or how one can hope to establish which model—if any—is a faithful description of nature.

## Contents



# 1. INTRODUCTION

The existence of neutrinos was first postulated by Pauli almost 90 years ago, and since that time neutrinos have continuously puzzled, surprised, and informed particle physicists and our understanding of the Universe and nature at the smallest distance scales (for a good overview of the history of neutrinos until the turn of the twenty-first century, see, for example, Reference 1). Over the years, we have established that neutrinos are fermions, have spin 1/2, have no internal structure—that is, are fundamental particles—and carry no electric charge. As far as we can tell, neutrinos participate only in gravitational and weak interactions, rendering them very difficult to detect. The study of neutrino properties requires significant ingenuity and patience.

Neutrinos have been demonstrated to be priceless probes when it comes to understanding the weak interactions and revealing the internal structure of nucleons and nuclei. Neutrinos are produced abundantly in nature, most notably in circumstances where the weak interactions play a decisive role. These include radioactive decay processes and the nuclear fusion processes that occur inside stars. The measurement of neutrinos produced in the core of the Sun helped establish that the Sun's energy is a consequence of nuclear fusion dominated by the so-called *pp* chain (2). Neutrinos are, by an enormous margin, the dominant energy-loss mechanism of core-collapse supernova explosions. The observation of a dozen or so neutrinos from supernova 1987A (3, 4) confirmed that our rough understanding of supernova explosions was correct and was followed by an avalanche of scientific activity in particle physics and astrophysics. The community anxiously awaits the arrival of neutrinos from the next nearby core-collapse supernova explosion. Finally, neutrinos are predicted to be relics of the Big Bang. The Standard Model of cosmology predicts that neutrinos are the most abundant matter particles in the Universe—they outnumber electrons and protons by approximately one billion to one—and have played a significant role in shaping the Universe observed today. Although this cosmic neutrino background has never been observed directly, there is strong indirect evidence that it indeed exists and that, on average, there are a few hundred neutrinos per cubic centimeter in the Universe.

Like all other fundamental fermions, neutrinos come in (at least) three different types, or flavors. A convenient way of describing neutrino production via charged-current weak interactions is to classify the different neutrino flavors according to the flavor of the charged lepton that is produced or destroyed along with the neutrino. Thus, electron-type neutrinos ($\nu_e$) are produced and destroyed along with electrons ($e$), as are muon-type neutrinos ($\nu_\mu$) and tau-type neutrinos ($\nu_\tau$)

along with muons ($\mu$) and taus ($\tau$), respectively. Since 1998, experiments with neutrinos produced in the Sun (5, 6), in the atmosphere (7), in nuclear reactors (8, 9), and in particle accelerators (10, 11) have demonstrated beyond a reasonable doubt that neutrinos can change flavor as they propagate. The rate of flavor change depends on the neutrino energy, the distance traveled, and the propagation environment. The only hypothesis capable of explaining all the neutrino data collected during the last few decades is that at least two of the neutrino masses are not zero and are different from one another, and that leptons mix. In this case, neutrinos change flavor as a function of distance and energy through the phenomenon of neutrino oscillation.

Specifically, the neutrino flavor eigenstates are linear combinations of the neutrino mass eigenstates $\nu_1$, $\nu_2$, and $\nu_3$ (assuming that there are in fact only three types of neutrinos) with masses $m_1, m_2$, and $m_3$, respectively, and the two bases are related by a unitary transformation

$$\nu_\alpha = U_{\alpha i}\nu_i, \qquad 1.$$

where $\alpha = e, \mu$ or $\tau$; $i = 1, 2$, or 3; and $U_{\alpha i}$ are the elements of a unitary $3 \times 3$ matrix.

Despite decades of experimental efforts, the only evidence we have that neutrino masses are not zero comes from neutrino oscillation experiments. These measure the neutrino mass-squared differences, which are $m_2^2 - m_1^2 \simeq 7.5 \times 10^{-5}$ eV$^2$ and $m_3^2 - m_1^2 \simeq \pm 2.5 \times 10^{-3}$ eV$^2$ (12). Precision measurements of the spectrum of tritium $\beta$ decay require $\sum_i |U_{ei}|^2 m_i^2 < 4.0$ eV$^2$ at 95% CL, whereas cosmic surveys constrain the sum of all neutrino masses to be at the sub-eV level. For example, $\sum_i m_i < 0.23$ eV at 95% CL (13). Although constraints from cosmic surveys are indirect, it is safe to say that all three neutrino masses are smaller than 1 eV.

The discovery of nonzero neutrino masses is among the most important particle physics results of the last two decades: It indicates that the Standard Model of particle physics (SM) is incomplete. Furthermore, after 20 years of intense experimental and theoretical research, we still do not know the physics that leads to nonzero neutrino masses. The purpose of this review is to discuss the different models for neutrino masses, as well as to discuss the differences between the models, how they connect to other aspects of particle physics, and whether or how one can hope to establish which model—if any—is a worthy description of nature. Before describing the different models, I first discuss in greater detail the meaning of the statement "the SM is incomplete." I also provide a brief description of the qualitatively different types of neutrino mass models.

The SM is a renormalizable, Lorentz-invariant quantum field theory characterized by a rather large gauge symmetry: $SU(3)_c \times SU(2)_L \times U(1)_Y$. It also includes several chiral fermion fields, $Q, u^c, d^c, L$, and $e^c$—three copies of each—and a fundamental scalar field, $H$, with nonzero $SU(2)_L \times U(1)_Y$ quantum numbers. I use the notation wherein all SM fermion fields are left-handed Weyl fermions. In this notation, one can refer to Dirac masses as the Lorentz contraction of different fields (e.g., the electron mass term, which is present after electroweak symmetry breaking, would be of the form $m_e e e^c$), while Majorana masses are the Lorentz contraction of the same field (e.g., neutrino Majorana mass terms would be of the form $m_\mu \nu\nu$). After electroweak symmetry breaking ($\langle H \rangle \neq 0$), the $W$ boson and the $Z$ boson become massive, along with all electrically charged fermions. The particle content of the SM, along with the Higgs sector and different imposed symmetries, does not allow for nonzero neutrino masses. In fact, one can turn the picture around and say that the minimal SM predicts all neutrino masses to be zero. Nonzero neutrino masses imply, therefore, new ingredients of the quantum field theory of particle physics. These might come in the form of new light particles, new heavy particles, new mass scales, new symmetries (or a reclassification of existing symmetries, as I hope to make clear below), and so on. I argue that, regardless of the mechanism nature has chosen to provide masses to the neutrinos, its implementation will lead to a new SM that is qualitatively different from the SM minimalistically described above.

There is more information in the data than the fact that there are neutrino masses. Experiments have revealed not only that some neutrino masses are nonzero but also that neutrino masses are tiny. **Figure 1** depicts all known fundamental fermion masses. The figure shows that whereas fermion masses are very different from one another—they span at least 13 orders of magnitude—neutrino masses are qualitatively different. The charged-fermion mass space, even though it spans more than five orders of magnitude, is homogeneously populated; in other words, there are charged fermions with masses of order 1, 10, $10^2$, $10^3$, $10^4$, and $10^5$ MeV. By contrast, neutrino masses are not only tiny—they are below $10^{-6}$ MeV—but also "lonely." There are no known fundamental fermions with masses between 1 eV and 511 keV. This observation provides more guidance for the construction of neutrino mass models. Not only must they yield nonzero neutrino masses; they should also explain why neutrino masses are so small and different.

Neutrino mass models fall into three broad categories, discussed in detail in the following sections. These are described as follows.

