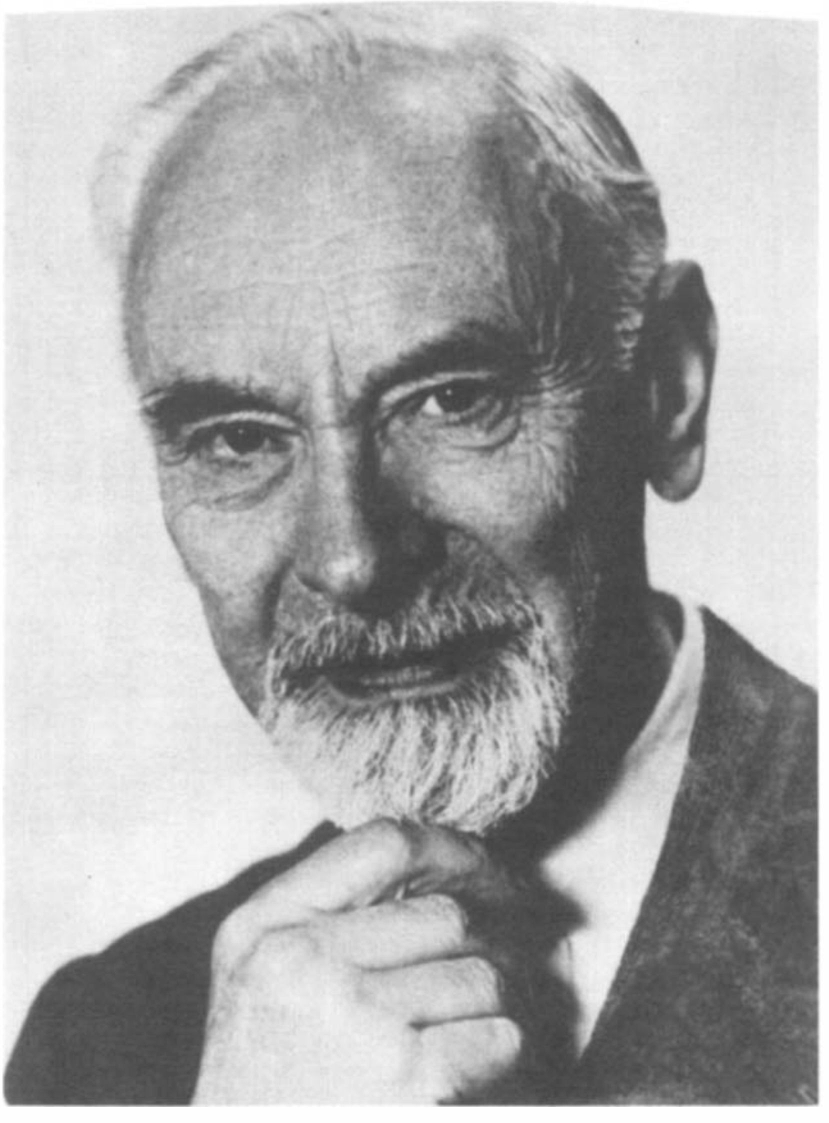

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LUDWIG PRANDTL AND HIS KAISER-WILHELM-INSTITUT

K. Oswatitsch

Institut für Strömungslehre und Wärmeübertragung, Technische Universität, Wien, Austria

K. Wieghardt

Institut für Schiffbau, Universität Hamburg, Hamburg, West Germany

1. INTRODUCTION

To describe the work of Ludwig Prandtl, we have had to restrict ourselves to activities under his direct influence and responsibility, i.e. his own publications (Prandtl 1961), and those of his immediate coworkers and pupils as long as they stayed in Göttingen. Some of these individuals later on became well known for their own work elsewhere in Germany or in various other countries, especially after the war. To cite Flügge-Lotz & Flügge (1973), "The seeds sown by Prandtl have sprouted in many places, and there are now many 'second-growth' Göttingers who do not even know that they are." Even with the restriction to Göttingen papers, we could not strive for completeness, although we have tried to depict the variety of Prandtl's interests. In particular, we have had to omit all the extensive work done at the *Aerodynamische Versuchsanstalt* (AVA), directed by A. Betz since 1937, although there too worked many members of the Göttingen school who have won for themselves a name in aerodynamics. A short history of the AVA from 1907 to 1982 is given by Wuest (1982).

2. PRANDTL'S FIRST 30 YEARS

Ludwig Prandtl was born on 4 February 1875 in Freising, north of Munich, as son of a professor at the Agricultural Central School there.

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To get a feeling for this time, a few events of the year 1875 may be recalled: Hans Christian Andersen and Eduard Mörike died, Thomas Mann and Albert Schweitzer were born, Mark Twain's *The Adventures of Tom Sawyer* came out, there were first performances of Bizet's *Carmen* at Paris and of Tchaikovsky's piano Concerto No. 1 at Boston. Last but not least, London's sewerage system was completed (Grun 1975).

As to science: at the same time a young student, M. Planck (1858–1947), asked Ph. von Jolly (1809–84) whether he should study physics or rather music, in which he was also interested. The answer he received was that, in general, physics is a completed science, although there are little niches here and there where further work might be useful. Yet, as we know now, Planck's quantum theory came in 1900, Einstein's relativity in 1905, and, which is of interest here, Prandtl's boundary layer in 1904 after the first trial flight of Zeppelin in 1900 and the first successful flight of a powered airplane by Orville and Wilbur Wright in 1903.

Starting in 1894, Prandtl studied mechanical engineering at the *Technische Hochschule* (TH) in Munich and became an assistant of A. Föppl (1854–1924), who was also his doctoral father. His thesis was "On Tilting Phenomena, an Example of Unstable Elastic Equilibrium" (Prandtl 1900), and his graduation to Dr. phil. took place at the University of Munich on 29 January 1900. (The degrees Dipl.-Ing. and Dr.-Ing. had only just been introduced at the TH in 1899.)

He then worked for less than two years at a factory in Nuremberg (later called MAN). One of his tasks there was to improve the suction of sawdust from a wood-cutting machine; he failed because he used a diffuser with too large an opening angle. However, this little mishap haunted him for years, even after he had left the firm, until at last he "invented" boundary layers and separation in 1904 (Prandtl 1904a).

