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Leonardo da Vinci and Fluid Mechanics

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Abstract

This review focuses on Leonardo da Vinci's work and thought related to fluid mechanics as it is presented in a lifetime of notebooks, letters, and artwork. It shows how Leonardo's remaining works offer a complicated picture of unfinished, scattered, and frequently revisited hypotheses and conclusions. It argues that experimentation formed an important mechanism for Leonardo's thought about natural fluid flows, which was an innovation to the scientific thinking of his day, but which did not always lead him to the conclusions of modern fluid mechanics. It highlights the multiple and ambiguous meanings of turbulence in his works. It examines his thinking suggestive of modern concepts such as the no-slip condition, hydraulic jump, cardiovascular vortices, conservation of volume, and the distinctive path of ascending bubbles we now term Leonardo's paradox, among others. It demonstrates how Leonardo thought through analogies, building-block flow patterns, and synthesis, leading both to successes—especially in the management of water—and to failures, perhaps most obviously in his pursuit of human flight.

1. INTRODUCTION

The wide-ranging interests of Leonardo da Vinci (1452–1519), expressed in his art, notebooks, sketches, and writings, continue to intrigue modern scholars. Despite the wide array of analyses of his work, there is as yet no comprehensive examination that both situates Leonardo's work and thought related to fluid mechanics in its historical contexts and interprets it from the perspective of a trained fluid mechanician. The combination of these two axes of analysis is important, for we cannot read his work without, on the one hand, an awareness of why it was being compiled, how Leonardo worked, and how his ideas have come down to us, and, on the other hand, the trained eye of a specialist who can interpret his texts and their connection to modern theories and practice of fluid mechanics. We hope to keep these two perspectives in mind as we present an overview and analysis of Leonardo's work and thought. We do not approach his work for evidence of modern fluid mechanics but instead analyze it and its pursuit of ideas on its own terms and in its own contexts.

This review surveys Leonardo's work to highlight his key thoughts of interest to the modern fluid mechanics community. Leonardo's engagement with fluid flows has been noted even in recent popular literature (Isaacson 2017) and thus warrants fresh investigation and analysis from the original writings and drawings. We do not provide a detailed analysis of all Leonardo's ideas about fluids and their various contexts, but we point readers in the direction of further materials on each topic. Many scholars have observed Leonardo's interest in natural world systems that we would now investigate as fluid mechanics. Perhaps the most thorough series of studies of this area of Leonardo's thought that has been undertaken by a hydraulic engineer was that of Enzo O. Macagno, who was particularly interested in Leonardo's hydraulic theories. In more than 20 papers and monographs published by the Iowa Institute for Hydraulic Research from 1986 to 2006, Macagno conducted a widespread survey of Leonardo's writings about fluid mechanics as they related to water. This provides an important resource for scholars, but it is not a comprehensive study of all Leonardo's thought about fluid flows in all natural systems. Moreover, it did not consider Leonardo's works in the contexts in which they were written, drawn, or constructed, which has been the important contribution of historians and of art scholars such as Carlo Pedretti, particularly in relation to Leonardo's art and architectural contributions (see Ludmer 1985).

In this study, we present Leonardo's thought through his images and through his texts. In both cases, it should be recognized that we are presenting these images and texts outside of their context on the page, frame, or architectural setting in which they were created, and we present his texts in English, rather than in the Italian vernacular in which Leonardo wrote. Any act of translation is also an act of interpretation. For this analysis, we have translated Leonardo's words into modern English, keeping as close to the historical context and concepts as possible and providing the original text, links to online images of the pages, and updated references in a **Supplemental Appendix**. We have then interpreted this translated wording in our discussion with an eye to topics that are of interest to the fluid mechanics community. This is an important distinction from much previous discussion of Leonardo's fluid mechanics, where translations using such terms as "velocity" and "friction," rather than more mundane terms such as "speed" and "rubbing," may inadvertently suggest that Leonardo was operating with modern fluid mechanics concepts. We also retain strike-throughs and insertions where they occur in Leonardo's original text so that readers can gain a sense of the process of his thought as he recorded it. For the sake of readability, however, we have introduced some grammar and punctuation.

Supplemental Material >

2. BEAUTIFUL DIAGRAMS: CONCEPTUALIZING LEONARDO'S PROCESS AND PRACTICE

In a series of studies, the art historians Ernst Gombrich, Carlo Pedretti, and Martin Kemp have highlighted the importance of understanding the artistic and conceptual elements of Leonardo's production alongside its scientific considerations (Gombrich 1969; Kemp 2004, 2007; Pedretti 1978). This is an essential point in interpreting Leonardo's work as it relates to fluid mechanics. Firstly, from his education, training, and life experiences, Leonardo came to study natural world systems with many of the preconceived notions of his era about how nature functioned and what was possible to discover about it. His visual texts are not, therefore, a form of proto-photographic representation. As Kemp (1992, 2004) has argued, "Leonardo's observations were structured through inherited concepts of how nature was thought to operate" (Kemp 2004, p. 65).

Secondly, what are frequently termed Leonardo's observations can more usefully be understood as diagrams, that is, visual and written texts that reflected both the assumptions of his times and his analysis of what he experienced in the world, including what he observed through his own experiments (Gombrich 1969). In Macagno's (1986, 1987) account of Leonardo's methodology for the study of fluid flows, he concluded that Leonardo's sketches were not purely observational, but rather included conceptual elements that reflected his thinking about the structure of the flow. As such, Macagno adopted a terminology of "rheograms" to describe the visual texts contained in Leonardo's notebooks [Macagno did not incorporate Leonardo's artwork in his studies; see Macagno (2002)]. The term "rheogram," Macagno (1987) argued, conveys the nonphysical nature of many of Leonardo's drawings in the sense that one will never see in nature what Leonardo drew. While this terminology goes some way toward clarifying Leonardo's process, as we explain below, it does not fully describe his practice.

Importantly, Leonardo brought to his work, whether in his notebooks or his artworks, the background of a formal artistic apprenticeship and a profound consideration of what was beautiful at the time. As Kemp (2004, p. 65) has argued, for Leonardo, "good art relied upon selection in the cause of beauty," a practice that led to a hypernaturalist style. We thus see nature in Leonardo's visual texts as he thought it could be interpreted most beautifully (and perhaps also most emotionally satisfyingly). While this consideration may have been more influential in his artwork, we cannot assume that he simply turned off his artistic faculty in considering nature elsewhere in his work. Thus, we adopt the terminology of "beautiful diagrams" to describe Leonardo's processes and production, and our analysis below reflects these understandings of his practice.

3. ANALYZING LEONARDO'S WORK AS A REFLECTION OF HIS THOUGHT

Our study of Leonardo's work canvasses his entire production and considers artworks alongside his many notebooks. This is consistent with our view that Leonardo did not separate his thinking on these matters. Thus, we can derive valuable information by surveying and comparing presentations of his thought across his productions, while keeping their contexts of creation in view. Moreover, consistent with the argument that Leonardo's beautiful diagrams represent his analysis of experiences and engagement with natural world systems, we suggest that his works should be interpreted as an evolving corpus of thought, rather than a static compilation of fixed ideas.

Close study of the contours of Leonardo's life make clear that he continued to return to consideration of natural world fluid flows as he moved between different states, patrons, and commissions for works ranging from engineering designs to portraits, and as he garnered experiences of the world through them. Read through the lens of lifelong learning, the ideas expressed in Leonardo's texts change and develop, contradicting prior productions and sometimes causing him to amend them. Leonardo's interpretation of the neck muscles of the saint in his unfinished painting *Saint Jerome in the Wilderness*, commenced around the 1480s, may be an example of this process, with the underlying drawing under its painted layers suggesting a development in his understanding of anatomy, most likely as a result of his later attendance at dissections in Rome (Clayton 2016, Kemp 1972).

Furthermore, we do not understand Leonardo's work as a complete presentation of his thinking on these subjects. What remains to us are a series of notebooks, letters, and artworks, along with suggestions, echoes, and sketches of others, that have been distributed in pieces, separated from their original order and context, since the time of his death. We are not studying Leonardo's thought as he wished it to be presented, but as his heirs and subsequent collectors felt fit to circulate and curate it (Marinoni 1954). Indeed, among his notebooks are elusive references to a "Book of Water" (*Libro dell'acqua*) that has not come down to us in a recognizable form as such. Macagno dedicates seven volumes (e.g., Macagno 2000–2006) to a reconstructed compilation of what this work might have contained. That it took Macagno so many volumes to do so is indicative of just how engaged with fluid flows Leonardo was throughout his life. As Macagno (2000, p. 2) notes, conceptually flow "is one of the dominant traits of Leonardo da Vinci's mind; in fact, it pervades all his life and work."

How do we situate Leonardo's thinking in its time? To make sense of his thought about natural flow systems, we must recognize that Leonardo's relatively low social origins, need for income, and early artistic training placed him outside the learning communities in which elite scientific knowledge of the era was being conceptualized. He engaged with this material in discussions and through extensive reading (Descendre 2010). However, his ideas were also developed by exposure to further contexts, such as the world of contemporary artists and through commissions and collaborations to undertake engineering projects and to present spectacular ceremonial pageantry for European leaders, for example. Leonardo's career spanning such varied courtly contexts afforded him an identity, financial support, and time in which he could pursue interests not available to many other contemporaries, which is particularly remarkable given the poor rate of completion of many of his projects.