1. Add to the SM new chiral fermion fields $n^c$. These allow for neutrino–Higgs Yukawa interactions that, after electroweak symmetry breaking, translate into Dirac masses for the neutrino states. This possibility is discussed in detail in Sections 2 and 3.1.
2. Allow for other sources of electroweak symmetry breaking. It is possible to add to the SM a new Higgs boson field with a nonzero vacuum expectation value that only contributes, as far as fermion masses are concerned, to nonzero neutrino masses. This possibility is discussed in detail in Section 3.3.
3. Add to the SM a new source of mass, independent from the electroweak symmetry–breaking scale. In the SM, all masses are proportional to the electroweak symmetry–breaking scale. In the limit that this scale vanishes, all SM particle masses vanish. Neutrino masses are no exception; they are also proportional to the electroweak symmetry–breaking scale. However, here neutrino masses have a dual nature; they are a consequence of two independent mass scales. In these types of scenarios, nonzero neutrino masses are evidence of a new mass scale of nature, and it may be that neutrinos are the only avenue through which this new physics can be explored. This topic is the main theme of Sections 3 and 4.

Neutrino oscillation data have also measured the parameters of the leptonic mixing matrix. It is possible that these contain information that is vital for uncovering the origin of nonzero neutrino masses. I discuss this issue in more detail in Section 5. Finally, Section 6 offers some concluding remarks.

## 2. MASSIVE DIRAC NEUTRINOS

As argued above, new degrees of freedom are required in order to render neutrinos massive. A very simple possibility is to postulate the existence of new left-handed Weyl fermions $n^c$ with no SM quantum numbers. SM gauge singlet fermions are a very benign addition to the SM Lagrangian. They do not affect SM gauge anomaly cancellations and are only modestly constrained by experiments. SM gauge singlet fermions couple to the SM only via neutrino–Higgs boson Yukawa interactions:

$$\mathcal{L}_{\nu\mathrm{Yukawa}} = y^{\alpha i} L_\alpha n^c_i H + \mathrm{H.c.}, \tag{2}$$

where $y^{\alpha i}$ are the Yukawa couplings; $\alpha = e, \mu$ or $\tau$; $i = 1, 2, \ldots, N_n$; and $Nn$ is the number of gauge singlet fermion fields. If this is the only new interaction, after electroweak symmetry breaking, the left-handed neutrino fields $\nu_\alpha$ in $L_\alpha$ and the $n^c$ fields combine into massive Dirac fermions, with mass matrix $(m_D)_{\alpha i} = y_{\alpha i} v$, where $v$ is the vacuum expectation value of the neutral

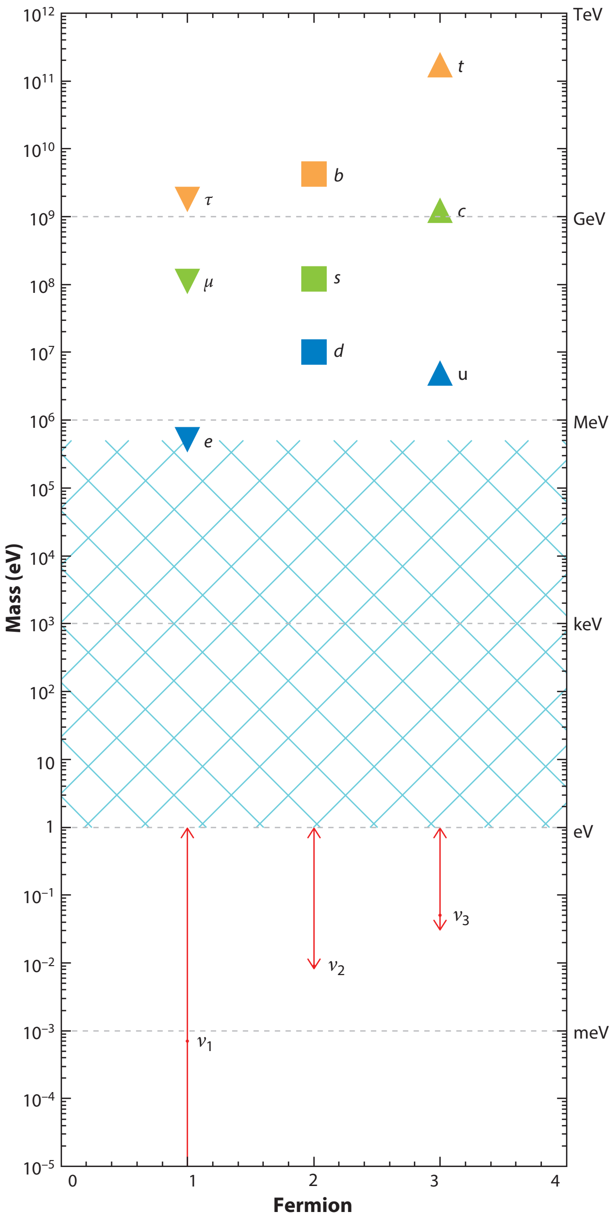


**Figure 1**

The Standard Model fermion masses. There are no known fermions in the blue-hatched mass region. The neutrino masses are not known; only the mass-squared differences are. The arrows indicate the allowed ranges for the neutrino masses, assuming a so-called normal mass ordering: $m_3^2 > m_2^2 > m_1^2$. The qualitative 1-eV upper bound is representative of information one can extract from precision measurements of the spectrum of tritium $\beta$ decay (12).

component of $H$. For this reason, the $n^c$ fields are also referred to as left-handed antineutrinos (or right-handed neutrinos).

Data require at least two $n^c$ fields so that at least two neutrinos are massive. They also require the new dimensionless Yukawa couplings to be tiny: $m_D \sim 0.1$ eV translates into $y \sim 10^{-12}$. This requirement raises a new question: Why are the neutrino Yukawa couplings so small? Although we do not know the answer, the community often interprets the tiny Yukawa couplings as a hint that there is more new physics behind Equation 2. As argued above, the main concern is not that the neutrino Yukawa couplings are tiny but rather that they are tiny compared with all other charged-fermion Yukawa couplings. The electron Yukawa coupling is $\mathcal{O}(10^{-5})$ and, thus, also tiny. Nonetheless, it is still one million times larger than the largest possible neutrino Yukawa coupling. In the remainder of this section, I review some models that aim to explain why neutrino Yukawa couplings are so small.

Before proceeding, I note that the existence of $n^c$ fields raises another question: Why aren't the $n^c$ fields allowed Majorana masses? If right-handed neutrinos existed, they would be unique among all known matter fields. The quantum numbers of $Q$, $u^c$, $d^c$, $L$, and $e^c$ are such that, before electroweak symmetry breaking, none of these objects are allowed to have masses. This is not the case for right-handed neutrinos. As far as the SM is concerned, there is no symmetry that forbids the presence of the following term:

$$\mathcal{L}_{\nu^c\,\mathrm{mass}} = -\frac{M^{ij}}{2} n_i^c n_j^c + \mathrm{H.c.}, \qquad 3.$$

where $M^{ij} = M^{ji}$ are the elements of the $N_n \times N_n$ right-handed neutrino Majorana mass matrix. There is nothing wrong with exploring the physics of nonzero $M$. I review this possibility in detail in Section 3.1. Nonetheless, if $M \neq 0$, then the observed neutrino masses are not $m_D$ and the neutrinos are not Dirac fermions. Dirac fermions with masses derived directly from Equation 2 require $M = 0$, which, in turn, requires an explanation in the form of a new symmetry. If we postulate that lepton number ($L$) conservation is a law of nature, then Equations 2 and 3 are not simultaneously allowed. Furthermore, if one chooses $L(L) = +1$, $L(n^c) = -1$, then Equation 2 is allowed whereas Equation 3 is forbidden; that is, $M \equiv 0$ because of lepton number conservation.

