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# The Long-Run Discount Rate Controversy

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## Abstract

The choice of the rate at which one should discount the long-term benefits of mitigating climate change is highly controversial. Both the level and the slope of the term structure of discount rates have been discussed intensively in relation to the determination of the social cost of carbon. Although some of the parameters of the problem are ethical and outside the scope of economic analysis, we claim that there are converging and convincing arguments in favor of using an annual real risk-free discount rate going from approximately 4% to approximately 1% for maturities going from zero to infinity. Investing in climate mitigation yields highly uncertain future benefits. Such uncertainty should also be taken into account in the selection of the discount rate, although the appropriate approach is highly controversial.

## 1. INTRODUCTION

How should one compare benefits occurring at different time horizons? A tradition has developed over the past two centuries to make such benefits comparable by discounting.[1] The discounted (or present) value of a future benefit is the immediate benefit that is considered to be socially equivalent to that future benefit. An action is considered desirable if its net present value (NPV), i.e., the sum of the present value of the benefits and costs that the action generates, is greater than zero. But although discounting and NPV are universally accepted tools to evaluate investment projects and public policies, there is still much controversy about which discount rate should be used in practice, particularly for the distant future.

The absence of consensus on this question is critical for many important applications from climate change to the preservation of natural resources, the evaluation of pension-fund liabilities, and the speed at which public debts should be reduced in the Western world. Experts and evaluators are well aware that the discount rate is a key parameter driving the evaluation of actions having long-lasting impacts. Indeed, the present value of $1 million received in 200 years is equal to either $137,000 or $1, depending upon whether one uses a discount rate of 1% or of 7%; these rates are within the range suggested by different experts. The emergence of global challenges to the sustainability of human growth has put pressure on economists to tell the scientific truth, if it exists, about which discount rate to use for long-run costs and benefits. The objective of this article is to summarize the recent key findings and the remaining disagreements on this question.

The discount rate and the market interest rate translate our collective values toward the future into key economic variables. Societies with a low discount rate value the future more than do societies with a higher discount rate, in a way that is made precise below. The way we discount long-term benefits expresses our responsibilities toward future generations. Common wisdom suggests that our market economy is too short-termist; it does not sufficiently value the long-term impacts of our actions. This perspective suggests that people use discount rates that are too high compared with what would be desirable from the point of view of intergenerational welfare. Yet this statement has never been seriously tested, probably because there is no consensus about what would be the socially desirable level of the discount rate.

We must recognize that economics cannot provide a full answer to this question, which involves deep ethical issues. Investing for the future implies a transfer of consumption from current generations to future generations. In a growing economy, this process means transferring consumption from the poor to the wealthy. If we recognize a collective ethical concern in favor of reducing inequalities, the discount rate can be seen as the minimum rate of return of the investment that is required to compensate for the increased intertemporal inequality that the investment generates. The larger our aversion to inequality, the higher is the discount rate. This is the essence of the famous Ramsey (1928) rule, which has become a focal point of the recent debate on discounting. Even though economists may document individual aversion to inequalities (e.g., by observing prosocial behavior in practice and in the lab), we have no unique legitimacy to determine our degree of inequality aversion at the collective level, which is a purely ethical parameter.

Although this controversy originates mostly from environmental issues, it is in essence a pricing question. Unsurprisingly, the modern theory of finance and asset valuation provides key elements for determination of the discount rate. However, this theory has primarily examined the short-

[1]See, for example, Moog & Bösch (2013), who provide an interesting overview of the history of thought on discounting in German forestry.

term pricing of assets, interest rates, and risk. Expanding the theory to explore its consequences for long time horizons remains a challenge. Finance scholars are interested in explaining the formation of the interest rate for a typical 3-month maturity, but we want to determine the socially desirable interest rate to be used for maturities expressed in centuries! Another challenge when we use modern finance theory is that the simplest versions of the theory fail to explain the interest rates and risk prices observed in financial markets.

Following the tradition of the literature that we survey, in most of the article we make the unrealistic assumption that the projects we want to evaluate are risk free. Thus, we characterize the determinants of the risk-free discount rate. But one should recognize that most actions have uncertain short- and long-term impacts. In the finance literature, this recognition led to the characterization of risk-adjusted discount rates, which we examine in Section 7.

## 2. DISCOUNTING IN PRACTICE

### 2.1. Market Interest Rates

Firms use their costly capital to invest. Consider a firm implementing a single investment project. This project is profitable if and only if its return is larger than the firm's cost of capital. To illustrate, consider a firm with a safe project. Competition in the capital market implies that the firm will offer its lenders a return on their capital that is not different from the risk-free market interest rate. To generate a profit, the return to the project must exceed this rate. Thus, firms should use the market interest rate as the discount rate to evaluate their safe investment projects. When projects are risky, the cost of capital to finance them will be adjusted for risk, and so will their associated discount rate (see Section 7).

Observed market interest rates are thus a good indicator of the discount rate used by firms. These rates have fluctuated through time and across countries. We have good estimates of real interest rates for a large set of Western countries since the late-nineteenth century. In **Table 1**, we present the average realized real returns of sovereign bills and bonds and market equity

**Table 1 Annualized real returns, 1900 to 2006**

| | Bill | Bond (10 year) | Equity |
|---|---|---|---|
| Australia | 0.6% | 1.3% | 7.8% |
| Canada | 1.6% | 2.0% | 6.3% |
| Denmark | 2.3% | 3.0% | 5.4% |
| France | −2.9% | −0.3% | 3.7% |
| Italy | −3.8% | −1.8% | 2.6% |
| Japan | −2.0% | −1.3% | 4.5% |
| The Netherlands | 0.7% | 1.3% | 5.4% |
| Sweden | 1.9% | 2.4% | 7.9% |
| Switzerland | 0.8% | 2.1% | 5.3% |
| United Kingdom | 1.0% | 1.3% | 5.6% |
| United States | 1.0% | 1.9% | 6.6% |

Reproduced from Gollier (2012).

over the period 1900–2006 for 11 countries.[2] Sovereign bills and bonds are considered the safest assets of the economy. Bills are debt contracts with maturities not exceeding 1 year; bonds may have maturities of up to 50 years. The interest rate is maturity dependent; i.e., there is a term structure for the rate of return to safe assets. The term structure is usually increasing: Safe projects with a longer maturity yield a larger cost of capital and a higher discount rate. For the very short maturities corresponding to bills, real rates averaged 1% during the past century in the United States and were even negative in France, Italy, and Japan. Sovereign bonds with a 10-year maturity generated a slightly larger average real return, approximately 2% for the United States. We conclude that market forces induced the private sector to use a rate of between 1% and 2% to discount risk-free projects during the past century, averaging over the business cycle.

## 2.2. Government Evaluation

Several countries routinely use benefit-cost analysis to evaluate public decisions, including transportation projects and environmental and safety regulations. Some have published guidance to standardize evaluation methods, including the discount rate.

In the United States, the guidance (OMB 2003) recognizes that the discount rate should depend on whether a regulation reduces investment or consumption. The return to investment (the opportunity cost of capital) is larger than the rate at which consumers trade current consumption for future consumption (the social rate of time preference) because of taxes on capital income.[3] OMB (2003) suggests that the shadow-price-of-capital approach, which converts reductions in investment to forgone future consumption increments and discounts these at the social rate of time preference, is analytically preferred but impractical because of uncertainty about the extent to which a regulation displaces investment versus consumption. As a default, the guidance requires that projects be evaluated using two rates: 7% and 3%. The first is the average before-tax rate of return to private capital, taken as an estimate of the opportunity cost of capital. The second is the average return to 10-year government bonds, taken as an estimate of the social rate of time preference. For projects with intergenerational effects, OMB suggests supplementing these two rates with an additional, lower-but-positive discount rate. This addition is justified by arguments about the illegitimacy of discounting future generations' well-being and uncertainty about future growth.