Nevertheless, before that, he was appointed professor at the TH in Hanover on 1 October 1901 at the age of 26. This might have been the summit of his career, but actually it was just the beginning. For in the meantime the famous mathematician Felix Klein (1849–1925) at Göttingen University had become interested in Prandtl. For many years to come he played a very important role in the life of Prandtl, who simply said of him, "He was my fate" and kept in his office a large portrait of Klein. To understand this influence of Klein, some peculiarities of the German system of higher education—especially for engineers—in thc nineteenth century should be outlined first (Manegold 1970).

A neat example of the deep roots of the "inferiority complex" of many engineers is to be seen already at the very beginning of engineering science. Probably the first chair in this field was established at Leyden University in 1600, a chair for fortification, on the recommendation of the Quartermaster-General Simon Stevin, with the stipulation that the lectures be given in Dutch because later on these students would have to deal mainly with humble workers. Only after exactly 100 years, in 1700, professor allowed to teach in Latin, just like all his other colleagues.

Later on, German philosophers like Schelling and Schleiermacher of the early nineteenth century still had a following in Prandtl's lifetime. They maintained that if universities had to cultivate natural sciences besides philosophy at all, then it must be done only for their own sake without any practical or even technical purpose—no bread science! As Goethe said to Eckermann in 1829, "Whereas we Germans are drudging with the solution of philosophical problems, the English laugh at us and win the world with their common sense." Klein's opinion was that the real task of natural science is not to explain nature, a goal that would never be reached in the last resort, but rather to master nature.

Moreover, the Industrial Revolution had come to Germany somewhat belatedly. It is true that polytechnical schools or institutes were founded after the model of Monge's École Polytechnique in Paris (1794), first in Prague (1806),[1] Vienna, and Graz (1815), and then in Karlsruhe (1832) and many other German-speaking states. For some decades the Viennese school had the best reputation, and even young men of noble birth did not feel ashamed of studying there. Only at the end of the century did all these schools become THs and so, at least officially, of the same rank as the older universities. On the other hand, many THs in Central Europe and Scandinavia had the same structure as the German ones, and there the language mostly used for publications on engineering sciences was German (e.g. Oseen 1927), becoming English after the Second World War.

At the end of the nineteenth century, Klein had already been fighting for many years, and in grand style, against the then common polarization between practical engineers and noble thinkers at universities. At first he had tried to reach a general close cooperation between THs and universities. Especially after his second visit to the United States during the world exhibition in Chicago (1893), he was much impressed by the superiority of American technology and of the pragmatic teaching at the Massachusetts Institute of Technology. His idea was—roughly—that the "front-line officers" for industry should come from THs and their "general staff" from universities. But both groups protested violently. The engineers again felt subordinated, and the war cry of the others was "Against materialism and Americanism!"

Hence, Klein pursued the less ambitious idea that at least his Göttingen

[1] The first German universities were Prague (1348) and Vienna (1365) after the Sorbonne in Paris (1255).

University should have a few chairs for technical applied sciences as well as for pure sciences—in their mutual interest. This plan was supported strongly in Berlin by *Ministerialdirektor* F. Althoff. (At this time all education was directed from the center in Berlin. The Kaiser himself signed the letter of appointment for each professor.) To find financial support, professors and interested directors of industry founded in 1898 the *Göttinger Vereinigung zur Förderung der angewandten Physik und Mathematik*.
director of *Farbwerke Elberfeld*,
1975).

As early as 1900, Klein had made inquiries about Prandtl, as shown in still existing letters from A. Föppl, A. Stodola, and the director of MAN (Rotta 1985). Meanwhile, Prandtl had become a full professor in Hanover, whereas Klein could offer him only an *Extraordinariat*,
associate professorship without faculty membership, although the difference in salary was to be paid by the *Göttinger Vereinigung*.
1904, yet still before Prandtl's lecture in Heidelberg on boundary layers, Klein succeeded in enticing Prandtl to come to Göttingen against the wishes of A. Föppl, his doctoral father, who later became his father-in-law in 1909. In due course, in 1907, Prandtl became full professor again, and his chair for mechanics was the only one at a German university for almost a half-century. And so the quiet university town of Göttingen later became the center of aerodynamic research in Germany.

3. PRANDTL'S FIRST TWO DECADES AT GÖTTINGEN

In Hanover, Prandtl met the mathematician C. Runge (1856–1927), who sometimes was called the father of numerics. Soon they became friends, and together they went to Göttingen in 1904/5, where they combined their two institutes into one *Institut für angewandte Mathemathik und Mechanik*.

It was the time of the Zeppelin and Parseval airships, and in 1906 the *Studiengesellschaft für Motorluftschiffahrt* was founded in Berlin. Prandtl proposed the erection of a test station corresponding to already existing ship model basins. In particular, he designed two wind tunnels, one of the open Eiffel type and the other a return-circuit tunnel, but still with closed test section. The latter tunnel was built in 1908 with a borrowed 30-HP motor in a tiny house on rented ground in Göttingen.

In those first years at Göttingen, Prandtl (1906) published the first estimate of the thickness of a shock wave. In his thesis, Magin (1908) showed by schlieren photographs of the flow in a Laval nozzle the limited upstream influence of supersonic flow, which was not taken for granted

80 years ago. Also, the supersonic Prandtl-Meyer flow around a corner was originated by Prandtl (1907) and Meyer (1908). H. Blasius (1907) calculated the first laminar boundary layer along a flat plate, and four years later Hiemenz (1911) did the same for a circular cylinder. Both works were dissertations. Prandtl (1910), who was not aware of an older paper by Reynolds (1874), showed again in mathematically improved form the analogy between resistance and heat transfer. Prandtl's pitot-static tube is described in Prandtl (1912).

Th. von Kármán (1881–1963) came to Göttingen in 1908. At first he finished his thesis on a buckling problem of his own (von Kármán 1909). Then he was employed by Prandtl to work on Zeppelin problems in the new wind tunnel. Just a year later he became *Privatdozent* at the university.