Massive Dirac neutrinos, therefore, imply that the symmetry structure of nature is larger than that of the SM. Not only is the Lagrangian invariant under the SM gauge symmetry; it is also invariant under lepton number transformations. Note that, for massless neutrinos, lepton number is indeed a conserved quantity,[1] but it is not regarded as fundamental symmetry. Along with baryon number, lepton number is an accidental symmetry of the SM, and there is no fundamental reason to expect that, at shorter distances, physics conserves lepton number. If the neutrinos are massive Dirac fermions, the situation is qualitatively different: Massive Dirac neutrinos imply that lepton number (or some subgroup of lepton number; see, for example, Reference 14) is a fundamental symmetry of nature. In summary, if the neutrinos are massive Dirac fermions, the status of lepton number needs to be upgraded from an accidental symmetry to a fundamental symmetry.

## 2.1. New Symmetries

I argue above that in order to fit the neutrino oscillation data with Equation 2 Yukawa couplings of order $10^{-12}$ are required. One possibility that explains such small couplings is to postulate that,

[1] Technically, lepton number is not conserved in the SM; it is violated by nonperturbative effects. The same is true of baryon number ($B$). The combination $B - L$ (baryon number minus lepton number) is an exact accidental global symmetry of the SM. Every time I refer to lepton number, I am in fact referring to the nonanomalous $B - L$ symmetry.

in fact, Equation 2 is forbidden by a new symmetry. If the $L$, $n^c$, and $H$ fields are characterized by more quantum numbers, it is straightforward to choose them such that the neutrino Yukawa couplings are forbidden. If, however, the new symmetry is spontaneously broken, Equation 2 might appear after symmetry breaking. A thorough discussion of this topic can be found in, for example, Reference 15.

Imagine a new gauge interaction $U(1)_\nu$ under which only the $n^c$ fields are charged (with charge $-1$). Imagine there is a new scalar field $\phi$ with $U(1)_\nu$ charge $+1$ and that $\langle\phi\rangle = v_\nu \neq 0$. In this case, Equation 2 is forbidden by the $U(1)_\nu$ symmetry. By contrast, the dimension-five operator

$$\mathcal{L}_{5\mathrm{Dirac}} = \frac{y^{\alpha i}}{\Lambda}\left(L_\alpha H\right)\left(n_i^c\phi\right) + \mathrm{H.c.}, \qquad 4.$$

where $\Lambda$ is some new effective energy scale, is allowed. After $U(1)_\nu$ and electroweak symmetry breaking, the neutrino Dirac masses are

$$m_D = y\frac{v v_\nu}{\Lambda}. \qquad 5.$$

Equation 5 states that neutrino masses occur only after two instances of spontaneous symmetry breaking. It also states that they are proportional to $v$ multiplied by $v_\nu/\Lambda$. If $\Lambda \gg v_\nu$, Equation 5 explains why neutrino masses are different from all charged-fermion masses. They are parametrically smaller than all other fermion masses by a factor $v_\nu/\Lambda$. A concrete, complete scenario was recently proposed (16), and this idea has also been explored in several different contexts (see, for example, Reference 17 and citations therein).

## 2.2. New Dimensions of Space

Very small neutrino masses might also be a consequence of the existence of new dimensions of space, of the flat (18) or warped (19) variety. Extra dimensions can manifest themselves as very small Yukawa couplings in a couple of different ways, which I only summarize, qualitatively, here.

One possibility is that the different SM fermion fields—say, $L$ and $n^c$—are localized at different positions in the extra dimension (20–22). In this case, the effective four-dimensional Yukawa couplings are proportional to the probability that the two higher-dimensional fermion fields overlap with one another. If the two fields lived in entirely separate regions of space, there would be no interaction between them. This possibility often leads to effective Yukawa couplings that are proportional to the exponential of the spatial separation of the two fields, $y \propto \exp[-|\ell_L - \ell_{n^c}|/\sigma]$, where $\ell$ are the average positions of the two different fermion fields and $\sigma$ measures the widths of the fermion wave functions in the extradimensional space. The exponential dependency on the parameters of the theory allows for very small (and very different) Yukawa couplings even if one restricts all parameters of the higher-dimensional theory to be of the same order of magnitude. Experimental consequences of this hypothesis have been discussed in the literature (see, for example, References 20–22 and citations therein).

Another possibility is that right-handed neutrinos are special. In the scenario first discussed in References 23 and 24, where the SM fermions are constrained to live in a four-dimensional subspace while right-handed neutrinos can access a large, extradimensional space, the four-dimensional neutrino Yukawa couplings are suppressed by the ratio (raised to some power) of the observed Planck scale $M_P \sim 10^{18}$ GeV and the fundamental string scale $M_F$, which could be of order 1 TeV (for a detailed discussion, see, for example, Reference 25). In other words, neutrino masses are small for the same reason gravity is feebly coupled: Both right-handed neutrinos and the graviton live in more space dimensions than the SM fields. An interesting side effect of these models is the existence of a very large number of new, very weakly coupled neutrino states—sterile neutrinos. The phenomenology of such scenarios has been studied in detail (see References 23, 25, and 26 and citations therein).

There are many other ways to explain why neutrino Yukawa interactions are very small and qualitatively different from those of the charged fermions. They, might, for example, arise only at the loop level, and they might be related to other ultraviolet physics. Concrete examples can be found in attempts to relate small neutrino Yukawa couplings to the physics of supersymmetry breaking (27, 28).

## 3. MASSIVE MAJORANA NEUTRINOS: THE STANDARD PARADIGM

Most of the literature on neutrino mass models is not related to the hypothesis that neutrinos are Dirac fermions or that there is a mechanism behind $\mathcal{O}(10^{-12})$ Yukawa couplings. Instead, most neutrino mass models rely on the hypothesis that neutrino masses are a high-energy phenomenon, and that the small neutrino masses are a consequence of the existence of heavy degrees of freedom whose masses are, in general, unrelated to electroweak symmetry breaking.

A powerful way to parameterize the effects of new, ultraviolet degrees of freedom is to add to the renormalizable SM Lagrangian nonrenormalizable operators. It turns out that there is only one dimension-five nonrenormalizable operator that one can construct out of SM fields that is consistent with gauge invariance (29):

$$\mathcal{L}_5 = -\frac{\lambda^{\alpha\beta}}{2\Lambda}\left(L_\alpha H\right)\left(L_\beta H\right) + \text{H.c.}, \qquad 6.$$

where $\Lambda$ is the effective new physics scale and $\lambda^{\alpha\beta} = \lambda^{\beta\alpha}$ are dimensionless coefficients. Note that $\lambda$ and $\Lambda$ are not independently defined. In order to remove this ambiguity, unless otherwise noted, I define $\Lambda$ such that the largest $\lambda^{\alpha\beta} \equiv 1$. Equation 6 is often referred to as the Weinberg operator. Note that it explicitly violates the lepton number symmetry. If $\Lambda$ is very large, it is reasonable to argue that Equation 6 captures the dominant low-energy effects of the high-energy physics. All other nonrenormalizable operators are of dimension six or higher and are, thus, most likely suppressed by higher powers of the same $\Lambda$.[2]

At low energies and for large enough $\Lambda$, Equation 6 leads to only one potentially observable effect. After electroweak symmetry breaking, Equation 6 translates into

$$\mathcal{L}_5 \to -\frac{m_{\alpha\beta}}{2}\nu^\alpha\nu^\beta + \text{H.c.}, \qquad 7.$$

where $m = \lambda v^2/\Lambda$. Therefore, Equation 6 leads to nonzero Majorana masses for the SM neutrino fields. This means that the presence of new physics associated to a new effective scale $\Lambda$ implies, somewhat generically, that neutrinos have nonzero Majorana masses. Furthermore, the neutrino Majorana masses are parametrically smaller, by a factor $v/\Lambda$, than the weak scale $v$ as long as $\Lambda \gg v$.