In the United Kingdom, the discount rate is based on the Ramsey rule (described in Section 4). The first term ($\delta = 1.5\%$) is interpreted as a combination of pure time preference and risk of catastrophe (under which the future effects would be eliminated or severely altered). The inequality-aversion parameter $\gamma$ is set to 1, and the economic growth rate $g$ is estimated as 2%, yielding a discount rate of 3.5%. For maturities greater than 30 years, the guidance specifies a stepwise decreasing discount rate motivated by uncertainty (Weitzman 1998, 2001; Gollier 2002). The discount rate is 3.0% for years 31–75 and falls to 1% for periods of more than 300 years in the future.

In France, the recommended discount rate also decreases with maturity. Following the recommendations of the Lebègue (2005) report, the rate is 4% for maturities up to 30 years and 2% for subsequent years. This schedule is a compromise that approximates the result of using the Ramsey rule with $\delta = 1\%$, $\gamma = 2$, and $g = 1.5\%$ for the first 30 years and representing uncertainty about growth as a binary lottery in which $g$ takes a common value for all future years, equal to 2% with probability 2/3 and 0.5% with probability 1/3.

[2] All interest and discount rates in this article are reported as annual percentages.

[3] Differences in risk may also contribute to observed differences, but not to the risk-free rates.

In Norway, current guidance specifies a risk-free discount rate of 2%, to which a risk premium of 2% is added for normal projects, yielding a discount rate of 4%. For projects viewed as having a high degree of systematic risk, the risk adjustment should be increased to 4%, yielding a discount rate of 6%. For major projects, systematic uncertainty about the costs, benefits, and how these are resolved over time is supposed to be analyzed and used to adjust the risk-free rate of 2%; however, following this guideline has proven difficult in practice. A recent expert committee charged with reviewing the guidance endorsed a rate of 4% for projects with maturities up to 40 years and recommended a schedule of declining rates: 3% for years 40–75 and 2% thereafter (Norwegian Ministry of Finance 2012).

## 3. POSITIVE APPROACH

We see in Section 2.1 that the private sector should use the market interest rate as the rate at which risk-free cash flows should be discounted. In this section, we discuss whether this recommendation should be universal.

Consider a (marginal) action that yields a cost $C$ today and a sure benefit $B$ in $t$ years. Is this project socially desirable? To answer this question, suppose that the market interest rate corresponding to that maturity is $r$. Suppose also that the project is financed by a reallocation of safe capital to the project. Disinvestment of $C$ units of capital today reduces the payoff of safe capital in the economy by $C\exp(rt)$ at time $t$. Reallocating safe capital to the project has no effect on current consumption and increases consumption at time $t$ by $B - C\exp(rt)$. This action is socially desirable if and only if the increase in consumption is positive, i.e., if the NPV $[-C + B\exp(-rt)]$ is positive. Thus, the efficient discount rate in this context is simply the interest rate. Under this argument, the discount rate can be interpreted as the opportunity cost of safe capital. The discount rate is closely related to the arbitrage argument so classical in the finance literature: If the NPV of the project with $r$ as the discount rate is positive, one can produce a safe positive profit by investing in the project and by going short on (i.e., borrowing at) the risk-free rate. This argument is positive because it is based on the revealed preferences expressed via market interest rates.

The same argument arises when economists explain why a zero discount rate cannot be socially desirable. Suppose that the interest rate on financial markets is positive. If a project with zero rate of return is financed by disinvestment of productive safe capital, its payoff does not compensate for the lost return, thereby destroying value for future generations. Said differently, if current generations want to improve the future, they should invest in the productive capital in the economy (yielding a positive marginal return) rather than in the project.

We believe that this positive argument provides a strong basis to discount safe projects whose maturities are within the range of maturities of safe assets actively traded on financial markets. Although we have estimates of historical real returns to safe assets (see **Table 1**), future real returns are necessarily uncertain. Interest rates on most government bonds are nominal, and so the real return will depend on realized inflation over the period to maturity. The positive approach cannot be applied for time horizons exceeding 20 or 30 years, because there are no safe assets traded on markets with such large maturities. Sovereign bonds with such large maturities of even the most financially reliable countries cannot be considered completely safe because of both inflation and default risk.

## 4. THE RAMSEY RULE AND THE NORMATIVE RISK-FREE-RATE PUZZLE

Suppose alternatively that the investment project considered in the previous section is financed through a reduction in consumption by the current generation, for the benefit of the future generation. In that case, there will be an (intergenerational) distributional effect. The evaluation of the project is more difficult than when one generation bears the net impact of the action, as assumed in the previous section. In this context, one needs to make ethical judgments about the intergenerational transfers. The standard approach inherited from public economics and public-choice theory is to evaluate the project in relation to its impact on the utilitarian intergenerational welfare function:

$$W = u(c_0) + e^{-\delta t} Eu(c_t), \tag{1}$$

where $c_0$ and $c_t$ are current consumption and future consumption, respectively, and $u$ is the (common) utility function. The parameter $\delta$ is the utility discount rate or rate of pure time preference; it is sometimes interpreted as the per-period probability of extinction. From Ramsey onward, many commentators have argued that ethics require $\delta = 0$, a position we accept here. A nonzero value of $\delta$ penalizes people on the basis of their birth date, which is as ethically unacceptable as racism (penalty based on the color of the skin) and sexism (penalty based on gender). Macroeconomists, classical growth theorists, and finance theorists have used a positive value as a technical trick to escape the problem of the potential unboundedness of intertemporal welfare in infinite-horizon models. Although individuals may discount their own future utility, that is not a reason to penalize future generations in the welfare function. To quote Solow (1974), "in solemn conclave assembled, so to speak, we ought to act as if the social rate of pure time preference were zero."

Intergenerational welfare is assumed to be the discounted sum of the flow of generational utility. Because the future is uncertain, future utility is evaluated through the expected utility of future consumption. Under this approach, a reduction in current consumption can be socially desirable if its impact on current utility is more than compensated by an increase in future expected utility.

If the project is marginal, its impact on $W$ is positive if its current social cost $Cu'(c_0)$ is smaller than its future social benefit $BEu'(c_t)$. This inequality can be rewritten as

$$-C + Be^{-R_t t} \geq 0 \quad \text{with} \quad e^{-R_t t} = \frac{Eu'(c_t)}{u'(c_0)}. \tag{2}$$

We can interpret this condition as a standard NPV test with a discount rate $R_t$ that incorporates both social preferences and beliefs about future growth. The efficient discount factor $\exp(-R_t t)$ is simply the expected marginal rate of substitution between current consumption and future consumption.

We assume that the utility function $u$ is increasing and concave. Under this assumption, the marginal utility of consumption is decreasing. In other words, if a social planner has the opportunity to transfer consumption from one agent to another, transferring from the richest individual to the poorest individual is optimal. Thus, the concavity of $u$ is a notion of aversion to inequalities. It is standard to consider the power specification $u(c) = c^{1-\gamma}/(1-\gamma)$, where $\gamma \geq 0$ measures the degree of inequality aversion in the economy. In the limit case of inequality neutrality, $\gamma = 0$ and $u'$ is a constant so that $Eu'(c_t)/u'(c_0)$ equals unity. Therefore, the efficient discount rate $R_t$ defined in Equation 2 is equal to zero. We hereafter assume that $\gamma$ is positive to incorporate inequality aversion.

Suppose first that there is no uncertainty surrounding the growth of the economy. Then, the definition of the efficient discount rate $R_t$ in Equation 2 implies the following characterization:

$$R_t = \gamma g_t \quad \text{with} \quad g_t = \frac{1}{t} \ln \frac{c_t}{c_0}. \tag{3}$$

This is the Ramsey rule, after Frank Ramsey (1928), who was the first to derive this condition from an optimal, dynamic consumption-saving problem.[4] The socially efficient discount associated

[4]The Ramsey rule is often written as $R_t = \delta + \gamma g_t$ to accommodate a nonzero value of $\delta$ in Equation 1.

with maturity $t$ equals the product of the degree of inequality aversion, $\gamma$, and the average growth rate of consumption from today to date $t$, $g_t$. In the special case with a constant consumption growth rate, $g(c_t) = c_0 \exp(gt)$, the Ramsey rule yields a constant discount rate $R = \gamma g$.