However, even if Leonardo the individual appears to have been perceived as a unique asset to the courtly environments in which he worked, the actual traction of his thought for contemporaries is far less clear. There is rather less evidence that his ideas, practices, and constructions were appreciated or adopted by others in his day. In fact, when a series of Leonardo's works—15 notebooks and the Codex Atlanticus—were offered to Cosimo II de' Medici, Grand Duke of Tuscany, in 1614, Cosimo had the material reviewed by Giovan Francesco Cantagallina, a civil and military engineer in his service. Cantagallina's report concluded that it did not warrant inclusion in the Grand Duke's collection, as it contained "nothing good" in it (Richter 1977, p. 397). This anecdote serves as a good reminder that even acclaimed polymaths can receive a negative review!

Furthermore, at least some of the ideas expressed in Leonardo's notebooks were not novel to his contemporaries. We know far more about Leonardo than others of his day through the preservation of his texts. Certainly some of the ideas that Leonardo expressed can already be found in works to which we know he had access, from classics to near-contemporary and contemporary engineers: Mariano di Jacopo (Il Taccola), Francesco di Giorgio Martini, and Giuliano da Sangallo, among others (Frommel 2016, Geddes 2020, Moffitt 1991). Di Giorgio's *Trattato di architettura*, for example, was a clear influence on Leonardo's architectural ideas and is heavily quoted in Madrid Codex 8936 (di Giorgio 1979, Pedretti 1978). A manuscript copy of di Giorgio (Codex Ashburnham 361) bears Leonardo's marginal notes. Di Giorgio's work was also foundational to Leonardo's work on fountains (Villermaux 1994), and a comparative analysis of di Giorgio and Leonardo's machines and technological knowledge has been conducted by Long (2004). We do not refer here therefore to Leonardo's discoveries but more simply to Leonardo's knowledge as we can interpret it through remaining sources. What we have then is an opportunity to study the thought of a significant and interesting individual, but we should not assume that his ideas, in this case about fluid mechanics, were necessarily anything more than representative of his wider world.

4. LEONARDO AS AN EXPERIMENTALIST

We have described Leonardo as a lifelong learner, whose curiosity saw him return again and again to consider natural flow systems. A key part of his method of self-education was experimentation. Indeed, it was a critical component of how he understood himself: "Leonardo da Vinci disciple of experience" was how he styled himself in the Codex Atlanticus, folio 520 recto (f. 520r). In so describing himself, Leonardo was explicitly attempting to set himself apart from a dominant, albeit increasingly challenged, intellectual culture that prided itself on its foundations in the worldview of antiquity and in developing its conceptualization of natural processes from classical suppositions. In the Codex Atlanticus, Leonardo, like others of his time, declared his rejection of this knowledge framework:

> Although I did not know how to cite authors, like them, citing a greater and more worthy thing, experience, the mistress of their masters. They go about deflated [sic: inflated] and pompous, dressed and adorned not with their own but with the efforts of others, and do not concede to me my own. (f. 323r)

He set out another (now familiar) epistemological methodology in Paris Manuscript (Ms.) E:

> First I will do some experiments before I ~~procee~~ proceeding beyond, because my intention is to cite first experience and then that reason to show why such experience is bound to behave in that way. And this is the real rule how speculators of natural effects have to proceed. (f. 55r)

Leonardo's experiments related to fluid flow appear to have been extensive. Most of his notes suggest that Leonardo worked primarily with air and water, but as Macagno (1986, p. 4) has documented, he also considered flow in fluids such as "oil, wine, blood, mud, sap, and granular materials like sand and seeds." Flow visualization appears to have been of particular interest to Leonardo, and his notebooks suggest that he employed dye, seeds, and other light materials to act as tracer particles.

Figure 1 shows examples of varied flow visualization experiments found in Leonardo's notebooks. These suggest that some experiments were designed to be conducted in nature, while others were experiments in controlled conditions. For example, **Figure 1*a*** from the Codex Leicester, f. 9 verso (f. 9v), shows the construction of flumes for the study of waves and wave interactions with a sediment bed. Leonardo's accompanying comments suggest that it was to be tested in the field: "Test in your pit, with the wind going from a to b, to see to which path the thing n on the bottom is pushed. I judge that it will return to m" (f. 9v). Further on in Codex Leicester, however, Leonardo described the construction of what is effectively a test tank for flow visualization experiments:

> If you want to see the motion of the air that is p penetrated by a moving [object] use the example in water, that is, under the surface, mix it with thin millet or other tiny seed that sustains itself in all levels of height, and then move inside this the ~~bol~~ moving object that sustains itself in the water and you will see the revolution of the water, which should be in a square glass vase for use as a box. (f. 29v)

Similarly, in Paris Ms. I, the text written within the tank (shown in **Figure 1*b***) notes that "This part is of glass, behind, it is of wood" (f. 41v). Beside the tank Leonardo wrote, "To experiment ~~that~~ how

Figure 1

(*a*) Drawings of flumes (Codex Leicester, f. 9v). Image reprinted with permission of the licensor through PLSclear from Laurenza & Kemp (2019), copyright 2019 Oxford University Press. (*b*) Experimental tank (Paris Manuscript I, f. 41v). Image reprinted with permission from RMN–Grand Palais (Institut de France), copyright RMN-Grand Palais/Michael Urtado.

water exits at the bottom of a tank, . . . make the vessel of straight sheets of glass, as you see, and stir up the water with paper, mashed up, and note the movement of these particles in their flow."

It is important to note that however recognizable such experiments might seem to modern fluid mechanicians, they were not always designed to test familiar concepts, nor did they lead Leonardo to the same conclusions. Leonardo's analysis was shaped by his access to appropriate technologies, his training, and his worldview about what was possible. In describing experiments that appear to test hydrostatic force at different depths, for example, Leonardo's general tendency to think mechanically (rather than mathematically) led him to sketch a tank (see **Figure 2**) where one wall had been replaced by horizontal planks of wood, each attached to pulleys and weights designed to measure the water force on that wall. Despite the revelatory potential suggested by such diagrams, Leonardo's thought on this topic across his oeuvre is inconsistent (see Macagno 1988c for a full discussion).

Leonardo's experiments and practical investigations extended to many fields of study beyond that of fluid flows, not least of which are his anatomical studies of animals and the human body.

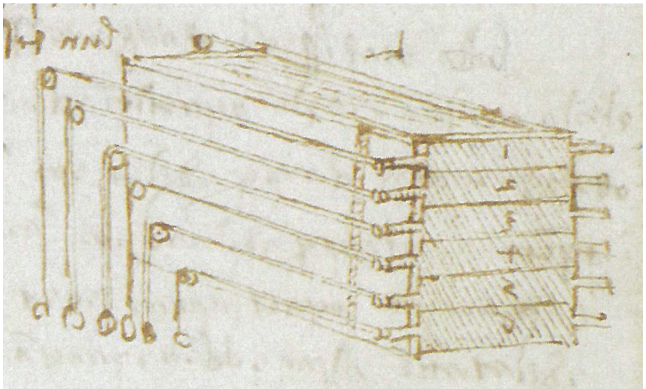

Figure 2

Sketch showing a mechanical measure of hydrostatic force (Codex Leicester, f. 6r). Image reprinted with permission of the licensor through PSclear from Laurenza & Kemp (2019), copyright 2019 Oxford University Press.

His experimental investigations of the aortic valve, for example, are of particular interest here, as they sit at the intersection of anatomy and fluid mechanics and are discussed further below. In the sections below, we identify and analyze several areas of significance to fluid mechanics within Leonardo's work.

5. TURBULENCE

There have been numerous references to Leonardo foreshadowing key concepts used in the modern study of turbulence physics. A sense of Leonardo's restless mind runs deep in the scholarship of this individual. Making comparisons to Vincent Van Gogh, Macagno (2000, p. 3) argued that "a deep and a vivid sense of kinematicity impregnates all [Leonardo's] work from art to zoology."

Moreover, the word *turbolenza* appears throughout Leonardo's writings, both as a noun and in its adjectival forms. Most commonly cited in fluid mechanics literature is Leonardo's listing of varied enquiries in the Codex Atlanticus, which includes topics such as "where the turbulence of the water is generated," "where the turbulence of the water is maintained for some time," and "where the turbulence of the water is lost" (f. 57v). In this context, the shared English and fluid mechanics term, "turbulence," seems to us appropriate. As such, it is no surprise that this is a commonly cited example within the technical community.