This remarkable yet simple idea can be made consistent with observations if $\Lambda = \mathcal{O}(10^{14}\text{ GeV})$. If Equation 6 is correct, one can make a robust prediction: Neutrinos are Majorana fermions, and lepton number is not an exact symmetry. Determining the nature of the neutrinos—Dirac or Majorana fermions—is a very high priority in fundamental particle physics. The most powerful probes of the nature of the neutrino are tests of lepton number conservation, especially searches for neutrinoless double-$\beta$ decay (for a review, see, for example, Reference 30). Other consequences

[2] I revisit this condition in Section 4 but assume for the remainder of this section that this is indeed the case.

of Equation 6 depend on its ultraviolet completion. Some of these consequences are discussed in the remainder of this section and in Section 4.

### 3.1. Type I (and III) Seesaw

As discussed in Section 2, the addition of right-handed neutrinos to the SM Lagrangian is accompanied, generically, by neutrino Yukawa couplings (Equation 2) and right-handed neutrino Majorana masses (Equation 3). In Section 2, I argue that if lepton number were an exact symmetry, either one or the other would be forbidden. Here, instead, I assume that lepton number is explicitly broken—in other words, that both $y$ and $M$ are nonzero—and discuss the consequences of this choice. By combining the two right-handed neutrino potentials, we examine the Lagrangian

$$\mathcal{L}_{\text{Type I}} = y^{\alpha i} L_\alpha n_i^c H - \frac{M^{ij}}{2} n_i^c n_j^c + \text{H.c.} \qquad 8.$$

After electroweak symmetry breaking, Equation 8 describes $3 + N_n$ potentially massive Majorana fermions, all of which are linear combinations of the $\nu_\alpha$ and $n_i^c$ fields. In this basis, the neutrino Majorana mass matrix [a symmetric, $(3 + N_n) \times (3 + N_n)$ matrix] is

$$m_\nu = \begin{pmatrix} 0 & m_D \\ m_D^t & M \end{pmatrix}, \qquad 9.$$

where $m_D$, the Dirac neutrino mass matrix, was introduced in the last section and the $t$ superscript indicates the transpose of the matrix $m_D$. For both $M$ and $m_D \neq 0$, the physics of Equation 9 falls into three qualitatively distinct categories. For the sake of simplicity, I assume, whenever relevant, that $N_n = 3$ henceforth.

The three categories are:

1. $M \ll m_D$. In this limit, the neutrino mass eigenvalues are pairwise quasi-degenerate with masses of order $m_D$ (and mass difference of order $M$), and the accompanying neutrino mass states are 50–50 linear combinations of $\nu_\alpha$ and $n^c$ states. Neutrino oscillation data require $m_D \sim 0.1$ eV, as in the case of Dirac neutrinos. In this limit, there is no simple, natural explanation for why neutrinos are much lighter than charged fermions. In this regime, the neutrinos are referred to as pseudo-Dirac fermions (31, 32). The reason is as follows: For small enough $M$, the two quasi-degenerate neutrino states behave, as far as observations are concerned, as one Dirac fermion. For sufficiently large values of $M$, one can observe the small mass splittings via neutrino oscillations. These are, in some sense, oscillations between the $\nu_\alpha$ states, which participate in weak interactions and are also referred to as active neutrinos, and the $n^c$ states, which do not participate in weak interactions and thus are also referred to as sterile neutrinos; and the relevant mixing is maximal. The study of solar neutrinos rules out these long-wavelength, large–mixing angle oscillations and constrains right-handed neutrino masses to be less than around $10^{-9}$ eV (33, 34). Smaller $M$ values may affect the flux of neutrinos from sources further away than the Sun (35).
2. $M \simeq m_D$. Here, all Majorana neutrino masses are roughly of order $m_D$, $M$, and all mass eigenstates are linear combinations of $\nu_\alpha$ and $n^c$ states with large coefficients. Data require $m_D \sim 0.1$ eV. In this case, there are six neutrino mass eigenstates, all of which participate in experimentally accessible neutrino oscillations. These six states are strong mixtures of active and sterile neutrinos. This scenario is ruled out. There is no evidence for more oscillation frequencies of the same order as the known atmospheric and solar oscillation frequencies. Roughly speaking, this means that right-handed neutrino masses smaller than ~0.1 eV are excluded (34, 36). This result, combined with constraints obtained in the pseudo-Dirac

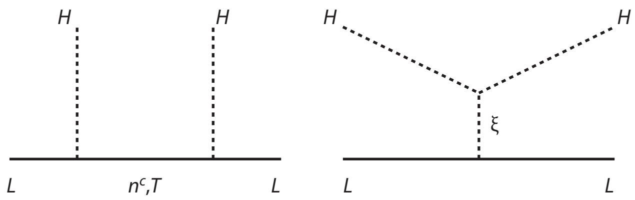


**Figure 2**

The three distinct scenarios that yield the Weinberg operator (Equation 6) once new heavy states are integrated out at the tree level.

regime, reveals that $M$ values between $10^{-9}$ eV and $10^{-1}$ eV are experimentally ruled out if neutrino masses are a consequence of Equation 8.

3. $M \gg m_D$. The neutrino masses split into two subsets: heavy neutrinos with masses $M$, which are made up mostly of the $n^c$ sterile states, and light neutrinos with masses of order $(m_D)^2/M$, which are made up mostly of the $\nu_\alpha$ active states. The mostly active neutrinos—the ones that are observed experimentally—are parametrically lighter than the weak scale by a factor $m_D/M$. This parametric suppression of the mostly active neutrino masses is referred to as the seesaw mechanism (37–42), and in this regime, Equation 8 is often referred to as the type I seesaw Lagrangian.

The seesaw Lagrangian is an ultraviolet-complete model for the Weinberg operator. In the limit $M \gg m_D$, as far as low-energy phenomena are concerned, one can integrate out (**Figure 2*a***) the right-handed neutrino fields and obtain Equation 6, where

$$\frac{\lambda}{\Lambda}v^2 = m_D \frac{1}{M} m_D^t. \qquad 10.$$

The consequences of the type I seesaw depend dramatically on the values of the Lagrangian parameters, $y$ and $M$. As discussed in cases 1 and 2, above, values of $M$ larger than 0.1 eV are not ruled out by observations, and one can always choose $y$ values such that the neutrino oscillation data can be explained. Very small values of $M$ either require tiny values of $y$ or require cancellations among the elements of $y$ and $M$ such that the values of $m_D M^{-1} m_D^t$ are much smaller than naïve expectations. Either way, low-energy seesaws do not provide a straightforward explanation of the small neutrino masses, as these are a consequence of either very small couplings or cancellations that require more explanation. Very large values of $M$, much larger than the weak scale, point to neutrino Yukawa couplings that are of the same order as those of the other charged fermions. In fact, the perturbativity of Equation 8, combined with our current understanding of neutrino masses, places an upper bound on $M$, which is required to be less than $\sim 10^{16}$ GeV (43).