The importance of the Ramsey rule to the debate on the discount rate should not be underestimated. It provides a crucial ethical argument in favor of discounting. In a growing economy, safely investing for the future is equivalent to transferring sure consumption from the current poor to the future wealthy. Under inequality aversion, this action is socially desirable only if the return to the project is large enough to compensate for the increased intergenerational inequality that it generates. The minimum rate of return is given by $R = \gamma g$, which is positive and increasing both in the degree of inequality aversion and in the growth rate.

Under this normative approach, calibration of the Ramsey rule requires information about both the degree of inequality aversion and the growth rate. As the future growth rate is uncertain, we need to use some estimate. In the parable of Lucas (1978), the production side of the economy is represented by trees in fixed supply (no capital accumulation) whose fruits are consumed by the representative agent, and the economic growth rate is exogenously given by the natural law of tree growth. In that case, $g$ can be considered as exogenous to the economic process. In classical growth theory (Solow 1956), economic growth is jointly generated by the accumulation of capital and by innovation. In the short run, capital accumulation and the interest rate are jointly determined from dynamic-equilibrium conditions.

Uncertainty about the future growth rate imposes an obvious limitation on what economists can say about the efficient discount rate for very long maturities. All we can say is that if we collectively agree that the average growth rate will be 2% per year through some time horizon, then using a discount rate of $2\gamma\%$ for this time horizon is sensible. A growth rate of 2% is an interesting benchmark because it has been the average growth rate of Western countries over the past two centuries.

The degree of inequality aversion $\gamma$ is an ethical parameter. The second column of **Table 2** documents some of the normative statements that various economists have made about its value. A degree of inequality aversion of between 1 and 4 seems to be a consensual proposition, with $\gamma = 2$ as a focal reference.

Understanding the meaning of the degree of inequality aversion is important. To illustrate, consider an economy with two social classes of equal population, in which the upper class consumes twice as much as the lower class. Consider a marginal redistributive policy to increase consumption in the lower class by \$1. What is the maximum sacrifice by the upper class that is justified to provide this benefit to the poor? If the answer is \$1, one is inequality neutral; a sacrifice of more than \$1 reveals some degree of inequality aversion. The maximum sacrifice $k = 2^\gamma$. Thus, a degree of inequality aversion of 2 means that one should be ready to give up as much as \$4 of consumption of the wealthy to give \$1 to the poor. The maximum sacrifice increases to \$16 if $\gamma = 4$, as has been recommended (**Table 2**). The power specification for $u$ implies that the problem is homothetic so that absolute wealth does not matter for determining the social efficiency of marginal transfers.

Disagreement about the level of $\gamma$ comes from the fact that this parameter plays many different roles in the discounted expected utility (DEU) model. For example, under the Rawlsian veil of ignorance, the level of inequality aversion should be equal to the degree of relative risk aversion of the representative consumer, thereby transforming an ethical parameter into a descriptive one. Quoting Ju & Miao (2012), "researchers in macroeconomics and finance generally believe that the risk aversion parameter is around 2," but a degree of relative risk aversion of between 1 and 4 is more representative of a soft consensus among economists. Parameter $\gamma$ also measures aversion to consumption fluctuations over time in the standard consumption-saving problem; i.e., $1/\gamma$ is the

**Table 2 Calibration of the discount rate based on the Ramsey equation (Equation 3)**

| Author | Inequality aversion $\gamma$ | Growth rate $G$ | Discount rate $\gamma g$ |
|---|---|---|---|
| Stern (1977) | 2 | | |
| Cline (1992) | 1.5 | 1% | 1.5% |
| IPCC (1995) | 1.5–2 | 1.6–8% | 2.4–16% |
| Arrow (1999) | 2 | 2% | 4% |
| HM Treasury (2003) | 1 | 2% | 2% |
| Lebègue (2005) | 2 | 2% | 4% |
| Arrow (2007) | 2–3 | | |
| Dasgupta (2007) | 2–4 | | |
| Stern (2007) | 1 | 1.3% | 1.3% |
| Weitzman (2007a) | 2 | 2% | 4% |
| Nordhaus (2008) | 2 | 2% | 4% |
| Pindyck (2013) | 1–3 | | |

Some of the authors add a rate of impatience $\delta$ to the Ramsey rule so that the last column is only a partial representation of what these authors recommend for the discount rate. Blank cells denote that data were not given.

elasticity of intertemporal substitution of consumption. Using estimates of demand systems, Stern (1977) finds a concentration of estimates of $\gamma$ of approximately 2, with a range of roughly 0–10. Epstein & Zin (1991), who propose a generalization of the DEU to disentangle aversion to risk and to fluctuations, find a value ranging from 1.25 to 5. Pearce & Ulph (1995) estimate a range from 0.7 to 1.5.

If we combine an index of inequality aversion of $\gamma = 2$ with a prospective average growth rate of $g = 2\%$, the Ramsey equation (Equation 3) gives us a normative discount rate of 4%. **Table 2** gives some variations in the calibration of this equation that are representative of the current literature on this question.

The Ramsey rule tells us the minimum return required to induce a marginal increase in savings. The above analysis suggests that if one believes that the growth rate of the economy will remain close to its historical trend since the Industrial Revolution, one should not invest at the margin in safe projects whose return is less than 4%. But we have seen that past generations in the twentieth century invested in safe projects whose return was as low as 1% in the United States. This investment led to a formidable accumulation of capital over the past century. This outcome was socially undesirable. If past generations believed in a bright future, why did they sacrifice so much of their production for the benefit of their much wealthier successors? The low return on safe assets during the period did not compensate for the large intergenerational inequalities that this generous saving and investment behavior generated. We refer to this observation as the normative risk-free-rate puzzle.[5] A possible explanation is that past generations were pessimistic or recognized that economic growth is an uncertain process (see the next section).

[5]Weil (1989) was the first to present the (positive) risk-free-rate puzzle, which states that the classical consumption-based capital asset pricing model (CCAPM) cannot explain why interest rates have been so low during the past century.

To this point, our survey leads to two contradictory recommendations for the discount rate: Use a positive market interest rate, or use a normative rate based on the Ramsey rule.[6] This contradiction can be resolved by recalling how these two recommendations have been obtained. Under the positive approach, we assume that the new risk-free project is financed through a reallocation of risk-free capital, so the opportunity cost of capital determines the discount rate. Under the normative approach, we alternatively assume that the new project is financed by an increase in savings from the current generation, so the marginal rate of intertemporal substitution determines the discount rate.

## 5. THE LONG-TERM DISCOUNT RATE CONTROVERSY

The emergence of global challenges to the long-term sustainability of economic growth has forced economists to reevaluate the power of their arguments and models concerning the efficient discount rate. We see in Section 3 that the positive approach based on the opportunity cost of capital does not yield an immediate answer for the discount rate to be used for long horizons for which there is no actively traded safe asset. In Section 4, we see that the normative approach relies on the prospective long-term growth rate of the economy, which is by nature highly uncertain. Thus, the two approaches raise specific questions when one wants to apply them to evaluate safe investment projects and public policies that have long-lasting impacts on the economy. Over the past 15 years, various arguments have been raised to justify using smaller rates to discount very distant costs and benefits. This controversial recommendation could bias our actions in favor of projects with distant benefits at the cost of reducing our efforts to improve the more immediate future. We reexamine these arguments here.[7]

### 5.1. The Extended Ramsey Rule

There is a simple argument in favor of a decreasing term structure of the safe discount rate, which is immediately apparent from the Ramsey equation (Equation 3). Observe that the argument leading to this equation can be applied for any time horizon $t$. Thus, the equation does not describe "the" discount rate, but rather the entire term structure of discount rates. To illustrate, consider a decelerating economy, i.e., an economy in which the average growth rate $g_t$ decreases with the time horizon $t$. In that case, Equation 3 tells us that the term structure of discount rates $R_t$ will inherit a negative slope from the term structure of $g_t$. Growth may be anticipated to decelerate for various reasons; Gordon (2012) provides an interesting overview. Deceleration is also typical of an economy entering the overheating phase of its business cycle. But since 2007, the Western world has been rather in the opposite situation, expecting acceleration after a recession. In such a context, one should use an increasing term structure of safe discount rates. This approach would have the advantage of biasing our collective actions toward those yielding immediate relief to citizens who are currently suffering because of the recession.