However, Leonardo employed this word in other contexts with a meaning that is not the modern fluid mechanics concept. For example, in Codex Arundel, Leonardo described water flow in terms that seem to suggest something other than the modern, technical sense: "So when it is murky and ruinous it rages, when it is clear and calm, ~~so it shows,~~ it bounces along with its gentle course between fresh grassland" (f. 57v). Here, Leonardo appears to be contrasting *turbulenta* against *lucida* ("clear"); hence, a meaning akin to murky or cloudy seems intended. Further, in his painterly recommendations on how to depict the biblical deluge among his loose sheets, Leonardo recommended the creation of a dramatic scene in which a "crumbling descent" and the "turbulent flow" of topsoil, brushwood, and rocks flow from the precipice of a rugged mountain [RCIN (Royal Collection Identification Number) 912665r]. This usage seems neither technical nor descriptive of water turbidity, but rather something more akin to such anthropomorphic meanings as "furious or raging movement of water," terms Leonardo used elsewhere. Likewise, in his treatise on painting, Leonardo provides direction on the depiction of soldiers at war, saying "the more the combatants are among the turbulence [i.e., melee], the less they will be visible and the less difference there will be between the light and shadow of them" (Farago et al. 2018, p. 651). Leonardo's sketch studies for his fresco *The Battle of Anghiari* are characterized by this strikingly dynamic sense of fluid flow among the men and animals at war (**Figure 3**). Macagno's awareness of Leonardo's multiple meanings for the term *turbolenza* across his work led him to employ a short form "TURB" to distinguish it explicitly from modern technical usage.

Others have focused upon Leonardo's extensive reference to a range of scales of eddies and their random nature at small scales when describing water and air flows. For example, Leonardo wrote in Paris Ms. G,

> Running water has within itself an infinite number of movements greater or lesser than its principal course. [. . .] This movement being sometimes rapid and sometimes slow, and turning sometimes to the right, sometimes to the left, now up, and now down, turning over and turning back upon itself, now one direction, now in the other, obeying all its forces ~~in the sa~~ and in the battles made by such forces, remaining always the prey of the victor. (f. 93r)

And in Paris Ms. F, he wrote, "The small eddies are almost innumerable, and large things are only turned round by large eddies and not by small ones, small things revolve both in small eddies and large" (f. 3r).

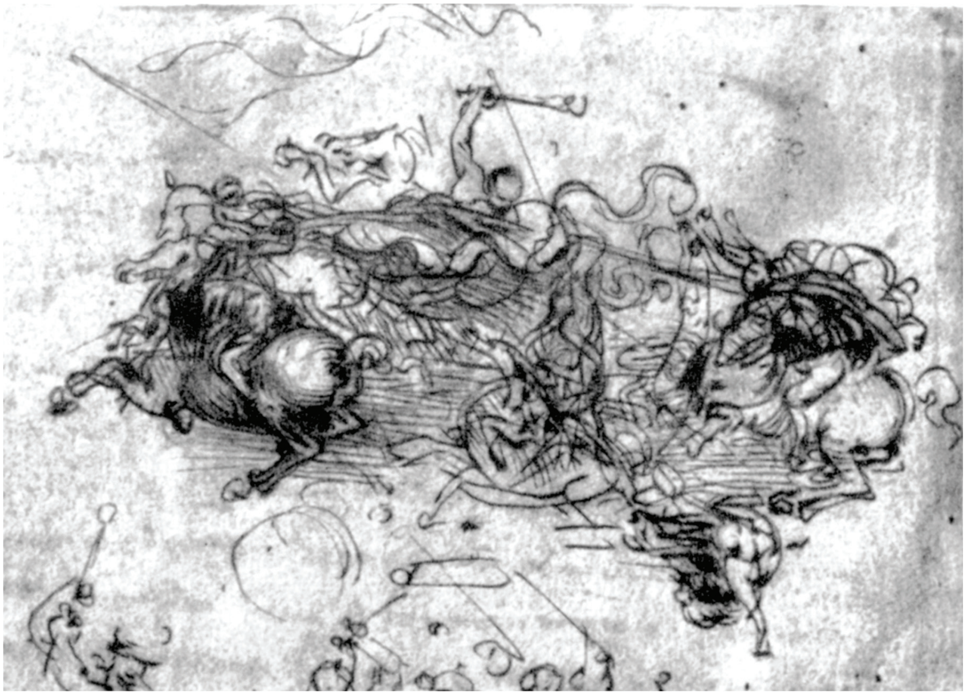

Figure 3

Sketch study for *The Battle of Anghiari* (circa 1503–1504). Pen and ink on paper, Gallerie dell'Accademia, Venice. Image reprinted with permission from World History Archive/Alamy Stock Photo.

Such descriptions have led Gad-el-Hak (2000) and others to conclude that they presage the concept of coherent structures and the Richardson–Kolmogorov cascade. Indeed, there are similarities between Leonardo's descriptions and that expressed in L.F. Richardson's adapted version of Augustus De Morgan's poem *Siphonaptera*, which was itself an adaptation of a poem by Jonathan Swift (Richardson 1922):

> Big whirls have little whirls
> That feed on their velocity,
> And little whirls have lesser whirls
> And so on to viscosity.

Figure 4 shows another commonly cited text with sketches of an old man and of turbulent wakes behind obstacles in a stream. The text reads

> Note the movement of the water level which does as hair, which has two motions, one of which depends on the weight of the fleece, the other on the way of the twists [ie curls], so too water has intense turning twists, one part of which depends on the impetus of the main course, the other on incidental and refractory motion.

Taylor (1974, p. 2) suggested that Leonardo "seems to be thinking about ways of separating flow into steady and turbulent components," and Lumley (1992, p. 203) concluded that the text seems to be "a clear prefiguring of Reynolds decomposition."

A rich example of multiscale turbulent flow is shown in the suite of Leonardo's so-called deluge drawings depicting the cataclysmic biblical event as storms, which date from the later period of

Figure 4

Sketches and notes of wake flows. Royal Collection at Windsor (RCIN 912579r). Image reprinted with permission of Royal Collection Trust, copyright 2020 Her Majesty Queen Elizabeth II.

his life (1517–1518). **Figure 5** shows one of these drawings. When Leonardo wrote down his thoughts about how to visualize a biblical deluge, central to his thinking were distinctions between the different forms of water's movements: "The swollen waters will swirl around the pool that encloses them, striking with dizzying eddies against the different objects, and throwing out into the air muddy foam, then falling back, making the beaten water refract in the air" (RCIN 912665r). It is interesting to consider the nature of these eddies whose spiraling shape appears to be repeated across a range of scales—this is discussed below in Section 9.

6. VELOCITY GRADIENT IN PROXIMITY OF A SURFACE: THE NOTION OF A BOUNDARY LAYER

The term "boundary layer" was first introduced in the literature by Prandtl (1904), who also presented the theoretical and mathematical basis of boundary layer theory. However, the effects of a boundary layer were previously discussed by others. For example, Froude (1874) carried out a series of towing tank experiments to study the frictional resistance of thin flat plates with varying roughness, and rightly concluded that the velocity of flow must vary with distance from the wall. In reference to Froude's studies, Taylor (1937) used the term "skin friction belt" to refer to the layer of fluid adjacent to wall that produces the skin friction drag.

Leonardo had certainly not theorized boundary layers as we understand them today, but his notebooks suggest that he was aware of the retarding effect of a fixed solid boundary on an adjacent liquid current (e.g., Codex Arundel, f. 136r; Codex Atlanticus, ff. 81v, 124r; Paris Ms. G, f. 14v; Paris Ms. M, f. 45r). Therefore, his work appears to contain the first known reference to the effect of the no-slip condition. In Paris Ms. F, for example, Leonardo listed 18 topics of water flow that he planned to write about, the first listed being: "Of the varied speeds of streams from the surface

Figure 5

One of Leonardo's series of deluge drawings. Royal Collection at Windsor (RCIN 912382). Image reprinted with permission of Royal Collection Trust, copyright 2020 Her Majesty Queen Elizabeth II.

of the water to the bottom" (f. 23v). Another notable example is found in his writings on the aortic heart valve (see Section 7), where Leonardo recognized that the solid surface of the valve retards the flow along the valve leaflet (Gharib et al. 2002, Keele 1973). **Figure 6** shows varied interpretations of pipes taken from a crammed notebook page devoted to the analogy between flow in pipes and the outflow of blood from the aorta (RCIN 919117). The text in the **Figure 6*a*** reads: "The water that rises through a pipe, that which rises highest will be furthest away from the walls of the pipe." Further, in considering a horizontal flow (**Figure 6*b***), Leonardo commented, "Of the water that pours out of a horizontal pipe, the part of its intersection that will be most remote from the mouth of the pipe [is] that which originates closest to the center of the mouth of the pipe."

The varying profile of velocity across the diameter of a pipe is revisited in other places. For example, the upper diagram in **Figure 1*b*** (Paris Ms. I, f. 41v) shows Leonardo's interpretation of how the central part of water issuing from a pipe flows further than that originating at the sides. Further, in the Codex Arundel (ff. 136r–137v), Leonardo appeared to refer to a velocity distribution across the flow near an interface: "The part of the liquid will be faster in its movement [is] that which is furthest from rubbing with [material] denser than it" (f. 136r). And in Paris Ms. L, he wrote, "If here water moved as fast as air, the boat would move like the wind without sails, but because wind is faster higher up than lower down but more powerful, it is wind in the sail than in the water [that is operating]" (f. 47v). As noted by Macagno (1989), Leonardo seemed to be considering here the property of the atmospheric boundary layer adjacent to the sea surface. In the context of boundary layers, it is also interesting to note in the Paris Ms. L, Leonardo appeared to mention the effect of surface roughness leading to increased (skin-friction) drag: "the ball being poorly cleaned has a difficult curved rubbing [friction] with the air that surrounds it" (f. 43r).