Whereas low-energy seesaws appear to be a more convoluted explanation for why neutrino masses are small, they are experimentally testable and may allow one to address other puzzles in particle physics. $M$ values around 1 eV (36, 44) may help solve the short-baseline anomalies (45–50), whereas $M$ values around 100 keV (51) may help address the dark matter puzzle. Consequences for cosmology and other laboratory experiments were also recently explored in the literature (see, for example, References 52 and 53, respectively). The key point is that, for small $M$ values, the mostly sterile neutrinos can be produced and detected because there is nonzero (albeit small) active–sterile mixing. The magnitude of the active–sterile mixing angle is approximately $\sin^2\theta_{as} \sim m_\nu/M$, where $m_\nu$ are the masses of the mostly active neutrinos. **Figure 3** illustrates what is currently known about right-handed neutrino masses, along with different estimates for the strength of active–sterile mixing.

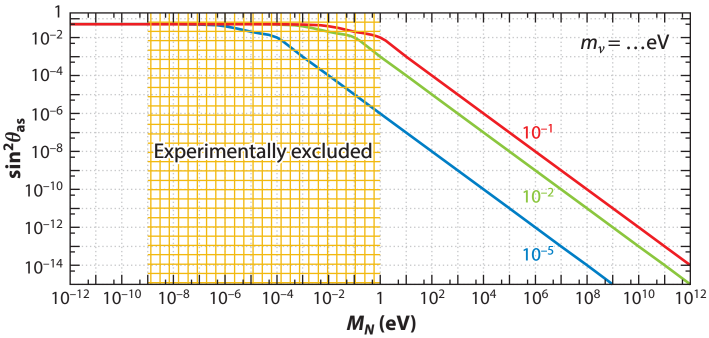


**Figure 3**

Current knowledge of right-handed neutrino masses $M_N$, along with estimates for the strength of active–sterile mixing $\sin^2\theta_{as}$, for different values of the masses of the mostly active neutrino states. $M$ values larger than $\sim 10^{25}$ eV (not shown) are also excluded. Modified from Reference 33.

High-energy seesaws, by contrast, may be impossible to test experimentally. They do, however, provide a very appealing mechanism for the dynamical generation of a baryon–antibaryon asymmetry in the early universe: leptogenesis (54; for a pedagogical overview, see Reference 55).

It is easy to see that in Equation 8 one can replace the singlet fermions $n^c$ with $SU(2)_L$ triplet fields $T$ with zero hypercharge, as first pointed out in Reference 56. This is referred to as the type III seesaw. **Figure 2*a*** shows that the Weinberg operator is generated once one integrates out the neutral component of the fermion triplet. The neutrino phenomenology of these scenarios is identical to that of the type I seesaw, but the existence of new charged fields—lepton-like states with electric charge $\pm 1$—rules out the possibility that the triplet Majorana masses $M$ are smaller than hundreds of GeV.

## 3.2. Right-Handed Neutrinos: Origins

Phenomenologically, the introduction of gauge singlet chiral fermions is mostly benign. As discussed above, these only couple to the SM via the neutrino Yukawa interactions and are mostly constrained by weak probes involving neutrinos. Their existence, however, raises several questions. Are gauge singlet fermions a side effect of a more fundamental understanding of nature at small distance scales? How many of these gauge singlet fermions are there? Is their number $N_n$ related to the number of SM fermion generations, or is it a consequence of unknown ultraviolet physics? Finally, if the right-handed neutrino Majorana masses $M$ are not zero, what are they? Is $M$ related to other scales in nature, including the weak scale, the Grand Unification scale, the Planck scale, or the string scale? In this section, I only comment on some the challenges and ideas related to the top-down view of nonzero neutrino masses (for a recent detailed review, see, for example, Reference 57).

A strong argument for the existence of right-handed neutrinos comes in the form of Grand Unified Theories (GUTs). GUT gauge groups larger than $SU(5)$, including $SO(10)$, predict the existence of chiral fermions that are singlets as far as the SM gauge group is concerned. These are all good right-handed neutrino candidates. If $SO(10)$ GUTs are realized in nature, then right-handed neutrinos exist inside the matter **16** representation, and there are as many right-handed neutrino generations as there are SM fermion generations (i.e., $N_n = 3$). Furthermore, in many models, right-handed neutrino Majorana masses can be directly related to the GUT spontaneous symmetry-breaking scale $M_{\mathrm{GUT}}$, and one naïvely expects $M \simeq M_{\mathrm{GUT}}$. The fact that the effective scale of the Weinberg operator is around $10^{14}$ GeV while the GUT scale (in the case of simple supersymmetric extensions of the SM) is around $10^{16}$ GeV has attracted a significant amount of attention and is often considered strong circumstantial evidence that nonzero neutrino masses and GUTs are closely related. GUTs also provide a powerful tool for relating quark and lepton mixing, and for constructing economical models of flavor. I comment on flavor models briefly in Section 5.

The leading theoretical construction that is potentially capable of realistically combining gravity with a quantum field theory description for the other fundamental interactions is string theory. String theories provide some of the ingredients widely used in understanding nonzero neutrino masses, including GUTs; supersymmetry; and new, compactified dimensions of space. The relation between string theory and nonzero neutrino masses, however, might be more direct. Whereas the string scale is usually expected to be close to the observed Planck scale $M_{\mathrm{P}} \sim 10^{18}$ GeV (modulo large or warped extradimensonal-type models), concrete realizations of string theory are well supplied with different objects—string moduli and other relics—which are both very light and very weakly coupled to the rest of the theory. From a quantum field theory point of view, some of these are chiral, gauge singlet fermions. Thus, it is natural to ask whether string theory–inspired scenarios have anything to say about the number, masses, and couplings of these states, and whether the seesaw Lagrangian, for example, is a natural consequence of string constructions, occurs only in specific string models, or is in some sense incompatible with string theory. General explorations of subsets of the string theory landscape have been attempted (see, for example, Reference 58), and the results are mixed. According to Giedt et al. (58, abstr.), "Surprisingly, we find that a simple see-saw mechanism does not arise. . . . Extended see-saw mechanisms may be allowed. . . . We briefly speculate . . . on the possibility of alternatives, such as small Dirac masses and triplet see-saws." String theory–inspired mechanisms for tiny Dirac neutrino masses have also been pursued (see, for example, Reference 59).

### 3.3. Type II Seesaw

The Weinberg operator can be generated by integrating out heavy fields at the tree level in three and only three distinct ways (60). In Section 3.1, I present two of those (the type I and type III seesaws), and here I discuss the third one, the type II seesaw. The idea is to add to the SM Lagrangian a new scalar field $\xi$, an $SU(2)_L$ triplet with hypercharge $-1$ ($\xi$ is usually referred to as a Higgs boson triplet).

As far as fermions are concerned, $\xi$ couples only to the lepton double fields $L^\alpha$ via the following Yukawa interactions:

$$\mathcal{L}_{\mathrm{triplet}} = \frac{f_{\alpha\beta}}{2} L^\alpha L^\beta \xi + \mathrm{H.c.}, \qquad 11.$$

where $f_{\alpha\beta}$ are dimensionless couplings and $\alpha, \beta = e, \mu$ or $\tau$. If the neutral component of $\xi$ were to acquire a vacuum expectation value $\langle\xi\rangle$, then Equation 11 would lead to Majorana neutrino masses $m_\nu = f\langle\xi\rangle$. These masses would be distinct from charged-fermion masses because they are proportional to another source of electroweak symmetry breaking. Small neutrino masses require $\langle\xi\rangle \ll 100$ GeV, much smaller than the vacuum expectation value of the SM Higgs doublet, or $f \ll 1$.