An obvious critique of the Ramsey rule is that the prospective growth rate of consumption is uncertain. This uncertainty is at the heart of our collective decision problems related to sustainable development. For example, we have to decide whether to fight climate change long before knowing whether our descendants who will benefit will live in a new Stone Age or in the Nirvana

[6]As implied by the normative risk-free-rate puzzle, our estimated normative rate (approximately 4%) exceeds the estimated positive rate (1–2%) for risk-free projects.

[7]Arrow et al. (2013, 2014) provide an alternative discussion of this controversial issue.

of an economy with a world GDP orders of magnitude larger than its current level. This uncertainty is an intrinsic ingredient of the problem that the Ramsey rule overlooks.

It is intuitive that uncertainty surrounding the future should induce society to take more care of it, i.e., to reduce the discount rate. At the micro level, this intuition is founded on the concepts of precautionary saving and prudence. Keynes (1930) was the first to suggest that individuals want to save more when their future income is more uncertain, and Drèze & Modigliani (1972), Leland (1968), and Kimball (1990) showed that this approach is indeed optimal if the marginal utility $u'$ is convex. Under the standard specification with $u'(c) = c^{-\gamma}$, marginal utility is convex, and so uncertainty about future consumption should indeed reduce the efficient discount rate. The intensity of this precautionary motive should be increasing in the degree of convexity of $u'$, which is measured by the index of relative prudence $-cu'''/u''$, which equals $\gamma + 1$ in this power specification.

Introducing uncertainty into the Ramsey model adds a degree of flexibility in the calibration of Equation 2. The simplest and most classical specification assumes that $c_t$ is lognormally distributed, with $\ln(c_t/c_0) \sim N\left(\mu_t, \sigma_t^2\right)$ so that the expected average growth rate is equal to $Eg_t = t^{-1} \ln(Ec_t/c_0) = \left(\mu_t + 0.5\sigma_t^2\right)/t$.[8] Under this specification, Equation 2 can be rewritten as follows:

$$R_t = -\frac{1}{t} \ln E\left(\frac{c_t}{c_0}\right)^{-\gamma} = \gamma\left(Eg_t - 0.5(\gamma+1)\frac{\mathrm{Var}(\ln c_t/c_0)}{t}\right). \tag{4}$$

This extension of the Ramsey formula to an uncertain future (obtained by Gollier 2002) is intuitive. It tells us that the Ramsey rule should be adapted to an uncertain future by reducing the expected growth rate by half the product of the degree of prudence $\gamma + 1$ and the annualized variance $\sigma_t^2/t$ of log consumption. Hansen & Singleton (1983) examine the special case in which the growth of log consumption follows a Brownian motion with trend $\mu$ and volatility $\sigma$. In that case, both the expected average growth and the annualized variance of log consumption are independent of the maturity and are equal to $Eg_t = \overline{g} = \mu + 0.5\sigma^2$ and $\mathrm{Var}(\ln c_t/c_0)/t = \sigma^2$, respectively, so that Equation 4 can be rewritten as

$$R_t = \gamma\left(\overline{g} - 0.5(\gamma+1)\sigma^2\right) = \gamma\left(\mu - 0.5\gamma\sigma^2\right). \tag{5}$$

In this special case, the impact of uncertainty is to uniformly reduce the discount rate at all maturities by the same constant $0.5\gamma(\gamma+1)\sigma^2$. This result is rather disappointing for two reasons. First, this model cannot justify using a smaller discount rate for longer maturities, thus weighing against the controversial recommendation to use a decreasing term structure of discount rates. Second, the effect of uncertainty on the efficient discount rate is small. In the Western world, the volatility $\sigma$ of the growth rate of consumption per capita has been approximately 3% per year during the past century. Thus, with $\gamma = 2$, the prudence effect reduces the efficient discount rate by only $0.5 \times 2 \times 3 \times (0.03)^2 = 0.27\%$. The uncertainty affecting growth reduces the discount rate from 4% to 3.73%. This reduction cannot solve the normative risk-free-rate puzzle described in the previous section.

Barro (2006, 2009) claims that using the observed volatility of US economic growth over the past century is not the right way to represent the uncertainty that we face in the future. This method

[8]We use several times the following property, which is ubiquitous in the modern theory of finance: $x \sim N(\mu, \sigma^2)$ implies that $E\exp(ax) = \exp\left(a(\mu + 0.5a\sigma^2)\right)$.

fails to recognize the possibility of low-probability macro catastrophes of the order of magnitude that other countries experienced during the twentieth century. Germany, Japan, and France, for example, lost approximately 60% of their GDP during World War II. Monetary crises may also have dramatic effects, such as the 21% loss of GDP in Argentina between 1998 and 2002. Recognizing that the United States is not immune to this type of event reduces the expected growth rate and increases the uncertainty, so it reduces the efficient discount rates. Recognition of the plausibility of catastrophes may reconcile the positive and normative approaches.

## 5.2. Parameter Uncertainty and the Dismal Theorem

Gollier (2008, 2012) examines an alternative road to reconciliation. The result that the term structure of efficient discount rates is flat relies on the assumption that there is no persistence in the growth process. The specification leading to Equation 5 includes the assumption that the change in consumption in the year 2101 is unrelated to the situation prevailing in 2100. We believe that this is an unrealistic representation of our collective beliefs about the long-term-growth process. The growth process is likely to be sensitive to random shocks with persistent effects, such as nonmarginal innovations, geopolitical instabilities, and exhaustion and deterioration of natural assets. The history of civilizations provides numerous examples of persistent waves of growth and decline of human societies. The Brownian motion that is behind Equation 5 cannot describe such systemic uncertainty.

The persistence of shocks to economic growth generates a positive correlation in the time series of the growth process $\left(x_t = \ln(c_{t+1}/c_t)\right)_{t=0,1,\ldots}$. It has no effect on the short-term uncertainty measured by the variance of $x_0$ but magnifies the uncertainty affecting distant consumption levels measured by the variance of $\ln(c_t/c_0) = x_0 + x_1 + \ldots + x_{t-1}$. From Equation 4, persistence of shocks has no effect on the efficient short-term discount rate but reduces the discount rate to be used for long maturities. This result provides a strong argument in favor of using a decreasing term structure of risk-free discount rates. Gollier (2008, 2012) provides various illustrations of this result by considering growth processes entailing mean-reversion, two-regime Markov switches or parametric uncertainty.

Weitzman (2007b) was the first to note that, even if we assume that the economy evolves as a Brownian motion, we should recognize that there is a high degree of uncertainty about the long-term trend and volatility. Let us examine the case of an uncertain trend, which is a clear example of a positive statistical relationship between $x_t$ and $x_{t'}$: If the trend happens to be high, it will raise the mean of all future growth rates. As a result, the term structure of discount rates will decrease. The simplest case is when our beliefs about the trend are characterized by a normal distribution. Gollier (2008) shows that this assumption implies a linearly decreasing term structure, which implies that the efficient discount rate goes to minus infinity for superlong maturities. Informally, this result means that if we have a feasible option today to transfer a sure infinitesimal unit of consumption to the very distant future, we should do so at any cost. The intuition is that the impossibility of excluding a very negative growth trend yields such a terrible risk for distant generations that one should do everything possible to escape the risk that they will face zero consumption. (In the standard model, marginal utility goes to infinity when consumption goes to zero.)