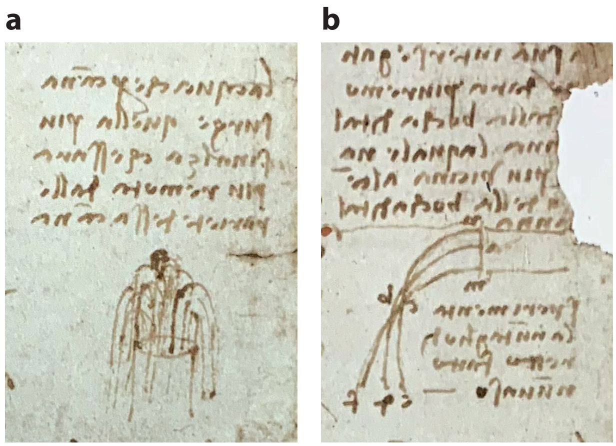

Figure 6

Pipe flows showing a varying velocity gradient. Royal Collection at Windsor (RCIN 919117r). Images reprinted with permission of Royal Collection Trust, copyright 2020 Her Majesty Queen Elizabeth II.

7. VORTICES AND THE HEART

Leonardo's anatomical studies have been described and discussed across an extensive literature. In the latter phase of these studies (circa 1508–1509 and 1513), Leonardo turned his attention to the cardiovascular system. In a period in which dissection as an educational practice was primarily prohibited by the Catholic Church, scholarly discussion about the function of this system was largely theoretical, derived from the authoritative work of Galen (circa 200 CE). Leonardo's own contributions to this subject through his detailed anatomical drawings were not known to his contemporaries.

For modern scholars, perhaps Leonardo's most celebrated achievement is his visual interpretation of flow through the part of the aorta known as the sinus of Valsalva, creating a vortex that serves to close the valve of a beating heart. This was carefully and fully discussed by Gharib et al. (2002). Leonardo's description and analysis are found across six loose sheets in the Royal Collection at Windsor, consisting of some 20 drawings and extensive notations (Keele 1973, Keele & Pedretti 1979). As discussed in Section 6, some of these drawings are of flow through pipes (see **Figure 6**) and relate to Leonardo's consideration that flow nearest the pipe walls is retarded compared to flow in its center. From this, he seems to have concluded that the flow through the valve will result in a spiral vortex (see **Figure 7*a*,*b***):

> the middle of the blood which surges through the triangle a b c acquires much more height than that which surges along the sides of the triangle, for that which in the middle of the triangle sends its impetus straight up and that which rises up from the sides spreads its impetus by lateral motion and hits the front of the arches of the hemicycle and follows the concavity of this hemicycle always declining until it hits the concavity of the bottom of this hemicycle and then turns up with reflected motion and goes on ~~revolving~~ in a fickle turning motion revolving until it consumes its impetus. (RCIN 919117r)

Gharib et al. (2002) highlight a second interesting point concerning Leonardo's description, that he recognized that the impulsive motion along a surface would generate a layer that could possess eddying motion, and he correlated the formation of vortices with the separated shear layer from the lips of the leaflets. Importantly, Leonardo identified the vortex formation in the sinus as the mechanism responsible for the closing of the valve. In vivo verification of this explanation had to

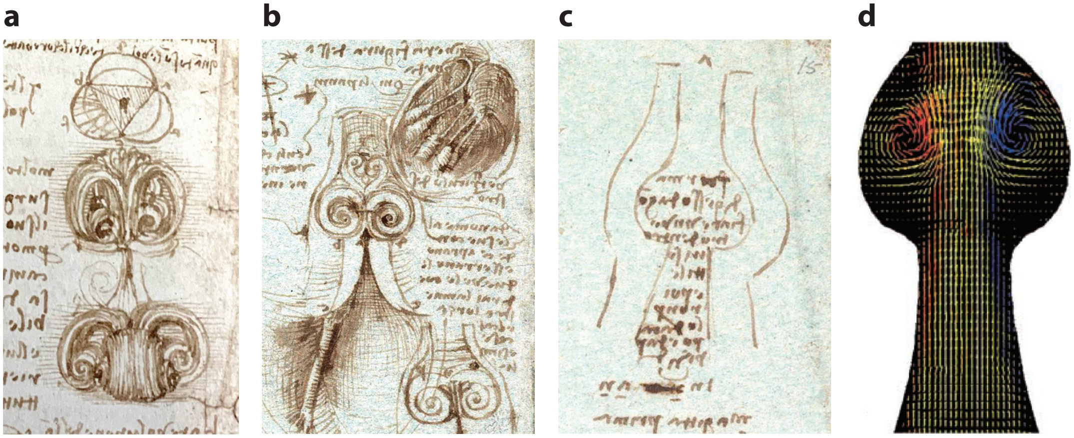

Figure 7

(*a*,*b*) Drawings of vortices in the aorta. Royal Collection at Windsor (*a*, RCIN 919117r; *b*, RCIN 919082r). (*c*) Sketch of a glass model of the base of the aorta (RCIN 919082r). (*d*) Particle image velocimetry measurements. Images reprinted with permission from (*a*–*c*) Royal Collection Trust, copyright 2020 Her Majesty Queen Elizabeth II, and (*d*) Gharib et al. (2002), copyright 2002 Springer Nature.

await the application of magnetic resonance imaging techniques to map the velocity field inside the sinus of Valsalva (see Bellhouse & Talbot 1969, Bissell et al. 2014).

There is only one record of Leonardo conducting vivisection himself (on a frog) and some evidence that he was not favorable to such practices. However, he did visit an abattoir to analyze the flow of blood from the hearts of slaughtered pigs (Keele 1961). Leonardo was also able to draw upon his training as a sculptor, having apprenticed in Verrocchio's Florence studio during the 1460s. Keele & Pedretti (1979), Gharib et al. (2002), and others have discussed the important role flow visualization played in Leonardo's thought on the human cardiovascular system. **Figure 7*c*** shows a sketch of a cast of the base of the aorta, which Leonardo describes as being for the purpose of visualizing flow within the cavity. The text within the illustration of the cast reads: "A gypsum shape [i.e., mould] to shape inside it a thin glass and then break it from head to foot at ~~n a~~ n n. But first of all put wax in the door [i.e., valve] of a bull's heart to see the true figure of its door" (RCIN 919082r). Similarly, Leonardo proposed elsewhere: "The shape of the glass to see in glass what the blood of the heart does when is closes the little doors of the heart" (RCIN 919076v). By seeding the flow within the glass model, an observer would have been able to see the vortices that are formed. Accordingly, Leonardo interpreted the closure of the aortic valves by the eddies of issuing blood (**Figure 7*b***) (Keele & Pedretti 1979).

Gharib et al. (2002) repeated the experiment from Leonardo's notebook, constructing a glass model according to the sketch in **Figure 7*c***. **Figure 7*d*** shows the resulting velocity field from a particle image velocimetry measurement where a pulsatile pump was connected to the outlet of the glass model. These quantitative measurements are seen to have striking similarities to Leonardo's sketches of the flow.

8. LEONARDO'S PARADOX

Another topic of fluid mechanics that has become associated with Leonardo is that of the motion of bubbles in a still liquid, now commonly termed Leonardo's paradox (Ohl et al. 2003, Prosperetti 2004). The paradox is one where, as expected, gas bubbles of sufficiently small size rise along a

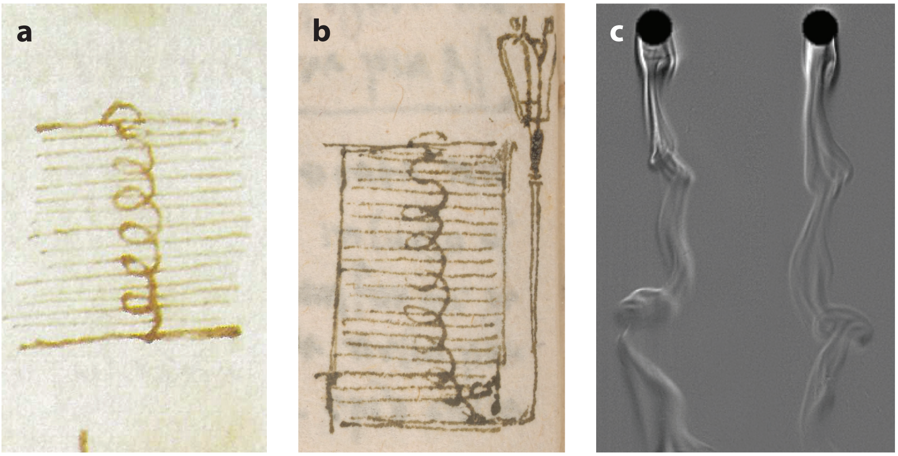

Figure 8

Rising bubbles in water in (*a*) Codex Leicester (f. 25r) and (*b*) Paris Manuscript F (f. 37v). (*c*) Unsteady path of spheres. Panel *a* reprinted with permission of the licensor through PLSclear from Laurenza & Kemp (2019), copyright 2019 Oxford University Press; panels *b*,*c* reprinted with permission from (*b*) RMN–Grand Palais (Institut de France), copyright RMN-Grand Palais (Institut de France)/Michael Urtado, and (*c*) Veldhuis et al. (2005), copyright 2005 IOP Publishing Ltd.

rectilinear path. Larger bubbles, however, rise along a spiral or zigzag path. Leonardo's work is the first known account to document the phenomena.