In some Central European universities one can apply, after the doctor's degree, for "habilitation" in order to become a *Privatdozent*.
a special habilitation paper and a colloquium before the faculty and results in permission—and then the duty—to teach and examine in a special field. This right is the so called *venia legendi*.
a salary; usually he earns his living as a scientific assistant or practicing physician, lawyer, and so on.[2] To become professor later on at one's own university or somewhere else it is, of course, helpful to be a *Privatdozent* at a well-known faculty and to have a certain repertoire of lectures.

In 1908 and 1909, F. W. Lanchester paid two short visits to Göttingen that greatly influenced the later development of wing theory. Almost 20 years later, when Prandtl delivered the Wilbur Wright Memorial Lecture in 1927, Lanchester remembered, "Professor Runge, who introduced us, acted as interpreter; Professor Prandtl and I could only smile at each other, for neither could speak the other's language" (Prandtl 1927).

The work at the test station became more and more important. Hence, the university took over the wind tunnel previously under the management of the aforementioned *Göttinger Vereinigung*.
Betz (1885–1968), who had just obtained his degree Dipl.-Ing. in naval architecture from the TH in Berlin. Prandtl and Betz complemented each other ideally. Betz was not only a scientist of high rank but also an efficient administrator—quite in contrast to Prandtl. Very soon he became the right hand of Prandtl, and in 1924 he was appointed vice-chairman of the institution that later became the *Aerodynamische Versuchsanstalt* (AVA), now the Göttingen center of the *Deutsche Forschungs-*
Anstalt für Luft-

In 1911 the *Wissenschaftliche Vereinigung für*

[2] For example, young Schopenhauer was a *Privatdozent*. Yet, since he offered his lectures at exactly the same hours as his great enemy Hegel, he attracted only a few students.

now called the *Deutsche Gesellschaft für Luft- und Raumfahrt* (DGLR). In an expertise for the establishment of the *Deutsche Versuchsanstalt für Luftfahrt* (DVL) in Berlin-Adlershof in 1912, Prandtl warned against the appointment of too many *Beamte*,
Since one could not foretell the future of aviation, the director should be given free hand as much as possible to cope with yet unexpected developments.

Von Kármán's paper on his famous vortex street was presented at the Göttingen Academy (von Kármán 1911, 1912, von Kármán & Rubach 1912). In 1913 he became professor at the TH Aachen, where he worked until 1929, except for five years of war service in the Austro-Hungarian air force (where, e.g., R. von Mises and K. von Terzaghi were other famous members). Communication between Göttingen and the new center in Aachen remained close, of course.

Wind-tunnel tests at this time yielded many new insights. For example, the drag coefficient of a sphere found at Göttingen was about twice the value G. Eiffel (1832–1923) had found in Paris. Hence, Eiffel (1912) made more tests with three spheres of various sizes and at various speeds and discovered the then shocking dependence of the drag coefficient on Reynolds number. In searching for an explanation, Prandtl (1914) supposed that the French boundary layer had become turbulent. Hence, he forced the boundary layer of his own sphere to also become turbulent by a trip wire, which indeed decreased the drag drastically. Incidentally, Prandtl often regretted that Eiffel did not want to continue their correspondence after the war.

During the First World War a bigger wind tunnel (*Kanal* 1) was built with an open test section of 4 m^2 and 300 HP; it was the first one of the Göttingen type. Regular test series with a three-component balance were started in 1918. The older tunnel of 1908—now called *Kanal* 2—was rebuilt,[3] also with an open test section, on the grounds of the new institute, *Modellversuchsanstalt für Aerodynamik*,
was the *Kaiser-Wilhelm-Gesellschaft* (KWG)—now the *Max-Gesellschaft* (MPG)—which was founded in 1911. The land was donated by H. Th. Böttinger, and so the institute address was *Böttinger-Str.*

Tests with airfoils and wings, together with the earlier conversations with Lanchester, stimulated Prandtl (1918, 1919a) to propound his wing theory. At the same time two theses came out: that of Munk (1918) on elliptic lift distribution as an optimum and that of Betz (1919a) on the lift of a rectangular wing. Betz (1919b) also proposed the first theory for

[3] Both tunnels were dismantled in 1948.

propellers with minimum energy loss, which Prandtl (1919b) presented to the academy with a supplement.

Another important thesis was that of K. Pohlhausen (1921) on the approximate calculation of laminar boundary layers by using the momentum equation that von Kármán (1921) had proposed at Aachen. It was the first of the now so-called integral methods.

Initiated by Prandtl, E. Pohlhausen (1921), the brother of K. Pohlhausen, wrote on the heat transfer on a plate with laminar boundary layer. Concerning this topic Prandtl always mentioned in his lectures and in his book that the "Prandtl number" had been formulated already by W. Nusselt (1909).

The thesis of Birnbaum (1924) deals with unsteady profile theory. Better known is his previously published paper (Birnbaum 1923) on the influence of camber on a profile in steady flow. Here, by request of Prandtl, work was finished that had come to a stop at the end of the war.

J. Ackeret (1927) reports in the *Handbuch der Physik*,
had given a formula expressing the compressibility effect on profile lift in subsonic flow in his mechanics seminar in 1922; this formula was later rediscovered by Glauert (1927). These were the first formulations of what is now known in more general form as the Prandtl-Glauert analogy [cf. Busemann (1928), where also test results of Göttingen and NACA are shown].

4. ORGANIZATION PROBLEMS IN SCIENCE

The Kaiser-Wilhelm-Institut (KWI) and the Aerodynamische Versuchsanstalt (AVA)

In 1907 Prandtl was offered a position at the TH Stuttgart, which he did not accept. Later, in 1920 and again in 1922, he was asked to accept the chair of his father-in-law, A. Föppl, at the TH Munich. But then the KWG proposed the foundation of a *KWI für Hydro-*
Göttingen with a regular budget, whereas the AVA essentially had to earn its own money.[4] Already in 1911, when the *KWG zur Förderung der Wissenschaften* was founded, Prandtl had written—again encouraged by Klein—a memorandum for such a KWI. But the KWG had previously promised to support the aforementioned DVL at Berlin-Adlershof,

[4] It might be mentioned here that most German research institutes such as, e.g., DVL (now DFVLR), AVA, and KWG (now MPG) are only registered associations (*eingetragene Vereine*). Insofar as they use public money, they are controlled by the state audit office. The individual KWIs (MPIs) are not corporate bodies, but the mother society [KWG (MPG)] is.

founded in 1912. And then came the First World War and inflation. But now, after the promise of KWG, Prandtl remained in Göttingen.