One can eliminate this paradox if the set of plausible growth trends is bounded below. What should be the optimal short/long spread of the safe discount rates in that case? Gollier (2008, 2013) shows that the asymptotic value of the discount rate for maturities tending to infinity is equal to the one coming from Equation 5, where $\mu$ is replaced by its smallest plausible value. In **Figure 1**, we illustrate this theory by examining an economy with an index of inequality aversion $\gamma = 2$; a volatility $\sigma = 3\%$; and a mean growth of log consumption that is unknown, but with a mean of

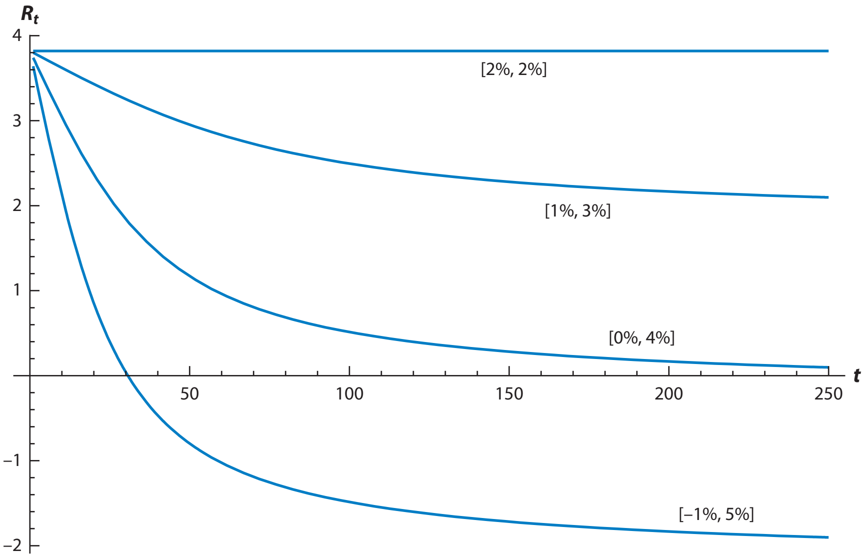


**Figure 1**

Term structure of efficient discount rates, assuming that $\gamma = 2$; $\sigma = 3\%$; and the trend of growth is $[a, b]$, which means that it is either $a$ or $b$ with equal probabilities.

$\mu_0 = 2\%$. When the trend $\mu$ is 2% for sure, the efficient discount rate is $R_t = 3.82\%$ for all maturities. In contrast, if the trend is either 0% or 4% with equal probabilities, this parameter uncertainty does not affect the rate at which short-term payoffs must be discounted, but it reduces the efficient discount rate for superlong maturities to $R_\infty = -0.18\%$.

Weitzman (2007b) considers a model in which the trend of the economy is known but its volatility is not. He argues that it is natural to represent our collective beliefs about this volatility by an inverse gamma distribution, and he shows that the efficient safe discount rate then goes to minus infinity for all maturities. Weitzman (2009) generalizes this result in a controversial dismal theorem. The theorem states that, under this fundamental uncertainty about the volatility of economic growth, we should do everything we can to transfer infinitesimally small, sure amounts of consumption to each future generation. This result comes again from the assumption of an unbounded marginal utility at zero and the existence of a fat left tail in the distribution of future consumption.[9] Horowitz & Lange (2009), Karp (2009), Nordhaus (2011), and Millner (2013) criticize the dismal theorem for various reasons; for example, marginal utility is likely to be bounded, and the unbounded support of plausible volatility is unrealistic.

## 5.3. The Weitzman-Gollier Controversy

Weitzman visited the University of Toulouse in 1996. On this occasion, Gollier presented an initial argument for a decreasing term structure that was based on the extended Ramsey rule (Equation 4) combined with the assumption of an index of inequality aversion $-cu''(c)/u'(c)$ that is decreasing rather than constant (Gollier 2002). Weitzman reacted by providing a much simpler argument. This interaction led to the publication of two controversial papers by Weitzman (1998, 2001). At first sight, this argument looks quite distant from the theory presented above. The argument goes

[9]In this case, log consumption has a Student's $t$-distribution rather than a normal distribution.

as follows. Let us reconsider a risk-free project with immediate cost $C$ and future benefit $B$ occurring in $t$ years. Its NPV equals $-C + B\exp(-R_t t)$. Suppose now that the discount rate is uncertain, i.e., that $R_t$ is a random variable. In that case, which decision criterion should be applied is not clear. Weitzman (1998, 2001) assumes that the project should be implemented if and only if the expected NPV is positive. This approach is equivalent to applying the standard NPV rule with a certainty-equivalent discount rate $R_t^W$ that takes the following form:

$$R_t^W = -\frac{1}{t}\ln Ee^{-R_t t}. \tag{6}$$

Assuming that the random variable $R_t$ is independent of $t$, Weitzman (1998, 2001) shows that the certainty-equivalent rate $R_t^W$ decreases from the expected rate $ER_t$ to its smallest plausible value when the maturity goes from zero to infinity.

Various interpretations of this model have been proposed. Weitzman (2001) posits that individuals use heterogeneous $R_t$. This paper does not make clear whether this heterogeneity comes from differences in preferences or from beliefs about future growth. If the problem is to aggregate heterogeneous beliefs, it will be important to know whether this heterogeneity comes from asymmetric information or whether people agree to disagree on their fundamental beliefs about the future of civilization.[10] If the problem is to aggregate heterogeneous preferences, a better aggregate rule may be to use the preferences of the median voter (see Weitzman 2013). Weitzman (2001) also assumes that the sample of 2,160 values of $R_t$ coming from a questionnaire sent indiscriminately to academic economists provides a valid estimate of the collective uncertainty we face about the "true" discount rate to calibrate the certainty-equivalent equation.

Weitzman (1998) considers a positive approach so that $R_t$ is interpreted as the market long interest rate, or the average opportunity cost of capital that will prevail over the period $[0, t]$. Newell & Pizer (2003, 2004) and Groom et al. (2007) calibrate this model by using long time series of interest rates in different countries. This positive interpretation is also problematic. It ignores an old debate in finance theory about how long forward interest rates should be determined when future short-term interest rates are uncertain. If one interprets $R_t$ in Equation 6 as the future short-term rate, which is assumed to be constant through time but unknown today, then this equation is nothing other than the expectations hypothesis of the term structure of interest rates that has existed in the literature since at least Macaulay (1938). This hypothesis has been highly controversial. To quote Froot (1989), "if the attractiveness of an economic hypothesis is measured by the number of papers which statistically reject it, the expectations theory of the term structure is a knockout." Surely one reason for the empirical failure of this hypothesis is that it assumes investors are risk neutral: When comparing the strategy to purchase a long bond with a strategy of rolling over investments in short-term bonds yielding a reinvestment risk, investors just compare the expected final return.

In line with earlier work by Pazner & Razin (1975), Gollier (2004) provides another critique that led to the so-called Weitzman-Gollier puzzle. Under certainty, the NPV rule is exactly equivalent to the net future value (NFV) rule, which states that one should invest in the project if and only if its NFV $-Ce^{R_t t} + B$ is positive. Obviously, the NFV and the NPV have the same sign, so they lead to the same final decision. The logic of the NPV is to transfer the future benefit $B$ to the present; the logic of the NFV is to transfer the current cost $C$ to the future. Although the two rules

[10] Freeman & Groom (2010) reexamine various interpretations and calibrations of the argument in Weitzman (2001).

are equivalent if the interest rate $R_t$ is certain, they are not if $R_t$ is uncertain. The criterion to invest if and only if the expected NFV of the project is positive is equivalent to the standard NPV approach with a discount rate $R_t^G$ defined as follows:

$$R_t^G = \frac{1}{t}\ln Ee^{R_t t}. \tag{7}$$

It is easy to show that these alternative certainty-equivalent discount rates have a term structure that is increasing, going from $ER_t$ up to the largest plausible $R_t$ for maturities going from zero to infinity. So we have a puzzle because there is no a priori reason to prefer one approach over the other. As Gollier (2004) explains, his aim is not to suggest that one should use the NFV approach to long-term discounting but to demonstrate that Weitzman (1998, 2001) fails to provide a convincing economic argument.