Figure 8*a*,*b* shows sketches from the Codex Leicester (f. 25r) and Paris Ms. F (f. 37v) of an air bubble rising through still water. The text that appears along **Figure 8*b*** describes the trajectory of air escaping from under water as "twisted like a snake in its elevation" (f. 37v). Interestingly, this paradox has persisted for centuries with the current explanation for the nonrectilinear path caused by a wake instability leading to a double-threaded wake (de Vries et al. 2002, Mougin & Magnaudet 2002, Ohl et al. 2003). This replaces earlier hypotheses based on the interaction between the instability of the rectilinear motion and a periodic oscillation of the wake (see, for example, Meiron 1989, Saffman 1956). The same mechanism, with a double-threaded wake structure, has been used to explain the unsteady path of solid spheres rising or falling freely in liquids, under the action of gravity, as shown in **Figure 8*c*** (Veldhius et al. 2004).

Leonardo's thinking appears strongly influenced by the general law, "Every impetuous movement bends towards the less resistance, fleeing from the greater," which was recorded in another hand in Codex Atlanticus (f. 865r), where Leonardo himself wrote, "Impetus is the impression of local movement transmitted from the mover to the mobile [thing]" (f. 460v). Leonardo applied this principle to explain curved paths of bubbles and other phenomena such as lightning. For example, in Paris Ms. F, he wrote

> If each moving object follows its motion by the line of its principle, what is it that makes the motion of the arrow or lightning bolt be crooked and bendable in many ways being in the same air? What is said ~~is due~~ to can be due to two ~~places~~ causes, one of which is that the air that condenses them in front of its furious impetus makes them resistant, and thus such motion is bending and is of the nature of a refractory motion but is not rectilinear and goes like the third [point] of the fifth [book] on waters, where it shows that air that sometimes comes from the bottom of marshes in the form of a rattle [snake] to the surface in a curvilinear crooked motion. (f. 52r)

The topic of forces and motion was one that Leonardo visited often in his notebooks but the concepts available to him predated Newtonian mechanics (Hart 1925, Ohl et al. 2003).

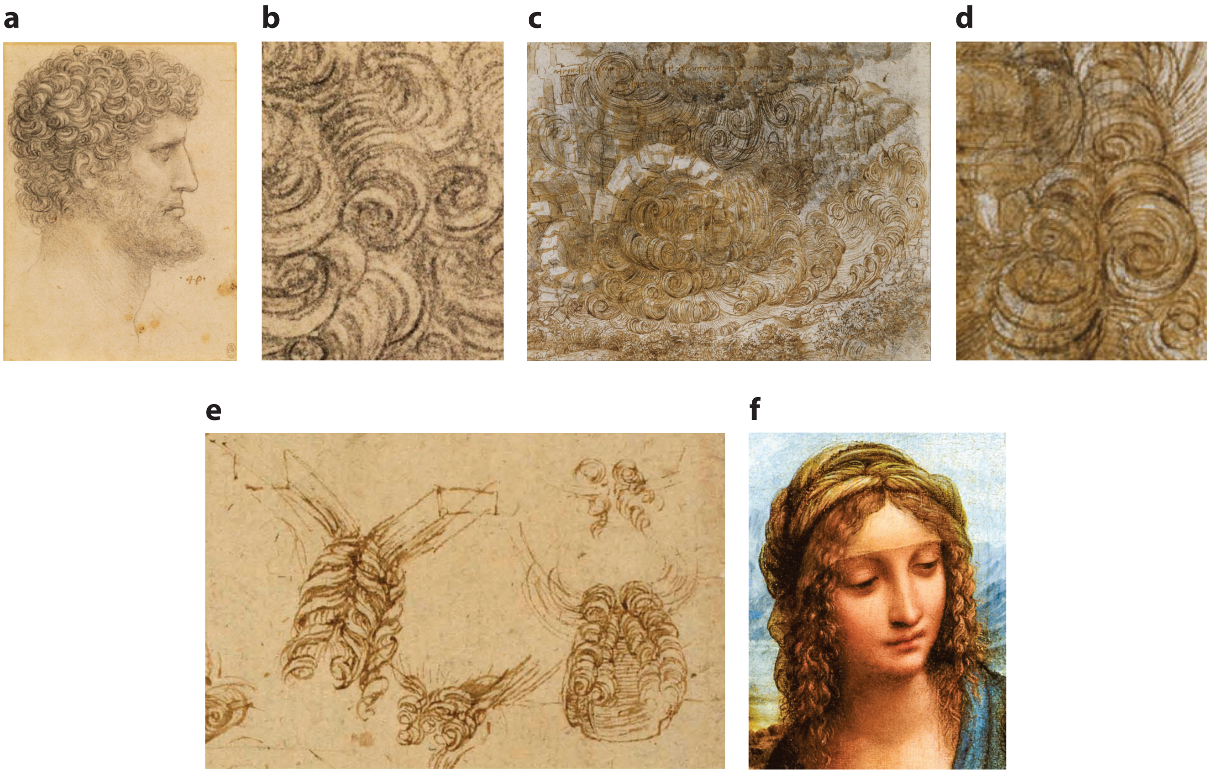

Figure 9

(*a*) Sketch of a bearded man. Royal Collection at Windsor (RCIN 912553). (*b*) Zoomed-in crop of hair sketch in panel *a*. (*c*) Deluge drawing (RCIN 912380r). (*d*) Zoomed-in crop of turbulent flow drawing in panel *c*. (*e*) Wake flow (Codex Atlanticus, f. 1,098r). (*f*) Zoomed-in view of *Madonna of the Yarnwinder* (circa 1510). Images reprinted with permission from (*a–d*) Royal Collection Trust, copyright 2020 Her Majesty Queen Elizabeth II; (*e*) World History Archive/Alamy Stock Photo; and (*f*) Biblioteca Ambrosiana Milan.

9. OBSERVATION AND SYNTHESIS

Macagno (1986, 1987) identifies Leonardo's use of analogies as central to his method of analysis. This is worthy of further consideration as it provides insights into Leonardo's drawings and notes. In Section 5, it was noted that Leonardo commented on the motion of the surface of the water resembling that of hair. This is further highlighted in **Figure 9**.

Figure 9*b* is a zoomed-in crop of a sketch of a bearded man shown in full as **Figure 9*a***, while **Figure 9*d*** is a zoomed-in crop of the deluge drawing shown in **Figure 9*c***. **Figure 9*b*,*d*** are seen to have spiralling patterns that share almost identical characteristics. **Figure 9*a–d*** are from the latter period of Leonardo's life (circa 1517) but almost the same patterns are found in the locks of hair of the Archangel Gabriel in the *Annunciation* (circa 1472), Leonardo's first major painting completed while still working in the workshop of Andrea del Verrocchio. The analogy between hair and water continues with Leonardo's characterization of helicoidal spiral vortices in fluid flow, as shown in **Figure 9*e***, which have a clear resemblance to the curls of hair in *Madonna of the Yarnwinder* (circa 1510) shown in **Figure 9*f***, as well as in other portraits such as *Salvator Mundi* (circa 1500).

Such analogies reflect a broader approach where Leonardo appears to start with the simplifying hypothesis that elements in nature follow a common pattern. For example, in Paris Ms. A, Leonardo drew a direct analogy between the flow of blood in the human body and the flow of water in the environment: "As from the lake of blood flow the veins which branch out across the human body, so too the ocean fills the body of the earth with an infinite number of veins of water" (f. 55v). Importantly, Leonardo tested this thinking through experiment and observation and modified his hypotheses as needed. Analogy between natural world systems and the human body are a common knowledge framework in Leonardo's time, as suggested by di Giorgio in his *Trattato di architettura*: "All art and reason are derived from a well-composed and proportioned human body" (di Giorgio 1967, pp. 3–4). Leonardo recorded similar comments in Paris Ms. A, noting: "Man is called by the ancients a little world . . . they are very similar" (f. 55v). As art historian Jill Burke (2018a, p. 200; 2018b) has noted, Leonardo blends a reconsideration of Vitruvian proportions with the tradition of thinking about proportion stemming from the medieval painters' workshops, one of which Taccola before him had explored in drawing. The dimensions of the Vitruvian man that is now so closely associated with Leonardo (see **Figure 10*a***) are similarly geometrically self-similar, so that if one provides the length of any body part, then all other lengths are known by a given scaling factor. In his thinking toward this conceptualization, Leonardo made and documented extensive measurements of many subjects, as demonstrated in **Figure 10*b*,*c***.