His *KWI für Strömungsforschung* came into existence in 1925. At the opening ceremony, the founder and president of KWG, Adolf von Harnack, explained: "With great care do we look for a good researcher. When we have found him we direct currents of good will to him, procure as much financial aid as possible, and leave it to him to do what he thinks right. We ourselves no longer exercise any influence on him. Our task is finished with his selection and the provision of means" (Betz 1957).

The new institute had, e.g., two high-pressure steel vessels, each with a volume of 10 m^3 for high subsonic and supersonic tests as well as cavitation experiments, and a rotating chamber of 3-m diameter and up to 60 rpm for the study of Coriolis forces in fluid flow. Unfortunately, most prospective investigators fell seasick in this closed room without a view of a fixed horizon as in a merry-go-round.

Prandtl directed his KWI with a staff of up to about 40 until 1945. His university institute for mechanics he handed over in 1934 to M. Schuler, well known for his work on gyroscopes. The name of the *Versuchsanstalt für Aerodynamik* was changed in 1919 to the *Aerodynamische Versuchsanstalt* (AVA). It was on the same grounds as the KWI and became an independent registered association in 1937 under its director, Betz, with over 700 employees during the war. Prandtl was chairman of the AVA directorate. Despite these organizational differences, there was always close contact between members of both institutions, sometimes even leading to marriage.

The research topic of the AVA was aerodynamics of airplanes and their propulsion in the widest sense. After the First World War, sometimes its main income was earned by investigations ordered by foreign countries (Wuest 1982). On the other hand, Prandtl at his KWI could freely plan farsighted research in various directions such as, e.g., meteorology.

Nowadays, when all activities are required to be of "relevance to society," whatever that means, the importance of such completely free research is often not appreciated. Yet commissions appointed to fix specific directions for future research work usually do not show the foresight and versatility of top experts. Without Prandtl and his KWI, Göttingen would never have emerged as a main center for flow research.

Foundation of the Zeitschrift für Angewandte Mathematik und Mechanik (ZAMM) and the Gesellschaft für Angewandte Mathematik und Mechanik (GAMM)

Prandtl delivered his famous lecture on flows with very small friction at the Third Mathematical Congress in Heidelberg in 1904. But there he found no response at all, with the only interested listener being Klein. At that time in Germany, such a lecture would have been appropriate to either the *Deutsche Mathematiker-Vereinigung* (DMV) founded in 1890 or the *Verein Deutscher Ingenieure* (VDI) founded in 1856.[5]

A special journal, the *Zeitschrift für Angewandte Mathematik und Mechanik*
with R. von Mises as editor and published by VDI-Verlag. The idea of establishing a corresponding society emerged first in correspondence between von Mises, Prandtl, and H. J. Reissner, also in 1921. A few sentences out of a letter from Prandtl to von Mises might be mentioned here because they are characteristic of his scientific credo (Gericke 1972):

> Against your name proposal (Society of Applied Mathematics and Mechanics) I will stick to mine (Society of Technical Mechanics) because I wish that the main object of the new society be rather the field of mechanics and its applications in civil and mechanical engineering. Applied mathematicians, i.e. those doing useful mathematics[6] for the main purpose of this society, are very welcome. What I would like to prevent is the dominance of mathematics and the mathematical treatment of problems. I think experiments ought to be stressed at least as much as theories.

Yet eventually, the *Gesellschaft für Angewandte Mathematik und Mechanik* (GAMM) was founded in 1923 at Marburg, where the first executive committee (Prandtl president, H. J. Reissner vice-president, von Mises secretary) and the first scientific committee (Emde, Finsterwalder, von Kármán, Knoblauch, and Trefftz) were elected.

The objective of GAMM is the cultivation and advancement of scientific work and international cooperation in the field of applied mathematics as well as in all branches of mechanics and physics that number among the foundations of engineering sciences. The Society pursues this goal chiefly by organizing scientific meetings (once a year).

For some unknown reason, GAMM has never applied for registration into the official *Vereinsregister* (register of associations)—perhaps simply because of the juristic inefficiency of its members. However, this semi-official status turned out to be helpful in 1933 for surviving and then in 1945 for reestablishing.

Already in 1922, von Kármán & Levi-Civita (1924) had initiated the first postwar international meeting of hydro- and aerodynamicists in Innsbruck, Austria. The first International Congress of Applied Mechanics in Delft in 1924 was initiated by C. B. Biezeno, J. M. Burgers, J. A.

[5] A time when, e.g., Prussia, Bavaria, and Hesse were still separate states, although members of a custom union since 1833.

[6] In 1931/32 Prandtl gave a lecture series in Göttingen with the provocative title "Intuitive and Useful Mathematics."

Schouten, and E. B. Wolff; there the idea of the International Union of Theoretical and Applied Mechanics was born.

5. THE FIRST TWELVE YEARS OF THE KWI

The young Swiss engineer J. Ackeret (1898–1981) came in 1921 to the AVA to continue his studies in flow research at his own cost. Yet, in 1922 he was given an assistantship when C. Wieselsberger (1887–1941) left for Japan. He proved extremely active and versatile; for example, he developed small electric motors with up to 30,000 rpm for model propellers (Ackeret 1924). When the planning for the new KWI began in 1922, Prandtl and Ackeret designed the supersonic and the cavitation tunnels, and he became *Abteilungsleiter* at the KWI. His work there can be described only sketchily here.

W. Tollmien (1900–68) became assistant to Prandtl in 1924. Prandtl & Tollmien (1924) generalized Ekman's spiral for the wind distribution above the rotating Earth to the much more realistic turbulent flow with a 1/7-power profile. This subject was later further developed in a thesis of Roux (1935). The thesis of Tollmien (1925) deals with an unsteady boundary layer, and that of Nikuradse (1925) with turbulent pipe flow.

Many other important developments took place in 1925. In that year the former naval architect Betz (1925) invented his wake-survey method to measure the drag of an airfoil in flight or in a wind tunnel. Yet, only in 1951 did the aeronautical engineer M. Tulin extend this method to ship models in a tank, mainly to separate viscous and wave resistance of a ship.