This controversy led to an active debate. Hepburn & Groom (2007) generalize the expected NPV/NFV analysis described above by showing that one can consider evaluation dates other than zero (for NPV) or $t$ (for NFV), each choice yielding a different term structure. Buchholz & Schumacher (2008) introduce risk aversion to the analysis and define the certainty-equivalent discount rate accordingly, but their recommendation remains sensitive to the date at which costs and benefits are borne.

In a series of papers (Gollier 2009, 2010; Gollier & Weitzman 2010; Weitzman 2010), Gollier and Weitzman converge to the conclusion that both risk aversion and the optimization of the allocation of the net benefits through time must be introduced into the picture to solve the puzzle. Suppose that the representative agent must decide whether to invest in the safe project at some date $t = -1$, immediately before learning what interest rate $R_t$ will prevail during the investment period $[0, t]$. Ex post (at $t = 0$), the agent will optimally determine her consumption-saving plan so that the state-dependent optimality condition $\exp(-R_t t) = u'(c_t)/u'(c_0)$ will prevail. This scenario implies that consumption growth rates are characterized by extreme persistence. In fact, under the specification of Weitzman (1998, 2001) and of all the papers that contributed to the controversy, the shock to consumption growth is permanent! If the agent contemplates consuming the net benefit at date zero, investing ex ante is optimal if $A = E\left[\left(-C + B\exp(-R_t t)\right)u'(c_0)\right]$ is positive. Alternatively, if the agent contemplates consuming the net benefit at the termination date $t$, investing ex ante is optimal if $B = E\left[\left(-C\exp(R_t t) + B\right)u'(c_t)\right]$ is positive. But the optimality condition makes immediately apparent that the two conditions are exactly the same: $A = B$. This observation solves the puzzle. In the special case with a logarithmic utility function, the optimal saving is independent of the interest rate. In this special case, $c_0$ is thus certain. In that case, condition $A \geq 0$ corresponds exactly to the Weitzman (1998, 2001) expected NPV rule, as shown by Gollier (2009). More generally, Gollier (2009) shows that the term structure of discount rates is decreasing and that the extended Ramsey rule (Equation 4) is compatible with this approach. Traeger (2013) provides the same conclusion, together with other insights.[11]

We conclude that the recommendation of a decreasing term structure initially made by Weitzman (1998) is correct, but for a different reason. His result is determined by the unrealistic assumption that shocks on the return to capital are permanent, and hence so are the induced shocks on consumption growth. If Weitzman would have assumed purely transitory shocks on the return of capital, then shocks on consumption growth would have been serially uncorrelated. In that case, we know from Section 5 that the term structure of the efficient discount rate would be perfectly flat.

[11]Freeman (2010) examines a similar model, but with Epstein-Zin preferences. He obtains qualitatively similar conclusions by assuming risk neutrality but aversion to consumption fluctuations.

Our conclusion on this controversy is twofold. First, as shown in Section 5, what matters for determining whether smaller rates should be used for longer maturities is the existence of a positive statistical relation in the time series of growth rates of consumption. This analysis provides a solid economic foundation and intuition for Weitzman's (1998) policy recommendation. Second, because of the absence of realism in Weitzman's assumption about the permanency of shocks on interest rates, it is not appropriate to use his famous rule (Equation 6) to recommend a specific term structure, as did the UK government, for example (HM Treasury 2003). Instead, we must go through the hard work of describing the stochastic process of the long-term-growth process, potentially with parametric uncertainty.

## 6. RELATIVE PRICES

Up to this point, we treat consumption at date $t$ as a single good $c_t$ and well-being as a function of consumption $u(c_t)$. In fact, people consume a diversity of goods and services, and the composition of consumption changes over time. Especially over long periods, the basket of goods that are consumed can shift dramatically; for example, most of the goods currently used for transportation, communication, and computation did not exist 100 years ago.

In addition to market goods, well-being depends on nonmarket goods such as health and ecosystem services. These goods are not included in conventional economic measures but are critical to well-being. Valuing the contribution of ecosystem services to well-being is notoriously difficult (Costanza et al. 1997, Bockstael et al. 2000, Kling et al. 2012), but the contribution of health improvements is more amenable to quantification. Estimates suggest that the value of improvements in life expectancy (not including improvements in health quality) over the twentieth century is comparable to the value of increased consumption of market goods over that period (Nordhaus 2003, Murphy & Topel 2006).

Relative prices of market goods and nonmarket goods can change dramatically. The prices of novel popular goods fall from infinite before they are introduced to affordable levels. Even for ancient goods and services, technological innovation can produce huge price changes. For example, Nordhaus (1997) estimates that the price of domestic lighting relative to a conventional consumer price index decreased by a factor of more than 1,000 over the period 1800–1992.

When real prices change, the effective discount rate varies by good. If the relative price of a good increases at a rate $d$ compared with the numéraire, which is discounted at rate $r$, then the effective rate of discount on the good is $r - d$. In the context of environmental quality, this effective rate is described as the ecological discount rate (Guesnerie 2004; Hoel & Sterner 2007; Gollier 2010, 2012). It measures the rate of substitution between the good at different dates. In health economics, the long-standing debate about whether future health and costs should be discounted at the same rate in cost-effectiveness analysis is essentially a question of whether the relative values of health and the goods that compose the cost term are constant (Gravelle & Smith 2001, Hammitt 2012).

The relative values of ecosystem services and of health seem likely to rise relative to those of material goods as the production and consumption of material goods increase. Because the value of mortality risk increases with real income, risk reductions (statistical lives saved) are effectively discounted at a lower rate than are other benefits. In its evaluation of the rules limiting use of stratospheric ozone–depleting substances, EPA included reductions in skin-cancer mortality risk to cohorts of US citizens ranging up to those born in 2075 (Hammitt 1997). In its base case, EPA assumed that the value of skin-cancer mortality risk would grow at the same rate as real income (assumed to be 1.7%); discounting the monetary value at the base-case discount rate of 2% yields an effective discount rate for mortality risk of 0.3%. In an alternative case, EPA assumed that the value of mortality risk would grow at twice the rate of income growth (3.4%) and discounted benefits at only 1%, yielding an effective discount rate of –2.4% (Hammitt 1997).

There are two equivalent methods for valuing future goods when relative prices may change. The standard approach is to estimate the monetary value of the increment to the good at the future date and then discount this value at the same rate as for all other monetary values. The alternative approach, suggested by Malinvaud (1953), is to estimate the present increment to the good that produces the same welfare effect as the specified future increment by using the ecological discount rate (for that good) and to value the present increment by using its current monetary value.

Let us generalize Equation 1 to a case in which utility $u(c_{1t}, c_{2t})$ depends on two goods, an aggregate consumption good $c_{1t}$ and another good or service such as health or environmental quality $c_{2t}$ (for concreteness we refer to $c_2$ as environment). We obtain

$$W = u(c_{10}, c_{20}) + Eu(c_{1t}, c_{2t}), \tag{8}$$

where the expectation reflects uncertainty about the future values $c_{1t}$ and $c_{2t}$ (and $\delta = 0$).

Consider a marginal project that would reduce current consumption $c_{10}$ by $\varepsilon \exp(-r_{1t}t)$ and increase consumption by a sure amount $\varepsilon$ at date $t$, with no effect on the environment. The economic discount rate is defined as the rate $r_1$ such that this project has no effect on $W$; it is given by

$$r_{1t} = -\frac{1}{t}\ln\frac{EU_1(c_{1t}, c_{2t})}{U_1(c_{10}, c_{20})}, \tag{9}$$

where $U_1(c_{1t}, c_{2t})$ is the derivative of $U$ with respect to its first argument. In contrast to the case with only one good (Equation 3), the economic discount rate depends on the evolution of the environment between dates zero and $t$ (unless the marginal utility of consumption is independent of environment, $U_{12} = 0$).