Leonardo, either purposely or intuitively, applied the concept of a repeating "universal" pattern when drawing spiralling eddies or curls. These patterns appear prolifically throughout his notebooks and paintings. **Figure 11** shows four examples from different applications. Superimposed upon **Figure 11** are logarithmic spirals, which agree remarkably well with Leonardo's freehand sketches. Logarithmic spirals are geometrically self-similar. [Interestingly, the similarity solution of the Birkhoff–Rot equation for describing the evolution of a vortex sheet is referred to as the Prandtl spiral, which is also a logarithmic spiral (see Eggers & Fontelos 2015).]

Leonardo could not know the mathematics of logarithms, first described by John Napier almost a century later. However, Leonardo would most likely have been aware of the golden spiral, which is a logarithmic spiral with an exponential growth factor related to the golden ratio, also known as the divine proportion. Leonardo was friends with Luca Pacioli, who tutored him in mathematics and was a fellow member of the Sforza Ducal court in Milan. In 1509, Pacioli published a book titled *De Divina Proportione*, for which Leonardo prepared 60 illustrations (incidentally, the only work Leonardo published during his lifetime).

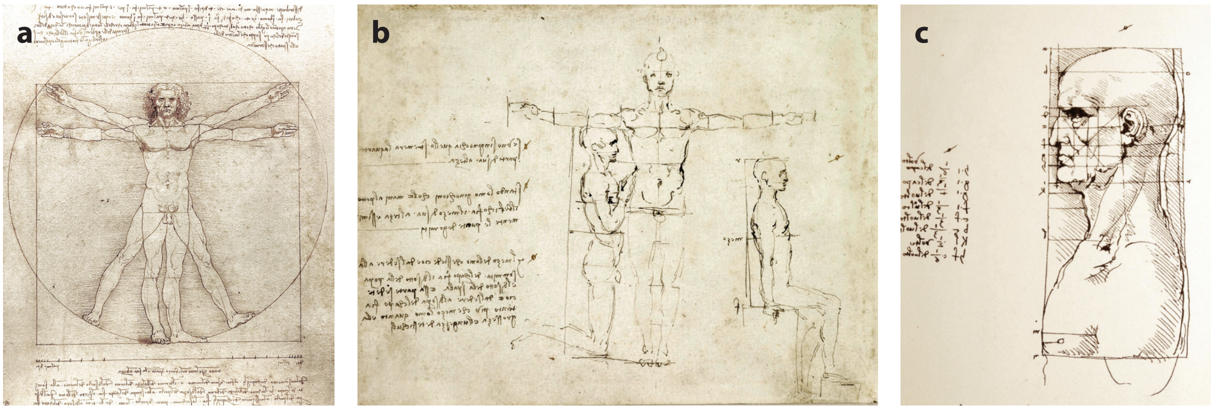

Figure 10

(*a*) *Vitruvian Man* (circa 1490). Gallerie dell'Accademia, Venice. (*b*) Length measurements of human subjects. Royal Collection at Windsor (RCIN 919132). (*c*) Male head in profile with proportions (circa 1490). Gallerie dell'Accademia, Venice. Images reprinted with permission from (*a*,*c*) World History Archive/Alamy Stock Photo and (*b*) Royal Collection Trust, copyright 2020 Her Majesty Queen Elizabeth II.

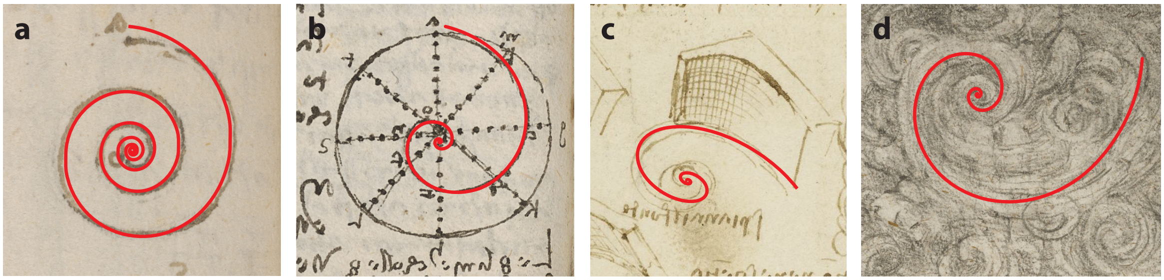

Figure 11

Logarithmic spirals. (*a*) Logarithmic spiral with growth factor 0.191 (Paris Manuscript E, f. 34v). (*b*) Golden spiral (Paris Manuscript G, f. 54v). (*c*) Golden spiral. Royal Collection at Windsor (RCIN 912666r). (*d*) Zoomed-in view of **Figure 5** with golden spiral. Images adapted with permission from (*a*,*b*) RMN–Grand Palais (Institut de France), copyright RMN-Grand Palais/Michael Urtado, and (*c*,*d*) Royal Collection Trust, copyright 2020 Her Majesty Queen Elizabeth II.

The golden spiral can be very well approximated geometrically using a compass, of which Leonardo was a master, with an ever-repeating subdivision of a rectangle whose aspect ratio is the golden ratio. Whether Leonardo used this directly in his visual texts is unknown, and he makes no specific mention of it in his surviving notebooks. Most likely, his sketches are intuitive but the similarities with the golden spiral in particular are marked. The spirals shown in **Figure 11*b*–*d*** are golden spirals, a pattern that we see repeatedly throughout his sketches, including in the deluge drawings (**Figures 5** and **9*c***) and in **Figure 12*a***, which has been noted as incorporating floral analogies. For example, the boils at the water surface closely resemble Leonardo's sketches of the Star of Bethlehem (Kemp 2007). These examples highlight the importance of analogies in Leonardo's thinking and the underlying idea that repeating patterns occur across nature.

Figure 12*a* is arguably among the most famous of Leonardo's beautiful diagrams that connects to modern fluid mechanics principles. It interprets the action of a plunging jet of water and the resulting multiphase turbulence with bubbles of air and water eddies interacting at a range of

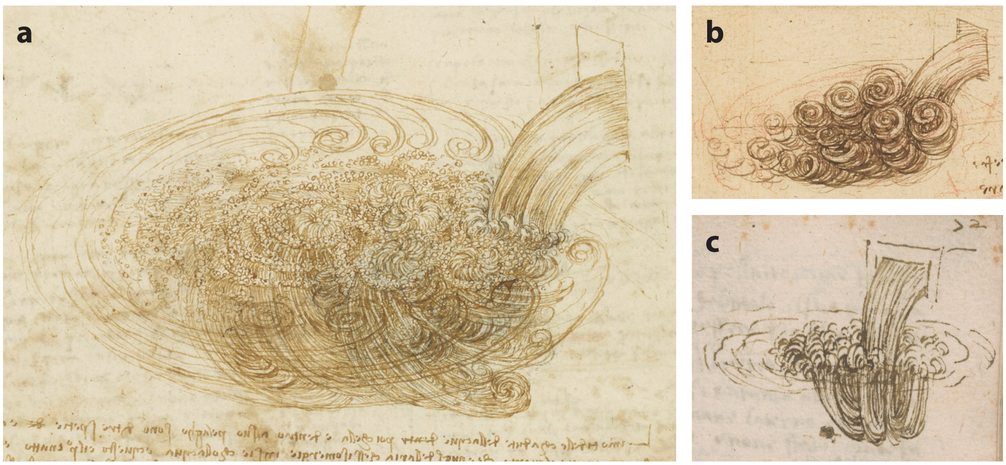

Figure 12

Sketches of a plunging water jet into a pool, with the resultant turbulent flow. (*a*) Royal Collection at Windsor (RCIN 912660v). (*b*) RCIN 912662. (*c*) Paris Manuscript F, f. 72r. Images reprinted with permission from (*a*,*b*) Royal Collection Trust, copyright 2020 Her Majesty Queen Elizabeth II, and (*c*) RMN–Grand Palais (Institut de France), copyright RMN-Grand Palais/Michael Urtado.