In 1925, Ackeret (1925) found his formulae for profile forces in supersonic flow.

Also in 1925, A. Busemann (b. 1901) came from Braunschweig, where he had received his Dr.-Ing. degree under O. Föppl. He built the rotating chamber, and Prandtl (1926a) gave a short report on first experiments; it was not much used, except in the thesis of Fabricius (1936) and by Stümke (1940).

Last but not least, the turbulent year of 1925 saw Prandtl (1925) propose the concept of the mixing length in turbulent flow, although he called it "stopping distance" (*Bremsweg*)
weg appeared a year later in Prandtl (1926b). It gave rise to an exciting "race" with von Kármán (1930), who produced similar and sometimes identical results by assumptions based on dimensional analysis only. Later on, G. I. Taylor (1935, 1936) gave in his statistical theory of turbulence the fundamentals for a really sound physical theory; here, mixing lengths turn up again as length integrals of correlation functions. He also initiated the hot-wire technique for measuring velocity fluctuations.

However, this theory was complicated and not helpful for quick practical applications such as pipe flow or plate resistance. One important "practical" result of Taylor (1936), the dependence of sphere drag on turbulence of the oncoming flow, could also be derived by dimensional analysis only, following Taylor's arguments (Wieghardt 1940).

Ackeret (1926) published tests on boundary-layer suction. He also made cavitation tests in the new facility as shown in his article on cavitation in the *Handbuch der Experimentalphysik*,

tome gives in several treatises by various authors an excellent survey of fluid mechanics of that time.

Tollmien (1926) used the mixing length to calculate free turbulent flow. Betz (1927) gave the first extensive aerodynamics of the windmill, which has now become topical again.

In connection with the Prandtl-Glauert analogy, a paper by Busemann (1928) has been previously mentioned. In this paper the small tunnel for high subsonic and supersonic flow is described; it was the first tunnel of the blow-down type. It had a test section of only 6 cm^2 and was always meant as a prototype for a later, bigger tunnel. By request of Prandtl, the title of his paper had the supplement, "(With Regard to Airscrews)," just as Glauert was also thinking of this application only. It would have seemed hubris to think of planes flying at such speeds. At that time, only the Mach angle was used; the designation "Mach number" was introduced by Ackeret in his habilitation paper at ETH Zürich in 1929.

Ackeret (1927) wrote the first monograph on gasdynamics in the *Hand-*

Figure 1 Photo showing (left to right) J. Ackeret, L. Prandtl, A. Betz, and R. Seiferth, taken in 1923.

buch der Physik; it was his last paper written in Göttingen. Early in 1927 he left for a post in industry (Escher-Wyss), and then he became *ausserordentlicher* (associate) professor in 1931 and later, in 1934, full professor and director of the newly founded institute for aerodynamics at ETH Zürich. His successor at the KWI was Busemann, with whom he had cooperated intimately.

Another remarkable year was 1929. Prandtl & Busemann (1929) found the first characteristics method for supersonic flow, which—in a modern sense—is not approximate as stated in its title. Admittedly, it is a panel method that, however, tends toward the exact solution for small panels. With it, profile flows, oscillating free jets, and parallel-flow nozzles were constructed; these results appeared later in many publications.

In 1929, Busemann (1929a) published his shock-polar diagram and a short paper (1929b) on the pressure on a conical tip, which gave all flow essentials and the appropriate calculation method.

Also in 1929, the work of Tollmien (1929) on the instability of the laminar boundary layer along a plate was presented by Prandtl at the academy. Whereas the thesis of Tietjens (1923) had in fact yielded amplification of disturbances, Tollmien calculated for the first time a critical Reynolds number for the beginning of instability. Following this work, Tollmien traveled to the United States and spent three years (1930–33) at Caltech in Pasadena.

Figure 2 The Kaiser-Wilhelm-Institut für Strömungsforschung at Göttingen about 1937.

Besides all these original papers, experimental work in wind tunnels was continually in progress. The main results were collected and published by Prandtl & Betz (1921, 1923, 1927, 1932). Also at this time, Prandtl's lectures were published (Prandtl & Tietjens 1929, 1931).

Busemann did his habilitation in 1930 at Göttingen and became lecturer for fluid- and thermodynamics at the TH Dresden in 1931. Later on, in 1936, he became professor and director of a new institute for gasdynamics at the *Luftfahrtforschungsanstalt* in Braunschweig.

At the end of the twenties, H. Schlichting (1907–82) studied at Göttingen. In 1930 he wrote his thesis on free turbulence in a wake (Schlichting 1930) and joined the KWI.

Semiempirical integral methods for the calculation of turbulent boundary layers at a given pressure distribution were developed in two theses by Buri (1931) and Gruschwitz (1931). The latter was much used for some time.

Nikuradse (1932, 1933) published his test results on turbulent flow through smooth and rough pipes; in order to define a special but reproducible roughness, the so-called sand grain roughness was invented. For many technical applications these two papers proved to be very important and were widely acknowledged. Unfortunately, this increased his self-esteem to such a height that he tried to replace Prandtl as director after Hitler had come to power. It was, indeed, a dangerous attack, for Nikuradse knew at least one man high up in the Nazi regime, whereas neither Prandtl nor Betz ever became party members in spite of their important positions. Luckily Prandtl was victorious. Nikuradse had to leave KWI and—without Prandtl's guidance—he never again wrote a paper worth mentioning.

Prandtl & Schlichting (1934) used these results of pipe flow to compute the resistance of a rough plate. This was of great interest, e.g., for airplanes and ships, and in particular for the extrapolation of model tests.

Schlichting (1932a,b, 1933, 1935) extended Tollmien's stability theory in various directions, especially in later years, when he had left Göttingen in 1935 for a post in industry (Dornier) and then later (since 1939) as professor at TH Braunschweig. While at Göttingen he also wrote on the rectangular wing in supersonic flow (Schlichting 1936); it was the first paper on a finite supersonic wing. For this he used the acceleration potential introduced by Prandtl (1936a).

Tollmien (1935) proved that for inviscid flow to be unstable, Rayleigh's condition of an inflection point in the velocity profile of a boundary layer is not only necessary but also sufficient. He did his habilitation in 1936 and became the successor of Trefftz at TH Dresden in 1937.