Depending on the scalar potential of the theory, it is possible that the SM Lagrangian augmented by the scalar field $\xi$ preserves lepton number. Equation 11, for example, preserves lepton number if one assigns lepton number $-2$ to the Higgs boson triplet field, $L(\xi) = -2$. If this were the case, $\langle\xi\rangle \neq 0$ would spontaneously break lepton number symmetry (61), a scenario that is challenged experimentally in a variety of ways (e.g., the massless Goldstone boson associated with the breaking of lepton number is experimentally severely constrained in a variety of ways; see Reference 62 and citations therein).

A different possibility is to assume that lepton number is explicitly broken. This occurs, for example, if the scalar potential includes a trilinear term $\kappa HH\xi$, where $\kappa$ is a coupling constant with dimensions of mass. If both $f$ and $\kappa$ are nonzero, lepton number is explicitly broken. In this case, and in the limit where the triplet mass $M_\xi \gg v$, the triplet acquires a suppressed vacuum expectation value $\langle\xi\rangle \sim \kappa v^2/M_\xi^2$ (63). Another way of understanding this is to note that if $M_\xi$ is much larger than the weak scale, it can be integrated out (**Figure 2*b***), yielding the Weinberg operator of Equation 6, where $\Lambda^{-1} = f\kappa/M_\xi^2$.

Similar to the type I seesaw, the type II seesaw has new degrees of freedom and new mass scales. Here, the new degrees of freedom are of the scalar variety, and in contrast to the type I seesaw, these are charged under the electroweak symmetry and hence couple to the electroweak gauge bosons. Along with a new neutral boson, $\xi^0$, there are two new charged scalars, $\xi^+$ and $\xi^{++}$ (and their antiparticles, $\xi^-$ and $\xi^{--}$).

Phenomenological consequences depend on the parameters of the Lagrangian, most importantly the new Yukawa couplings $f$, the vacuum expectation value of the Higgs boson triplet $\langle\xi\rangle$, and the mass of the triplet $M_\xi$. At collider experiments, triplet production depends on the coupling between the new scalar degrees of freedom and the gauge bosons. The most relevant couplings, responsible for single $\xi^{0,+,++}$ production, are proportional to $\langle\xi\rangle$.[3] By contrast, large $M_\xi$ values (above hundreds of GeV) render Higgs boson triplet production at, say, the Large Hadron Collider inaccessible. The new scalars decay into other scalars, gauge bosons, and leptons. The leptonic decays are especially interesting. They violate lepton by two units and are easiest[4] to observe in the decay modes of the doubly charged scalar, for instance, $\xi^{++} \to \ell_\alpha^+ \ell_\beta^+$, where $\alpha, \beta = e, \mu$, or $\tau$ (and $\ell_e = e, \ell_\mu = \mu$, and $\ell_\tau = \tau$). These decay rates are proportional to the neutrino Yukawa couplings $f_{\alpha\beta}$, which are, in turn, directly proportional to the elements of the neutrino mass matrix in the flavor basis ($m \propto f$). Measurements of the $\xi$-decay branching ratios provide direct information about the flavor structure of the neutrino mass matrix, including Majorana phases. For a detailed discussion, see, for example, Reference 64. The main challenge, not surprisingly, is related to the fact that neutrino masses are very small. Here, this translates into either (*a*) very small $\langle\xi\rangle$, which renders the production of the Higgs triplet states very rare, or (*b*) very small Yukawa couplings $f$, which render the branching ratio for the decay of these states into leptons very small (compared with, say, the decay into gauge bosons, such as $\xi^{++} \to W^+W^+$).

Virtual processes involving Higgs triplets mediate several rate phenomena, including charged-lepton flavor-violating processes. In particular, $\xi$ exchange mediates, at the tree level, $\mu^+ \to e^+e^-e^+$ decays, which are severely constrained experimentally (12). More concretely,

$$\mathrm{Br}(\mu \to eee) \propto \left(\frac{M_W^2}{M_\xi^2}\right)^2 \frac{|(m_\nu^*)_{\mu e}(m_\nu)_{ee}|^2}{\langle\xi\rangle^4}. \quad 12.$$

There is a direct relationship between the branching ratio for rare lepton decays and the elements of the neutrino mass matrix (for detailed calculations and discussions, see, for example, References 65–68).

[3] Electroweak precision measurements already constrain $\langle\xi\rangle$ to be less than ~1 GeV (12) because a triplet vacuum expectation value modifies the tree-level SM prediction for the ratio of the $W$ boson and $Z$ boson masses.

[4] It is not possible to establish, in a collider experiment, that lepton number is violated when there are neutrinos in the final state.

# 4. MASSIVE MAJORANA NEUTRINOS: OTHER SCENARIOS

In Section 3, I discuss the three different ways in which, at the tree level, new, heavy degrees of freedom lead to Majorana neutrino masses via the Weinberg operator (Equation 6). It is useful to recast the general philosophy behind the seesaw mechanism in the following way. We postulate that at some energy scale $M$ there are new degrees of freedom and that the presence of these degrees of freedom violates lepton number—in other words, in the limit $M \to \infty$ lepton number is conserved so that lepton number–violating effects, including nonzero Majora neutrino masses, are proportional to inverse powers of $M$. The effective scale $\Lambda$ of the dimension-five operator in Equation 6 is related to $M$ and to the details of the lepton number–breaking new physics. For example, in the type I seesaw, heuristically speaking, $\Lambda = M/y^2$. Data translate into $\Lambda \sim 10^{14}$ GeV but do not directly constrain the new physics scale $M$ or the new couplings $y$. As discussed in detail in Section 3, large $\Lambda$ values might signify that the new physics scale is very heavy or that the new physics is very weakly coupled—that is, $y \ll 1$.

However, there is another possibility: The connection between the new lepton number–breaking physics and the Weinberg operator might be more indirect. It may, for example, occur at higher order in perturbation theory (at one loop, two loops, etc.). In these scenarios,

$$\frac{1}{\Lambda} \sim \Pi(y_i)\left(\frac{1}{16\pi^2}\right)^n \frac{1}{M}, \tag{13}$$

$\Pi(y_i)$ is a product of different new physics and SM couplings (gauge couplings and Yukawa couplings), and $n$ counts the number of loops required to generate the Weinberg operator. There are many models of this type. Among the best-known models are the one proposed by Zee (69) [and Wolfenstein (70)] in the early 1980s, which has been ruled out by neutrino oscillation data, and the Zee–Babu model (71, 72), where neutrinos masses are a two-loop effect.

For a concrete example, I discuss a model proposed by Ma (73). Assume that the SM particle content is extended to include the right-handed neutrino fields $n^c$ and a new scalar field $\eta$ with the same SM quantum numbers as the Higgs boson; in other words, $\eta$ is a doublet with hypercharge $+1/2$. We add all possible interactions consistent with the gauge symmetry but further require that the $n^c$ and $\eta$ fields be odd under a $Z_2$ symmetry (all other fields are even), which is not spontaneously broken. The portion of the Lagrangian that includes fermion fields is

$$\mathcal{L}_{\text{scotogenesis}} = y^{\alpha i} L_\alpha n^c_i \eta - \frac{M^{ij}}{2} n^c_i n^c_j + \text{H.c.} \tag{14}$$

Although there are right-handed neutrino Majorana masses $M$ and neutrino Yukawa interactions (with strength $y$), active neutrinos nonetheless do not, if these are the only new interactions, acquire Majorana masses. One way to appreciate this fact is to appreciate that lepton number is still conserved. If we assign $L(n^c) = 0$ and $L(\eta) = -1$, Equation 14 is lepton number invariant. Lepton number is explicitly broken once we consider the scalar potential. In particular, the interaction $g(H\eta^\dagger)^2$ (where $g$ is a dimensionless coupling) is allowed by the $Z_2$ symmetry. In summary, if $g$, $M$, or $y$ vanishes, lepton number is not broken, so nonzero Majorana neutrino masses are proportional to all three new physics parameters. It is easy to check that one can integrate out the $\eta$ and $n^c$ fields in order to generate, at the one-loop level, the Weinberg operator. The active neutrino Majorana masses are ultimately proportional to $(yv)^2 g/(16\pi^2 M_{\text{eff}})$, where $M_{\text{eff}}$ is a function of the $\eta$ and $n^c$ masses. In this model, which is referred to as scotogenic (73)—created from dark, the lightest component of the $\eta$ field is stable and, potentially, a good dark matter candidate.