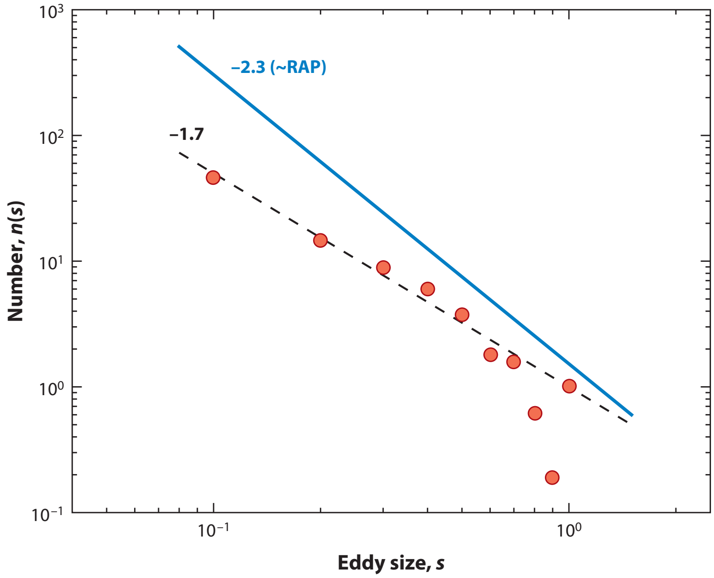

Figure 13

Spiral eddy number density distribution, $n(s)$, per eddy size, s, across 10 deluge drawings. The −1.7 power law slope is considerably less than the value of −2.3 expected for, e.g., random Apollonian packing (RAP).

scales. Art historian Martin Kemp describes it as "the most complete of all his water drawings. It is to his hydrodynamics what the 'great lady' anatomy is to his science of the human body, that is to say, a composite study in which cause and effects from many separate analyses are fused together in an astonishing synthesis" (Kemp 2007, p. 305). **Figure 12*b*,*c*** shows related sketches from preliminary stages in this synthesis.

Figure 12*a* is indeed an interpretive synthesis, not a snapshot of what the eye or a photograph would capture. The image reflects a theoretical construction, incorporating analogies, universality, and self-similarity. We have already mentioned that spiral eddies, well approximated with golden spirals, are prevalent, and they appear to make up the universal building blocks of the flow. **Figure 12*b*,*c*** shows two examples of this, giving insight into how Leonardo was thinking about vortices and how they can be used to describe the turbulent flow.

In Section 5 we referenced the nature of the eddies drawn in Leonardo's deluge drawings (circa 1517–1518). We make two observations here related to what was discussed above. The first is that the eddying patterns that Leonardo sketches are well approximated by a logarithmic spiral, and the same shape appears to be repeated across the range of scales—see **Figure 5** and its zoomed-in view, **Figure 11*d***. Accordingly, as discussed above, the eddies are well approximated by logarithmic spirals and therefore are nominally geometrically self-similar.

The second observation concerns the spatial distribution of the eddies. The eddies seen in **Figure 5** appear not to be purely space filling but have a degree of spatial intermittency. We can test this idea to some degree by analyzing the number density distribution per eddy size across several deluge drawings. In total, we analyzed 10 of the deluge drawings, where a template logarithmic spiral was fitted to all the observed eddies, resulting in 1,786 realizations. The eddy/spiral sizes, s, were normalized per drawing by the size of the largest eddy in the given drawing so that $0 < s \leq 1$. The resulting number distribution per size is shown in **Figure 13**. Since we are enforcing a self-similar structure by using one template pattern, we expect to have a power law distribution, and this is seen in the figure, confirming that we have a reasonable number of realizations for convergence. We note that the power law slope is approximately -1.7, considerably less than the value of $-(d_f + 1) \sim -2.3$ expected for a random space-filling pattern in a plane (as per random Apollonian packing of a plane, which has fractal dimension of $d_f \sim 1.3$; see Varrato & Foffi 2011).

As seen here in the case of the deluge drawings and in many other places, Leonardo repeats patterns, namely spirals that are geometrically self-similar, and he does so intuitively. The above analysis is limited in that we are not able to quantitatively compare it to real turbulent flows. However, we do note that Leonardo's deluge drawings are not simply space filling and demonstrate a degree of spatial intermittency, which is a feature qualitatively observed in modern visualizations of eddies from numerical simulations of turbulent flow.

10. FLIGHT

Leonardo's interest in flight and thus his engagement with fluid flow in air has been well explored (Bilancioni 1925, Giacomelli 1936, Gibbs-Smith 1967, Hart 1961, McCurdy 1910). In this section, we highlight some key elements of his thought pertinent to our wider review.

In some of his productions, Leonardo appears to consider both air and water as fluids to which equivalent fluid dynamic principles apply. In Paris Ms. E, he wrote:

> To give true knowledge of the movement of birds in the air it is ~~the~~ necessary to give first the knowledge of the winds, which we will prove by means of the movements of water in itself. This knowledge is attainable; it will provide a ladder to arrive at knowledge of birds in the air and the wind. (f. 54r)

Likewise, in Codex Atlanticus, we read, "Write of swimming underwater and you will have the flight of the bird through the air" (f. 571r). Yet Leonardo also seemed to have recognized that air was compressible while water is not, but this got him into some trouble when interpreting flight. Here, he interpreted the wing compressing the air as an essential mechanism for lift. In a short codex to the subject around 1505, he considered how birds gain altitude by pressurizing air:

> When the bird that by beating its wings wants to rise up, it raises its shoulders and beats the tips of the wings inverse of itself, and condenses the air that is between the tip of its wings and its breast which makes the bird rise upward. (f. 5v)

This seems to contradict his other statements above. Nonetheless, Leonardo made astute observations on the flight of birds and their maneuvers in thermals, updrafts, and wind shear. Richardson (2019) analyzed Leonardo's sketches and notes in Paris Ms. E (circa 1513–1515) and concluded that Leonardo documented the physics of what is now referred to as dynamic soaring, a technique described by and credited to Lord Rayleigh in 1883.

It is interesting to note in relation to describing changes in bird flight velocity, Leonardo describes movement in terms of *gradi*, which we interpret as "degrees." For example, in Paris Ms. L he wrote:

> More and much faster is the movement of volatiles [birds] than that of the wind, that if it were not thus, no bird would move against the wind but it moves so much less against this wind than its natural flow through still air that the degrees of the motion of the wind is less than the motion of the ~~b~~bird. (f. 55v)

This may suggest that Leonardo was adopting a nondimensional parameter, as deduced by Macagno (1989). However, it is likely that Leonardo intended a kind of relative unit of measure, although perhaps not a precise one.

Leonardo also extensively explored the idea of human-powered flight. This included designs for an ornithopter (see **Figure 14*a***). Integrating his study of avian anatomy and bone structure and

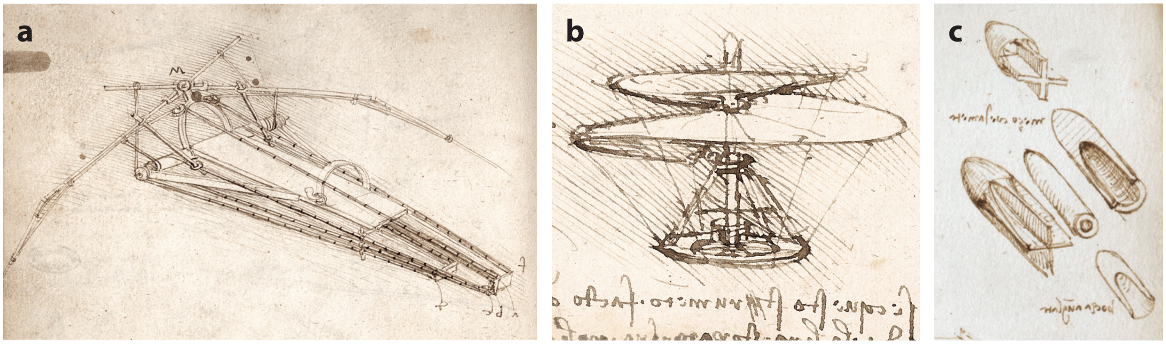

Figure 14

(*a*) Ornithopter (Paris Manuscript B, f. 74v). (*b*) Rotorcraft (Paris Manuscript B, f. 83v). (*c*) Streamlined bodies (Codex Arundel, f. 54r). Images reprinted with permission from (*a*,*b*) RMN–Grand Palais (Institut de France), copyright RMN-Grand Palais/Michael Urtado, and (*c*) the British Museum.

concluding that wings alone would not carry a human weight, he designed a complex machine in which the wings were coordinated with hand levers and foot pedals. Leonardo also contemplated vertical lift devices. Considering air as a fluid similar to water, he experimented with the conversion of an Archimedes screw in various forms as a mechanism to achieve lift. A sketch of such an air-screw rotorcraft is shown in **Figure 14*b***. In Paris Ms. B, he describes how, when "turned with speed, the said screw makes itself a screw in the air and will climb high" (f. 83v). While providing specific details about the dimensions and construction, there is no evidence that one was ever built and it would have faced the obvious challenge of attaining sufficient power to lift a human weight.

A very different application of Leonardo's ideas about the movement of objects through air can be found in Codex Arundel, where he sketched a series of projectile stabilizers and streamliners (see **Figure 14*c***). Unfortunately, the diagrams are only marked with the words describing their types of design ("angled mouth," for example), but no further commentary here reveals his specific thinking on this point.