W. Frössel (1936) reported on tests on pipe friction in subsonic and supersonic flow. A theoretical addition was given later in the thesis of Koppe (1947).

6. THE SEPARATION OF THE AVA AND THE LAST DECADE OF THE KAISER-WILHELM-INSTITUT

In 1937 the management of the AVA and KWI was separated because of the large expansion of AVA and its administration. Prandtl became chairman of the AVA directorate and Betz director of AVA, which had eight divisions or institutes. Cooperation with KWI was close, in particular with the institute for theoretical aerodynamics under Betz (deputy: F. Riegels) and with those for unsteady aerodynamics under W.-G. Küssner and for high-speed problems under O. Walchner. Most of the AVA institutes were on the same grounds as the KWI, yet there were two separate entrances. Luckily, control at the KWI entrance was less formal for its "golden horde."

In spite of the high prestige of Prandtl and Betz, Göttingen did not have the biggest wind tunnels in Germany. These were at the DVL in Berlin and at the *Luftfahrtforschungsanstalt* in Braunschweig, also of Göttingen type. The biggest supersonic tunnels, all of blow-down type, were in Braunschweig, Aachen, and Peenemünde. Hence, Betz and Walchner built such a tunnel from their own revenues.

At the much smaller KWI, the hierarchy was more of a family type. Prandtl's administrative deputy was H. Reichardt, who started in 1927 at AVA and came in 1930 to KWI; he worked mainly on turbulence and cavitation. A small bureau for design was led by W. Frössel, who also supervised the workshop. But, as he liked to point out, he was never officially appointed thereto. He had come to KWI as a locksmith apprentice, but then he performed tests as reported in his aforementioned paper of 1936. Despite his age, to become admitted to the university as a doctorand, he had to pass the high school examination. At last he took his doctoral degree with a thesis on lubrication (Frössel 1941), a field in which he carried on successful research.

In fluid mechanics, theoreticians can be useful of course, but left alone they would soon dry up. Hence, it seems appropriate to mention the nonscientific staff too. Prandtl's truly faithful secretary was Miss Eleonore von Seebach (called "Lorchen"), who also typed all of our papers. (At AVA, Miss Kreibohm worked as Betz' secretary for a half-century!) Bookkeeping was done by Mrs. Grüber and her help. There were also at least six girls using automatic calculating machines, sliderules, or graphical methods. In the workshop were several foremen, and the test work was done by a few but excellent technicians; for example, one specialized in hot-wire tests, another in photography and optical methods, and so on. Together with the two watchmen, Prandtl's crew had up to 40 people.

In the second half of the thirties, the KWI building and facilities were somewhat enlarged. At this time Dr.-Ing. F. Schultz-Grunow came to the institute because Prandtl wanted a representative of mechanical engineering; Prandtl liked to call himself a plain mechanical engineer. In 1937 he engaged Dr. phil. H. Görtler, a pure mathematician. K. Oswatitsch, who had done his doctorate in theoretical physics at Graz University, came in 1938, at first as a stipendiary of the *Deutsche Forschungsgemeinschaft* (German Research Association). The same year K. Wieghardt joined, although he had done his doctorate at Göttingen only with the grade "good." Dipl.-Ing. H. Schuh came in 1939 from Vienna.

Of Schultz-Grunow's papers we recall three (Schultz-Grunow 1939, 1940, 1942): The first one was his habilitation paper on inertia of turbulencc, the second was on the outer law for the turbulent boundary layer past a plate, and the third concerned his characteristics method for one-dimensional wave propagation, for which examples of applications reappeared in many textbooks. Before this last paper came out, he accepted a chair at TH Aachen in 1941.

As for Görtler, for his first official task Prandtl gave him a differential equation, saying "I want a solution starting at zero linearly, reaching a maximum, and then coming down to an asymptote. Please, figure it out." But Görtler could not find such a solution. And at last, as a good mathematician, he even succeeded in constructing a rigorous nonexistence proof for such a solution! But, when he proudly showed this to Prandtl, the latter only said, "Oh, is that so? Well, it might be that this constant here also depends a little bit on Reynolds number." Besides this experience, Görtler was the only one who became acquainted with fluid mechanics through private lessons by Prandtl. In his habilitation paper (Görtler 1940), he showed the instability of flow along a concave wall that generates longitudinal Görtler vortices, analogous to Taylor vorticcs between two rotating cylinders (Taylor 1923). In 1944 Görtler took a position at the University of Freiburg.

At the end of the thirties, a series of theses on wing theory came out. Three of these used the new acceleration potential: Kinner (1937) for the circular wing, Krienes (1940) for the elliptic wing, and Schade (1940) for the oscillating circular wing. The rectangular wing in Wieghardt's (1939) thesis was represented by four horseshoe vortices calculated numerically. In a footnote there, the integral equation for a continuous vortex distribution is mentioned, which is usually ascribed to E. Reissner (1949); it can also be found in Mattioli (1939). Later on, Kinner worked in industry, and Krienes and Schade at AVA.

To Oswatitsch, Prandtl gave a special problem "so that something is done again in gasdynamics"; at that time, this was—compared with wing theory, turbulence, and boundary-layer theory—a field of minor interest. Already in the supersonic part of the small KWI tunnel, there had appeared inexplicable schlieren called "the ghost" by Busemann. At first, by deriving a formula for the growth of droplets with time, Oswatitsch (1942) found out that sufficient condensation of humidity was impossible during the short stay of the air in the nozzle. But then, inspired by a paper on the breakdown of a supersaturated thermodynamic state, he was able to give a detailed theory for the ghost. Here a chance cluster of molecules can serve as the kernel of a droplet. Meanwhile, such humidity effects were avoided by using silicagel desiccators in front of all blow-down tunnels.

Another problem was the pressure recovery of a ramjet at Mach number 3. At that time, no better recovery seemed obtainable than that behind a single normal shock. However, Oswatitsch (1944) developed a concept in which the air is compressed first by a conical tip in one or several oblique shocks to a lower supersonic flow and only then in a normal shock to subsonic flow leading to the combustion chamber. At that time this was a complicated diffuser for which the KWI tunnel was too small. But at last, his experiments in the AVA tunnel of Walchner were successful.