Now consider a marginal environmental project that increases $c_{2t}$ by a sure amount $\varepsilon$. The standard method by which to value this improvement is to calculate the monetary value at $t$ by multiplying $\varepsilon$ by the future marginal rate of substitution between consumption and the environment,

$$v_t = -\left.\frac{dc_{1t}}{dc_{2t}}\right|_U = \frac{U_2(c_{1t}, c_{2t})}{U_1(c_{1t}, c_{2t})}. \tag{10}$$

The resulting monetary value at date $t$ is then discounted to the present by using the economic discount rate $r_{1t}$. A complication is that, seen from date zero, the future rate of substitution between consumption and the environment is uncertain.

The alternative approach is to calculate the ecological discount rate. Consider a marginal project that increases $c_{2t}$ by a sure amount $\varepsilon$ and reduces $c_{20}$ by $\varepsilon \exp(-r_{2t}t)$. This project has no effect on welfare if $r_{2t}$ is the ecological discount rate over the period,

$$r_{2t} = -\frac{1}{t}\ln\frac{EU_2(c_{1t}, c_{2t})}{U_2(c_{10}, c_{20})}. \tag{11}$$

The monetary value of this environmental increment is obtained by multiplying by the current rate of substitution between consumption and environment $v_0$ (Equation 10).

Both the economic and ecological discount rates depend on the growth in consumption and in environment over the period and on uncertainty about these growth rates. Let us examine the determinants of the ecological discount rate.

If $U$ is concave in $c_2$, then the ecological discount rate is higher if the environment improves (and is smaller if the environment worsens) over the period zero to $t$, because of diminishing marginal utility. This effect is analogous to the effect of economic growth on the discount rate in the single-good case.

The effect of uncertainty about the change in $c_2$ depends on the curvature of $U_2$. If $U_2$ is convex ($U_{22} > 0$), then uncertainty about the change in $c_2$ decreases the ecological discount rate. This effect is analogous to the precautionary effect in the extended Ramsey rule (Equation 4); i.e., one should do more to protect the future environment if its state is more uncertain.

The effect of growth in consumption $c_1$ on the ecological discount rate depends on whether consumption and environment are complements or substitutes. When $c_2$ represents environment, both possibilities seem plausible: Stronger technological growth suggests less reliance on environment for production, but its value for other uses (or nonuse value) may increase. If consumption and environment are substitutes ($U_{12} < 0$), stronger consumption growth decreases $U_2$ and hence increases the ecological discount rate. When $c_2$ represents health, a complementary relationship seems most plausible ($U_{12} > 0$) (Hammitt 2013). In this case, stronger consumption growth increases the marginal utility of health and decreases the health discount rate.

The effect of uncertainty about consumption growth on the ecological discount rate depends on the sign of $U_{211}$. If this derivative is positive, $U_2$ is convex in $c_{1t}$, and so adding a zero-mean risk to $c_{1t}$ increases $EU_2(c_{1t}, c_{2t})$. The condition that $U_{211} > 0$ is termed cross-prudence in consumption. It can be interpreted as the case in which harms to consumption and environment are mutually aggravating in the following sense (Eeckhoudt et al. 2007): Consider an arbitrary pair $(c_{1t}, c_{2t})$, a sure loss to environment $-l$, and a zero-mean risk to consumption $\varepsilon$. Lottery A is a fifty-fifty chance to face the consumption risk or the sure environmental loss; lottery B is a fifty-fifty chance to face the consumption risk and the environmental loss simultaneously or to face neither. Cross-prudence in consumption implies a preference for A over B, i.e., to face one harm for sure rather than risk facing both harms together.

Finally, the ecological discount rate also depends on the dependence between $c_{1t}$ and $c_{2t}$. If consumption and environment depend on each other and $U$ is cross-prudent in environment ($U_{221} > 0$), then a positive dependence of growth in consumption and environment increases $EU_2(c_{1t}, c_{2t})$ and hence decreases the ecological discount rate. Cross-prudence in environment implies that an environmental risk and a consumption loss are mutually aggravating, and hence one prefers a lottery with a fifty-fifty chance to face one or the other over a lottery with equal chances of facing both harms simultaneously or facing neither.

A parallel analysis shows that the economic discount rate $r_{1t}$ also depends on the same factors: the changes in environmental quality and in consumption, uncertainty about the changes, and the dependence between them. The point that the economic discount rate depends on future environmental quality and other goods and services, and on uncertainty about them, is not widely appreciated.

## 7. RISKY PROJECTS

The arguments developed above are about the level and term structure of the risk-free discount rate, i.e., of the rate at which safe projects should be discounted. Most of these arguments have been developed in the context of climate change. They have also served in public reports in the United Kingdom (HM Treasury 2003), the United States (OMB 2003), and France (Lebègue 2005) to justify a unique all-purpose rate schedule to discount the expected net benefits of public policies. But most investment projects, particularly those involving the distant future, have uncertain future benefits and costs.

At least three classical theories provide recommendations about how to treat the riskiness of future costs and benefits in the evaluation of projects and policies. The most basic theory is based on the so-called Arrow-Lind theorem (Arrow & Lind 1970). This theorem states that when an investment project yields net benefits that are independent of the systematic risk of the economy, these benefits should be discounted at the risk-free rate. The intuition is that risk can be spread among a large population of stakeholders. Because the risk premium goes to zero as the square of the size of risk in the EU framework, this dissemination virtually eliminates the risk. Although Arrow & Lind (1970) recognize in their paper that their result holds in theory only for idiosyncratic risks, they intend to apply it to a much broader domain. They claim that "the government undertakes a wide range of public investments and it appears reasonable to assume that their returns are independent." This is a logical mistake because the net benefits of most public projects are affected by some common factors, such as global economic activity. The fallacious interpretation of the Arrow-Lind theorem overlooks the cost of risk in public projects. This interpretation is a problem because the valuation of projects in the private sector puts a high premium on risky projects, as can be seen by the large difference in the cost of safe capital and risky capital (**Table 1**). By reducing the discount rate to evaluate risky projects in the public sphere, this fallacy has contributed to the expansion of the public sector in many Western countries over the past four decades. In the United States, this problem may help to explain why a high discount rate of 7% has been selected and maintained since 1992 (revised downward from 10% in the 1980s).

An alternative method to treat the riskiness of future benefits is clearly stated in the second report of the IPCC (1995): "Most economists believe that considerations of risk can be treated by converting outcomes into certainty equivalents, amounts that reflect the degree of risk in an investment, and discounting these certainty equivalents." By converting uncertain benefits into certainty-equivalent benefits, the evaluator puts himself back into the framework examined in this article, so risk-free discount rates can be used. The difficulty comes, of course, from the specification of the certainty-equivalent operator. Let us consider an agent who will consume $c_t$ at date $t$ and who contemplates a marginal outcome $B_t$ occurring at that time. In the EU model, this marginal outcome has an effect on welfare that is equivalent to receiving the sure amount $P_t = E\left[B_t u'(c_t)\right]/Eu'(c_t)$ at date $t$. Thus, $P_t$ is the certainty-equivalent benefit that should be discounted at the risk-free rate $R_t$ described above. Observe that when $c_t$ and $B_t$ are independent, we immediately obtain that the certainty equivalent of $B_t$ is just the expectation of $B_t$. This is the Arrow-Lind theorem. Suppose alternatively that the future benefit $B_t$ is linked to future consumption through the following statistical relation: $B_t = \xi_t c_t^{\beta}$, where $\beta$ is a scalar and $\xi_t$ is a noise independent of $c_t$, with $E\xi_t = 1$. Observe that $\beta$ is a measure of the degree of correlation between the benefit of the project and economic growth. If we assume, as in the benchmark model that generated the extended Ramsey rule (Equation 5), that relative inequality aversion is constant and that log consumption follows a Brownian motion with drift $\mu$ and volatility $\sigma$, then the certainty-equivalent outcome at date $t$ can be rewritten as follows:

$$\begin{aligned} P_t &= \frac{EB_t u'(c_t)}{EB_t Eu'(c_t)} EB_t = \frac{Ee^{(\beta-\gamma)\ln c_t}}{Ee^{\beta \ln c_t} Ee^{-\gamma \ln c_t}} EB_t \\ &= \frac{e^{(\beta-\gamma)(\mu+0.5(\beta-\gamma)\sigma^2)t}}{e^{\beta(\mu+0.5\beta\sigma^2)t} e^{-\gamma(\mu-0.5\gamma\sigma^2)t}} EB_t = e^{-\beta\gamma\sigma^2 t} EB_t. \end{aligned} \tag{12}$$

The certainty-equivalent outcome is proportional to the expected outcome, and the factor of proportionality is exponentially decreasing at rate $\beta\pi$, where $\pi = \gamma\sigma^2$ is termed the systematic risk premium.