There is a simple and powerful way of studying a very large class of these indirect lepton number–violating models by using the language of effective field theories. The idea, first discussed in detail in Reference 74 and thoroughly explored in References 75 and 76, is as follows. Imagine

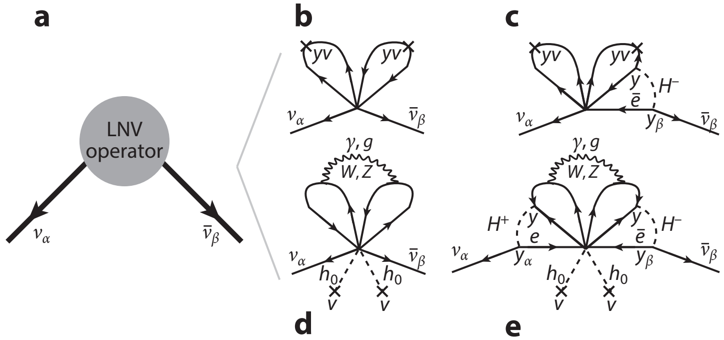


**Figure 4**

(*b*–*e*) Different scenarios that lead to (*a*) nonzero neutrino Majorana masses at the multiloop level. See Reference 75 for the proper definition of the different scenarios. Abbreviation: LNV, lepton number violation.

a combination of heavy fields that cause lepton number to be explicitly broken. Also assume that these fields, when integrated out at the tree level, manifest themselves, as far as lepton number violation is concerned, via effective operators with smallest mass dimension $D_{\text{tree}}$, $\mathcal{O}_{\text{D}_{\text{tree}}}$. One can use $\mathcal{O}_{D_{\text{tree}}}$ to estimate the contribution of this new physics to the Weinberg operator by computing loop contributions to the Weinberg operators from $\mathcal{O}_{D_{\text{tree}}}$ and the SM. Thus, we replace the new physics by $\mathcal{O}_{D_{\text{tree}}}$ and estimate its contribution to the neutrino masses without having to specify the details of the lepton number–breaking physics (**Figure 4**). Concrete examples are discussed in detail elsewhere (74–77).

This procedure allows one to estimate the effective new physics scale $\Lambda$ given the observed values of the neutrino masses. The presence of several known SM couplings—when we keep in mind that some of the charged-fermion Yukawa couplings are quite small—and loop suppressions of order $(16\pi^2)^n$ indicate that these loop models are associated with new physics scales that are much smaller than the $10^{14}$ GeV directly associated with the Weinberg operator. **Figure 5** depicts a histogram of different models and their effective new physic scales once one includes neutrino oscillation data. It is easy to see that the new physics scales $\Lambda$ span several orders of magnitude, from 100 GeV to $10^{14}$ GeV. The effective new physics scale, in turn, allows one to estimate whether the same new physics can be constrained by or discovered in different particle physics probes. These include collider experiments, which can "see" new particles as long as their masses are smaller than 1 TeV, and searches for rare or forbidden phenomena, which are sensitive to new particles with masses as large as hundreds of TeV (see, for example, Reference 78 for a discussion of the reach of charged-lepton flavor-violating experiments). The effective operator approach proposed in References 74 and 75 does not, however, allow one to estimate the new physics effects when it comes to lepton number–conserving processes. These require ultraviolet-complete models. For a complete example that concentrates on collider signatures, see Reference 77.

On one hand, the effective operator approach briefly discussed above can be extended in order to, for example, relate lepton number–violating and baryon number–violating phenomena if one assumes that Grand Unification is realized in nature (79). On the other hand, this approach is not complete; there are scenarios it does not capture. For example, another set of effective operators (80) captures the low-energy consequences of some new physics scenarios not captured by the operators highlighted in References 74 and 75.

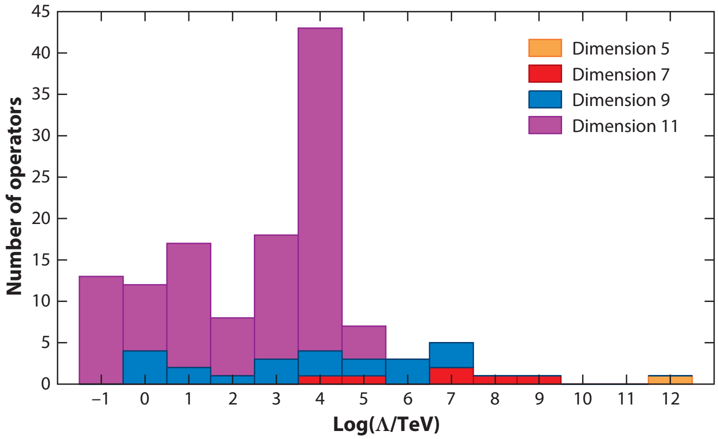


**Figure 5**

The effective physics scales $\Lambda$ required in order to fit the observed neutrino masses for different lepton number–violating scenarios. The colors indicate the mass dimension of the different $\mathcal{O}_{D_{\rm tree}}$ that characterize the different models. See Reference 75 for details. Modified from Reference 75.

## 5. COMMENTS ON LEPTON MIXING

Neutrino oscillation data also revealed that leptons mix. A great deal of particle physics research is dedicated to exploring whether there are hints to the existence and nature of new physics in the relative values of the different entries of the mixing matrix. Flavor mixing was first discovered, some 50 years ago, in the quark sector. The weak-interaction physics behind both quark and lepton mixing is the same, and both are captured by $3 \times 3$ unitary mixing matrices (**Figure 6**). The precise values are tabulated in, for example, the Particle Data Book (12).

**Figure 6** reveals a few interesting aspects of the mixing matrices. The quark mixing matrix is very structured. The diagonal elements are larger than the off-diagonal elements, and the latter are hierarchical: $|V_{us}|^2, |V_{cd}|^2 \gg |V_{ts}|^2, |V_{cb}|^2 \gg |V_{ub}|^2, |V_{td}|^2$. The lepton mixing matrix is very different from the quark one. With the exception of $U_{e3}$, all elements of $U$ have magnitudes of order one. The structure of $U$, assuming there is one, is more subtle. $U$ has several interesting features; for example, $|U_{\mu 3}|^2 \simeq |U_{\tau 3}|^2$, and $|U_{e2}|^2 \simeq |U_{\mu 2}|^2 \simeq |U_{\tau 2}|^2$. Alternatively, there may be no meaningful structure in the leptonic mixing matrix; it may be simply a random unitary matrix. This concept has been formalized and termed neutrino mixing anarchy (81, 82). Briefly stated, anarchy is the hypothesis that the observed lepton mixing matrix is consistent with a matrix drawn at random from a flat distribution of unitary matrices. This anarchical hypothesis can be tested (83) and provides a very good fit to the neutrino oscillation data (84).