In Oswatitsch (1945) the analytic formulation for the drag as an integral over the entropy flux is given.

Wieghardt measured in 1942 the drag increase of a plate due to single roughnesses such as rivets, holes, etc.; this wartime report was reprinted in Wieghardt (1953). He also measured the temperature distribution due to a point or line heat source at the bottom of a turbulent boundary layer; the results might be used in meteorology when the much greater eddy viscosity of the atmosphere is allowed for. This report of 1944 came out in Wieghardt (1948a). In his habilitation paper of 1944 (Wieghardt 1948b), he uses the energy equation for boundary layers for a calculation method. Only after the war was it found out that Leibenson (1935) had published this equation much earlier.

Schuh refined the hot-wire technique and worked also on heat transfer. His publications came out mainly after the war, including his thesis (Schuh 1946a) and Schuh (1946b).

Prandtl himself became more and more interested in dynamic meteorology. He considered it to be a main field of application of future fluid dynamics, whereas Sir G. I. Taylor (1886–1975) had come from meteorology to fluid mechanics. This is evident from some papers and lectures, as, e.g., in Prandtl (1932, 1936b). There are preliminary sketches of what was worked out later in two theses: Kropatschek (1935), on a simplified model of general circulation, and Stümke (1940), on generation of cyclones by source effects with a few tests in the rotating chamber. Further papers of this series are on atmospheric lee waves behind mountains by Lyra (1940, 1943) and on wind deflection due to Coriolis forces above a mountain chain by Görtler (1941) or above a single mountain by Rothstein (1943). Both Stümke and Rothstein remained at KWI during the war.

In 1942 Prandtl's book, *Führer durch die Strömungslehre,*
(Prandtl 1942a). It was also the third edition of his much smaller *Abriss der Strömungslehre.*
in German there is only one word for leader and guide; a *Führer durch London* is just a Baedeker. There were five editions and translations into English, French, Japanese, Polish, and Russian. After his death we carried on three further editions, the eighth in 1984. All of his articles in journals are reprinted in Prandtl (1961).

Initiated by the manifold and precise measurements of Reichardt (1941, 1942), Prandtl (1942b) proposed a theory for free turbulent flow such as jets and wakes. With it Görtler (1942) calculated some flows and found better experimental verification than previously, except for the edge regions.

In 1944 Prandtl listed seven problems "that should be investigated after the peace treaty." One of us still has a copy of this list with two pages of explanations. The problems are (*a*) influence of Richardson number on turbulence, (*b*) flow and heat transfer on a heated inclined plane for meteorological applications, (*c*) turbulence theory as in Prandtl (1945a), (*d*) generation of sand ripples in water or air, (*e*) boundary layer on rotating turbine blades, (*f*) generation of circulation by shaking a partly filled vessel, and (*g*) refined experimental study of turbulent correlation functions. On problems (*a*) and (*f*), he wrote two short papers (Prandtl 1944, 1949).

For problem (*c*) he advanced a transport equation for the turbulent kinetic energy when only one of the mean velocity gradients is predominant (Prandtl 1945a). This paper has indeed initiated almost all of modern turbulence modeling, especially after computers became available. But Boussinesq's exchange coefficient, nowadays called eddy viscosity, is still used as a scalar, in spite of Prandtl's warning that in general it should be a tensor.

Another paper (Prandtl 1945b) marks the beginning of three-dimensional boundary-layer theory. For 40 years, only two-dimensional boundary layers had been considered either in plane or axisymmetric flow!

Early in April 1945 American troops came to Göttingen, and some weeks later we belonged to the British occupation zone, about 10 km off the Russian zone. Previously the principals of the KGW and scientists from other KWIs—including celebrities like Hahn, Heisenberg, and Planck

—had come as guests to the grounds of KWI and AVA. Also, a huge mysterious machine had been stored there: the first Zuse computer.

For almost two years the British Ministry of Supply employed many KWI and AVA scientists for a special task. With Betz as editor, a review of all work done in Germany during the war in the field of aero- and hydrodynamics was written down on 7000 typed pages in German English: These were known as the Göttingen monographs of 1945/46. A similar, shorter review was done at the same time for the FIAT review of German sciences in the years 1939–46. These reviews were useful and enlightening—at least for the writers. Otherwise, a response did not seem to be forthcoming. On the other hand, many foreign colleagues had previously come for interviews, so that the main points of scientific progress in various countries had already been discussed, such as, e.g., sweptback wings (cf. Busemann 1971, Wuest 1982) or the experimental verification of instability theory by Schubauer & Skramstad (1947). Most of us saw von Kármán for the first time, in American uniform and with cigar.

Since AVA had belonged to the air ministry, it was dissolved. It was reestablished in 1953, now under the auspices of MPG. Betz retired in 1957 and Schlichting became director of AVA (Wuest 1982).

As it turned out, KWI had been under the ministry of education and so it could carry on. Only the name was changed—in 1948 it was renamed the *Max-Planck-Institut.*

retained only a department of MPI; two others were set up for Betz and Tollmien, who had come back from Dresden.

Prandtl engaged the meteorologist E. Kleinschmidt, who wrote a paper on tropical cyclones (Kleinschmidt 1951) that is still cited often; he also wrote the chapter on meteorology for the sixth and seventh editions of Prandtl's book. Another paper still much used is Ludwieg (AVA) & Tillmann (KWI) (1949); it gives a general formula for the wall friction when the size and the form parameters of a turbulent velocity profile are known.

Oswatitsch left Göttingen in 1946, Wieghardt in 1949.

Tollmien became successor to Prandtl's chair when Prandtl retired from the university in 1947.

7. PRANDTL'S WORK IN SOLID MECHANICS

So far we have mentioned only papers by Prandtl or his staff that are in the field of fluid mechanics. However, those working in solid mechanics also consider Prandtl as their colleague. Hence, a short survey of some of his work in this field is added here.

Prandtl (1903, 1904b) advanced his soap-film analogy for the elastic stress of a twisted bar. For a fully plastic material, the analogy of the sand hill was found by Nadai (1923a).

Prandtl (1920) determined the pressures under which blunt edges of ductile metals yield, and in Prandtl (1921) he generalized the concept of the ideally plastic substance (without strain hardening; cf. also Prandtl 1924).