This observation provides a nice introduction to the third method to integrate the risky payoffs into the evaluation. Under the standard specification presented above, we know that the risk-free discount rate $R_t$ is equal to the constant $R = \gamma\mu - 0.5\gamma^2\sigma^2$. Thus, the outcome occurring at date $t$

has a present value $P_t \exp(-Rt)$, which is equivalent to $EB_t \exp-(R+\beta\pi)t$. This means that the expected value of the uncertain outcome $B_t$ is discounted at the constant rate $\rho = R + \beta\pi$. Therefore, the third method consists of discounting the flow of expected payoffs at a rate $\rho$ that is adjusted for risk by adding a risk premium $\beta\pi$ to the risk-free rate $R$. This is the classical result of the consumption-based capital asset pricing model (CCAPM) (Rubinstein 1976, Lucas 1978, Breeden 1979). These risk-adjusted discount rates are commonly applied in the private sector. The evaluation of investment projects is based on their CCAPM-betas, which are typically ordinary-least-squares estimates of the equation $\ln \dot{B}_t = \alpha + \beta \ln \dot{c}_t + \varepsilon_t$.

Although the certainty-equivalent method is theoretically equivalent to the CCAPM, the latter has emerged as the common language and practice of economists over the past four decades. The CCAPM has mostly failed to explain how financial markets value risk. For example, the equity-premium puzzle (Mehra & Prescott 1985) shows that the CCAPM predicts a systematic risk premium of $\pi = 2\times(3\%)^2 = 0.18\%$ under the assumption of $\gamma = 2$ and $\sigma = 3\%$, which is an order of magnitude smaller than the observed risk premium of assets with $\beta = 1$. On a more normative ground, considering such a small systematic risk premium looks very counterintuitive because doing so makes the riskiness of projects nearly irrelevant to their evaluation. One possible resolution of the puzzle is offered by Barro's (2006, 2009) observation that calibrating a pricing formula on a volatility of consumption growth estimated from past data overlooks the possibility of large macro catastrophes. Barro shows that introducing low-probability catastrophes into the model can raise the systematic risk premium to 3–5% per year.[12]

What can be said about the term structure of the risk-adjusted discount rate $\rho$? We show above that this term structure is flat under the standard assumption of a random walk for the economic growth rate. The persistence of shocks to the consumption growth rate changes this pattern by magnifying the long-term risk, which magnifies the long-term precautionary effect and so reduces the long-term risk-free discount rate. But persistence has the opposite effect on the term structure of the systematic risk premium, as Gollier (2013) shows. Magnifying the risk on distant consumption magnifies the risk on the distant payoff of all projects with a positive beta. By risk aversion, this effect magnifies the risk premium for long maturities. The larger the beta of the project, the stronger is this countervailing effect. If the beta of the project is large enough, the net effect on the slope of the term structure of the risk-adjusted discount rate is positive.

The risk adjustment of discount rates is not common practice in public sectors; France and Norway are the only exceptions of which we are aware. This inefficiency is likely due to the difficulty of estimating betas. Let us illustrate this point with climate change. What is the beta of projects whose main objective is to reduce emissions of greenhouse gases? Some authors suggest that the climate beta is negative, i.e., that the future benefits of fighting climate change will be largest in states where future consumption will be smallest. Sandsmark & Vennemo (2007), Weitzman (2012), and Murphy & Topel (2013) develop the idea that if the main source of long-term uncertainty is the climate sensitivity, then a high climate sensitivity will at the same time reduce consumption and raise the benefit of reducing emissions. Under that story, the climate beta is negative, and reducing emissions has an insurance benefit that should be incorporated into a discount rate that is lower than the risk-free rate. In contrast, Nordhaus (2011), who

[12]In his report to the French government, Gollier (2011) uses this argument to recommend a systematic risk premium of approximately $\pi = 3\%$. Implementation of this new evaluation rule has been controversial in France because the application of the rule tends to raise the public discount rate. Industries with risky projects to be publicly financed are currently lobbying to use the certainty-equivalent approach. This development is interesting because, as explained in the text, the certainty-equivalent approach calibrated to historical data is equivalent to using a systematic risk premium that is an order of magnitude smaller than the official 3%. In short, the lobbies want to go back to the old, fallacious Arrow-Lind framework.

uses Monte-Carlo simulation of his integrated-assessment model, reaches the conclusion of a positive climate beta: "Those states in which the global temperature increase is particularly high are also ones in which we are on average richer in the future." This result is easy to understand: If the main source of long-term uncertainty is economic growth, then a high growth rate yields at the same time high consumption and a high concentration of $CO_2$ in the atmosphere, yielding high marginal climate damage. Gollier (2013) suggests that the relative uncertainty affecting long-term economic growth is much larger than the uncertainty affecting climate sensitivity, which implies that the net effect is a climate beta that is positive and larger than unity. He concludes that the term structure of the climate discount rates is increasing, from 3.5% for short maturities to up to 4.5% for long ones. But this conclusion is clearly very exploratory and controversial.

## 8. CONCLUSION

The discount rate is a measure of the relative importance of consequences occurring at different points in time. The estimated net benefits of projects whose benefits and costs are widely separated in time, such as climate mitigation, are highly sensitive to this rate. And yet there is no consensus on the correct discount rate to use for evaluating public projects, nor is there a single discount rate.

As we discuss here, the appropriate discount rate is likely to vary with the maturity, and so it is more useful to think of a schedule of discount rates for different maturities. The discount rate for a project with a specified maturity depends on the uncertainty about the state of the world in which the future benefits will occur and on the uncertainty about what the benefits will be. Uncertainty about the state of the world may be analyzed through the extended Ramsey rule, which highlights that shocks to growth are important for discounting only if they are persistent; transient shocks have little import. Uncertainty about the project's benefits implies the need to account for benefits by using certainty equivalents or a risk-adjusted discount rate; in each case, the adjustment for uncertainty depends on the degree of risk aversion and on the relationship between the project consequences and aggregate economic growth. Recognition that human well-being depends on a variety of market goods and nonmarket goods, and that the relative prices of these goods may shift dramatically over time, leads to the conclusions that different goods should be discounted at different rates and that these rates are interdependent. For example, both the economic discount rate for consumption and the ecological discount rate for environmental consequences depend on the changes in both consumption and environment and on uncertainty about these changes.

Finally, there are both positive and normative interpretations of the discount rate. From a positive perspective, the discount rate is a price that is observable for short- and medium-term maturities (up to approximately 30 years), although of course only the nominal rate is observable; because of uncertainty about inflation, the real rate is not known ex ante. From a normative perspective, the discount rate is a measure of an intertemporal rate of substitution, which may be intrapersonal or intergenerational, depending on the context. The literature has yet to resolve the discrepancies between the positive perspective and the normative perspective. The equity-premium puzzle highlights the large discrepancy between average returns to equity and risk-free sovereign debt; the normative risk-free-rate puzzle highlights the disparity between market interest rates and realized economic growth, which suggests that past generations have underestimated the growth that their investment would produce. For long-term and risky projects like climate mitigation, both puzzles contribute to uncertainty about the appropriate discount rate.

## DISCLOSURE STATEMENT

The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

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