In the quark sector, there are models that attempt to explain the hierarchy of the quark masses (see **Figure 1**) and the pattern of quark mixing. The same is true of the lepton sector. The level of theoretical activity in the lepton sector is high, and I do not attempt to summarize all the

$$|U_{\rm PMNS}| = \begin{pmatrix} 1 & 1 & 0.1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \qquad |V_{\rm CKM}| = \begin{pmatrix} 1 & 0.1 & 0.01 \\ 0.1 & 1 & 0.1 \\ 0.01 & 0.1 & 1 \end{pmatrix}$$

**Figure 6**

Order of magnitude of the different entries of the quark mixing matrix $V_{\rm CKM}$ and the lepton mixing matrix $U_{\rm PMNS}$.

different ideas under investigation. Recent overviews are available elsewhere (85–87). Instead, I point out, very briefly, that there may be connections between the mechanism responsible for nonzero neutrino masses and the fact that lepton mixing is very different from quark mixing.

It is important to ask why one would expect quark mixing and lepton mixing to be similar. Both quark masses and lepton masses (at least charged-lepton masses) are hierarchical, for example, and if Grand Unification is indeed realized in nature, at very high energies, quarks and leptons are different components of the same fundamental objects. The latter is, arguably, the most compelling argument. There have been several attempts to create complete (including flavor) grand unified models (see, for example, References 88 and 89 and citations therein). The idea is that the same flavor structure is present for quarks and leptons, but that these manifest themselves, at low energies, very differently in the quark and lepton sectors. The main source of difference, most of the time, is the fact that neutrino masses are qualitatively different from charged-fermion masses. The argument also works in the other direction. The fact that lepton mixing and quark mixing are very different is often presented as indirect evidence that neutrino masses are qualitatively different from charged-fermion masses.

Flavor models essentially rely on assigning different new quantum numbers to the three fermion generations. These new quantum numbers differentiate fields that are indistinguishable as far as SM gauge interactions are concerned and help explain why their masses are hierarchical and why, in the case of quarks, mixing is small. The fact that anarchy works might indicate that, if there is a flavor symmetry, all $L^\alpha$ fields have the same quantum number and hence that lepton mixing, which is governed by the physics of the $SU(2)$ doublet fermions, is large and neutrino masses, if they are of the Majorana type and come from, say, the Weinberg operator, are all expected to be of the same order. At the same time, it may be that the quark $SU(2)$ doublets $Q^\alpha$ have very different charges such that quark mixing is small. Curiously, these different charge assignments might be consistent with Grand Unification if the $L$ fields and the $Q$ fields are components of different fundamental objects, a fact that is explicitly realized in $SU(5)$ GUTs, as first discussed in Reference 90. Consequences of this idea, assuming that weak-scale supersymmetry also exists, were worked out in that paper and references therein.

The same physics that explains why neutrino masses are very small might also constrain the flavor structure of the lepton sector. In the case of certain scenarios where the neutrino masses are generated at the loop level, for example, the neutrino Majorana mass matrix ends up proportional to charged-lepton Yukawa couplings (in the basis where the charged-lepton Yukawa coupling matrix is diagonal), and there is a natural flavor structure imposed on the neutrino sector. These possibilities were studied in Reference 75. In the so-called flavor geography scenarios (20, 21) discussed in Section 2, both Dirac masses and lepton mixing are determined by the localization of the different fermion fields. It has been argued (21) that understanding small neutrino masses and small quark mixing is simple but that the large mixing angles in the lepton sector might require more structure in the extradimensional sector. Finally, there are scenarios in which neutrino Yukawa couplings are forbidden, at the renormalizable level, by gauge interactions that are flavor dependent (15). In these scenarios, the origin of nonzero neutrino masses may be entwined with the flavor structure observed in lepton mixing.

## 6. CONCLUDING REMARKS

Neutrino masses are known to be nonzero. They are also tiny when compared with all other mass scales in the SM. Nonzero neutrino masses are bona fide evidence for physics beyond the SM. New ingredients must be added to the our current understanding of particle physics in order to explain the phenomena revealed by neutrino oscillation experiments.

The mechanism behind neutrino masses is still unknown. There are many different ideas in the literature, many of which are discussed in this review. The different models are qualitatively different; they contain distinct ingredients and make divergent predictions about the nature of the neutrino. In some scenarios, neutrinos are Dirac fermions and their masses are generated in the same way that all other fermion masses are generated. In these types of scenarios, neutrino masses are very small because neutrinos are very weakly coupled to the source of electroweak symmetry breaking, the SM Higgs field. In many other scenarios, neutrinos are Majorana fermions. This might be a consequence of a more complicated electroweak symmetry–breaking sector—more Higgs-like bosons—or might indicate the existence of mass scales in nature that are not related to the electroweak scale. In these types of scenarios, new particles must exist. They may be as light as 1 eV or as heavy as $10^{15}$ GeV.

Progress will require significant input from experiments, and help may come from all different areas of particle physics. Understanding the fate of lepton number is of the utmost importance, as it will help reveal whether neutrinos are Majorana or Dirac fermions. This, in turn, will point neutrino mass model building in different directions. The most powerful probes of lepton number conservation are searches for neutrinoless double-$\beta$ decay (30). Precision measurements of charged-lepton properties and searches for charged-lepton flavor violation could also provide information about the origin of neutrino masses. Neutrinos and charged leptons are intimately related, and there are several scenarios in which the physics responsible for nonzero neutrino masses manifests itself in charged-lepton flavor violation (78) at experimentally accessible levels (91). High-energy collider experiments, including those at hadron colliders, may also contribute decisively to our understanding of the mechanism behind neutrino masses. In particular, some of the ingredients that go into nonzero neutrino masses could be directly produced and observed at, for example, the Large Hadron Collider. Similarly, cosmic surveys, and other astrophysical probes, will continue to explore, indirectly, the properties of the cosmic neutrino background, and they have the potential to access information about neutrino properties that cannot be accessed in the laboratory (92).

To date, however, all positive information about nonzero neutrino masses has come from neutrino oscillations. It is highly anticipated that these will continue to provide important clues about the origin of neutrino masses and lepton flavor. A comprehensive program to explore the physics of neutrino oscillations is currently being developed (93), so the future looks very promising. Neutrino oscillations will reveal, for example, the ordering of the neutrino masses. If this order turns out to be inverted—$m_2^2 > m_1^2 > m_3^2$—then we will learn that at least two of the neutrino masses are quasi-degenerate (at the few-percent level). Such a discovery would dramatically influence neutrino mass model building. None of the charged-fermion masses are degenerate, and quasi-degenerate masses are usually evidence for new symmetries (for example, the proton–neutron system, or the pion system, serves as very compelling evidence for isospin symmetry). Neutrino oscillation experiments will also reveal whether *CP* invariance is preserved in the lepton sector and may also find new neutrino states, new neutrino properties, or new weaker-than-weak neutrino–matter interactions.

The reader should note that this review is far from exhaustive. The literature is filled with many more intriguing explanations for why neutrino masses are small. These include variants of the seesaw mechanism, such as the inverse seesaw (94) and the linear seesaw (95, 96). Also, this review does not emphasize connections between nonzero neutrino masses and other shortcomings of the SM, including the dark matter and dark energy puzzles.

In summary, the neutrino mass puzzle is a central question in particle physics today. Piecing this puzzle together will require intense experimental and theoretical effort. Success will depend on the ingenuity and creativity of experimentalists and theorists, and a little help from nature.

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