Prandtl (1928) described an apparently simple model for solid bodies to calculate hysteresis, the dependence of yield stress upon strain velocity, and how temperature changes a state from solid to fluid. Reuss (1930) in Budapest also incorporated the elastic strains in the plastic domain, although in a more formal general manner; hence, this Prandtl body is often called a Prandtl-Reuss body. In particular, this model yields the hyperbolic sine speed law of deformation for which Eyring (1936) gave a more molecular explanation. Hence, rheologists call it now the Prandtl-Eyring model. For the pipe flow of such a fluid, see Prandtl (1950).

Prandtl (1933) extended his solid-body model of 1928 to investigate the time-dependent fracture of a brittle solid.

There were at least two young scientists at Prandtl's mechanics institute who later became well known all over the world. The first was A. Nadai (1883–1963), who came from Hungary in 1919; he became *Privatdozent* (Nadai 1923b) and left Göttingen in 1927 for the United States. The second was W. Prager (1903–80), from Karlsruhe, who did his doctorate and habilitation (1927) at the TH Darmstadt; he then worked at Göttingen from 1929–33. When Hitler came to power he went to the University of Istanbul, where he held the chair for theoretical mechanics until he left in 1941 for Brown University in Providence, Rhode Island.

8. PRANDTL'S PERSONALITY

Besides those pupils of Prandtl we have mentioned so far, there were, of course, many more who later on held important positions in the field of fluid dynamics. Some more of the over 80 students who did their doctorate under Prandtl's supervision are the following: C. Wieselsberger (1922), who worked in Japan before he became successor to von Kármán in Aachen; C. Tietjens (1923), who spent many years in India; H. Blenk (1924), the first director of the *Forschungsanstalt* in Braunschweig; D. Küchemann (1938), who joined the AVA and after the war the Royal Aircraft Establishment at Farnborough.

Those who were later on department heads at AVA were J. Stüper (1932), F. Riegels (1938), H. Ludwieg (1939), and W. Wuest (1941). D. T.

Dumitrescu (1941) became professor at Bucharest. B. Dolaptschiew (1937, 1938) wrote his thesis at Göttingen but took his degree from Sofia University, where he also became professor after the war.

These special examples might create the wrong impression that Prandtl had influenced the job hunting of his disciples, but actually we never even heard a rumor of this sort. He was always outspoken yet strictly neutral, also to his coworkers. He almost always had time to speak with them, but he did not obtrude his ideas, since he expected them to work as independently as possible. By way of conversation he just dropped scientific nuggets, which were, of course, collected with great care. Naturally, one had to report some progress from time to time, otherwise one was told to find another post somewhere else; yet, plenty of time was conceded in these very few cases. Most of his staff he had got to know beforehand as doctoral candidates for about two years time.

He expected his students to attend his lectures, or else he would ask, "Oh, do you know all that?" But he was not a fascinating lecturer, simply because he was often thinking too far ahead. Young students want to hear that this is black and that is white. So often, only the older ones understood why he formulated so cautiously, thinking of some far away exceptions. And the lecturer at the Wednesday colloquium had to expect harsh questions by Prandtl or Betz as soon as he became vague or tiresome.

Prandtl's working method is perhaps best described by von Kármán (1954):

> Prandtl, an engineer by training, was endowed with rare vision for the understanding of physical phenomena and unusual ability in putting them into relatively simple mathematical form. His control of mathematical methods and tricks was limited; many of his collaborators and followers surpassed him in solving difficult mathematical problems. But his ability to establish systems of simplified equations which expressed the essential physical relations and dropped the nonessentials was unique, I believe, even compared with his great predecessors in the field of mechanics—men like Leonhard Euler and D'Alembert.

Sometimes Prandtl could even bypass mathematics, at least qualitatively. For example, after some months, hard-working Dolaptschiew (1937, 1938) had calculated paths along which the vortices of a Kármán street move after a special periodic disturbance. Full of pride, he brought his results to Prandtl. But instead of listening to Dolaptschiew's broken German, Prandtl turned over the papers and began to design the paths the vortices ought to take. And indeed, after a few minutes his drawing looked very much like the calculated one. Similarly, he often solved scientific problems by visualizing the physical processes described by mathematical equations (Prandtl 1948).

The invisible foundation of Prandtl's ability in theory making was certainly a very intimate knowledge of mechanics acquired by many close observations of more or less intriguing processes, either in the laboratory or else in daily life, as certified by many anecdotes. On his way home he would explain to his companion the standing waves on the rainwater approaching the gully, or the different sound reflection of the footfalls when walking along a paling or a wall. Once, in the United States, he performed slight knee-bendings at the top floor of a skyscraper to measure the natural frequency of the building. Only when he had found his result confirmed roughly by thc formula for the fixcd-free bar was he ready to enter the restaurant. As a last example, we offer the following personal anecdote: Besides writing a thesis his doctoral candidates also had to give a colloquium lecture on a different subject. For this important event, one of us (KW) had bought new shoes with rubber soles. Unfortunately, they made a horrible squeaking noise on the parquet of the auditorium. And indeed, very soon KW noticed that Prandtl gazed fascinatedly at his shoes—alas, it was quite obvious that he was just advancing an acoustic theory of this interesting phenomenon.

Besides science, Prandtl had, of course, othcr interests, particularly music. He had perfect pitch and played the piano, mostly classical music; sometimes, he played a waltz for dancing when he had invited young people to his home. His mode of life in quiet Göttingen was quite unpretentious; he lived in a flat within 20 minutes walking distance of his institute. The only luxury was a little house in the Alps for vacations. He did know, of course, that he was exceptionally gifted and productive. Görtler (1975) once was present when Prandtl opened a letter announcing a new high honor. Prandtl showed him the letter, saying "Well, they might have thought of me a bit earlier."

In his family life the most tragic year was 1941. First his wife died, then the husband of his elder daughter lost his life as a soldier in Russia, and their baby also died.

In 1950 he could still enjoy three remarkable jubilees: his seventy-fifth birthday, the fiftieth anniversary of his doctorate, and the twenty-fifth anniversary of his institute. But in spring 1952, illness prevented him from further work and led to his death on 15 August 1953. Everyone who met Prandtl will always remember the dignity and kindheartedness of this great man.

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