11. LEONARDO, MASTER OF WATER

Leonardo was once famously named a *Master of Water* in the records of the Florentine government. In this role, he explored diverting the river Arno away from Pisa so as to cut access to the city, then Florence's enemy, from the sea. It was one of several jobs he held that were dedicated to controlling water as a way of wielding power. Leonardo's military contributions also included designs for the Venetian Republic to fortify the Isonzo River in the Friuli against possible Ottoman attack. Later, in France, working for Francis I, he reprised river diversion plans for the river Saudre. In 1482 Leonardo created a letter to present to Ludovico Il Moro Sforza, in essence a job application, which is contained in the Codex Atlanticus (f. 1,082r). Leonardo sketched for Ludovico an enticing array of military technologies to enhance his performance in war, but he also offered skills that would serve him in peace as well:

> In time of peace I believe I can give complete satisfaction, equal to any other man, . . . guiding water from one place to another
> I have ways of silently making underground tunnels and secret winding passages to arrive at ~~a certain~~ a desired [point], even if it is necessary to pass underneath trenches or a river. (f. 1,082r)

Leonardo's claims to specialist knowledge highlights his era's preoccupation with the power of water.

Leonardo's work on water was further extensively discussed in other treatises, including that of Pfister et al. (2009), who considered Leonardo's work in the context of modern hydrology; Kemp (2007); Geddes (2020); and others. Here we draw attention to how his extensive drawings, particularly in Paris Ms. H and the Codex Leicester, reflect his understanding of the power of flow around obstacles in a stream of water. **Figure 4** shows one such example, depicting a turbulent horseshoe vortex around an obstacle. Macagno (1988a) extensively discussed this topic and highlighted Leonardo's appreciation of the scouring and erosive power of these flows. Another important hydraulic phenomenon associated with the power of water is what in modern terms is termed a hydraulic jump, which Leonardo is often credited as the first to document and describe. He did indeed describe such flows, but he was not the first to have done so: Chanson (2000) described the use of steep chutes and dropshafts in Roman aqueducts, which invariably would have involved hydraulic jumps. Moreover, thirteenth-century sketches from China, such as *The Hangchow Bore in the Moonlight* by Li Sung (circa 1210, Sung Dynasty), depict tidal bores (H. Chanson, private communication). Even so, Leonardo independently described the phenomena in various places as documented by Macagno (2002). One example is in the Codex Atlanticus (f. 526v) where a small sketch is provided together with a description of flow down a steep-sloped channel. Leonardo also recognized that such abrupt jumps occur in simple flows such as for a laminar jet impinging on a surface (cf. tap flow into a sink). **Figure 15** shows a series of sketches of this flow for multiple jets and different surfaces. Interestingly, it has only recently been revealed that gravity plays no significant role in the circular hydraulic jumps of thin liquid films (Bhagat et al. 2018). Leonardo also studied the single-drop impact problem, which continues to be an active research area to this day. As noted by Villermaux & Bossa (2011), Leonardo's sketches of discrete liquid droplets impacting a sheet of paper in the Codex Leicester (f. 33r) show the radial fingers that result and note the axisymmetry of the impacted drop imprint pattern.

Leonardo also wanted to harness fluid flow as a source of power. In his plan for the river Arno, he had calculated how many men would be needed to create the ditches required and how many buckets a single man could dig in a day. He came up with a figure of some 54,000 man-days (Masters 1999). But Leonardo surmised that he could find alternatives to human labor, including both machines and natural forces. In the Codex Arundel (f. 29v), Leonardo appeared to consider

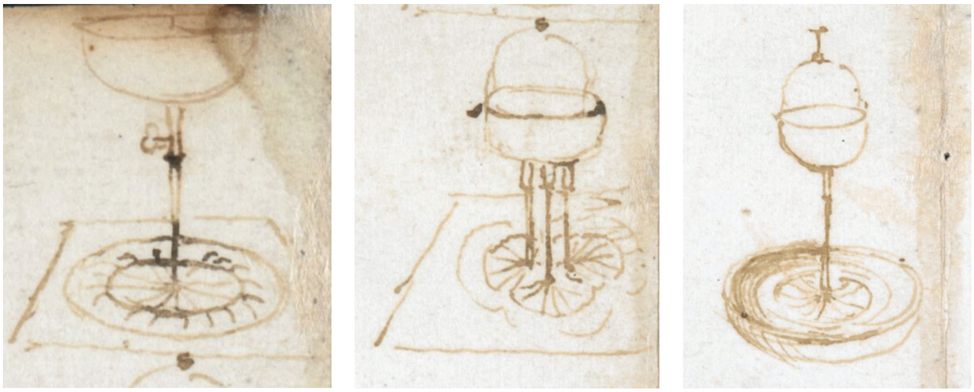

Figure 15

Hydraulic jumps (Codex Arundel, f. 167v). Images reprinted with permission from the British Museum.

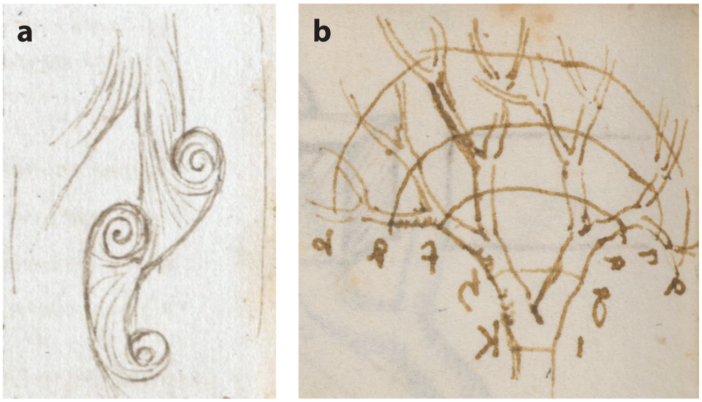

Figure 16

(*a*) Vortices strong enough to carve cavities in rock (Codex Arundel, f. 29v). (*b*) Conservation of volume for a branching tree (Paris Manuscript M, f. 78v). Images reprinted with permission from (*a*) the British Museum and (*b*) RMN–Grand Palais (Institut de France), copyright RMN-Grand Palais/Michael Urtado.

how vortices could be used to excavate channels (see **Figure 16*a***). In this image, he seemed to be reflecting on how vortices create bore holes.

Another important part of managing water causeways was understanding the relationship between the flow rate and the cross-sectional area of the ducts or open channels. Toward this, in several locations, Leonardo's notes suggest that he may have grasped the principle of the conservation of volume. For example, in Paris Ms. F, Leonardo wrote "Water that flows through a pipe which is empty and fills first the whole of its flat part, will fill up all the other parts, straight and oblique, and moving with equal speed" (f. 86v). An interesting analogy related to the conservation of volume, which Leonardo applied between different natural world systems, is that of the flow through branches of a tree and through rivers or closed conduits. For example, in Paris Ms. M, in reference to **Figure 16*b***, Leonardo wrote:

> each year when the branches of the plants have exhausted their growth, they comprise together as much as the size of their trunk, and in each degree of their [branch] growth, you will find the size of said trunk as in .i.K. .g.h. .e.f. .CD. .a.b. All of them will be the same the tree not being damaged; otherwise the rule does not fail. (f. 78v)

Just as he had observed of tree growth, Leonardo wrote of rivers in Paris Ms. I: "All the branches of trees, each degree of their height, combined, are equal to the size of their trunk—All the branches of waters, in each degree of their length being of equal movement, are equal to the size of their origin" (f. 12v).

Based on Leonardo's discussions, Macagno (1986) concluded that Leonardo can be credited with the first statements about conservation of volume, both in Eulerian and Lagrangian form, but it is unclear as to whether this extended to the conservation of mass. In the Codex Leicester, Leonardo presented detailed descriptions of water properties, suggesting that he considered water to be a purely incompressible liquid and thus that there would be no distinction between conservation of volume and mass. Leonardo did, however, recognize that water properties can change due to materials in suspension or in solution. For example, in Codex Leicester (f. 19v) he wrote that rainwater may have differences in density from summer to winter because it collects more dust during the summer. Similar variations are noted for turbid or saline waters, including variations due to changes in temperature (Macagno 1988b).

12. WEAPONIZING HIS THOUGHT

Finally, we should remember that Leonardo worked for some of the top military and political leaders in the Italian Wars, a major conflict that embroiled most of Western Europe. His patrons were numbered among Europe's leading families: Sforza and Borgia dukes and French Valois kings. As a designer, technician, and artist, Leonardo seized the professional and financial opportunities that war presented.

Leonardo created magnificent spectacles of his patrons' military achievements in festivities with advanced dramatic technologies. For instance, the festival Leonardo curated in France in May 1518 for his patron Francis I celebrated the king's military achievement. He staged an elaborate, multisensory reenactment of the Battle of Marignano complete with a siege and capture of a castle, during which falconets fired missiles of paper and mortars shot out balloons. Through these multimedia displays and performances, Leonardo helped to construct an identity for a man at war, shaped and defined by new technological advancements.

At other times, Leonardo followed the armies of his leaders as they waged war across Italy, but he did not fight on the front line as a soldier himself. His value to his patrons was not his body, but his mind. As we see in the letter to Ludovico Sforza above, he offered up his ideas for weaponization. Some of his commentary on weapon design concerned inflicting maximum psychological damage: a military campaign of shock and awe. Another idea that he explored was the use of poison gas on enemy combatants in two different notebooks (Codex Atlanticus, f. 346v; Paris Ms. B, f. 69v), evidence that is little discussed in the literature (Simms 1988).

Leonardo's 1482 letter to Ludovico is by no means unique among Leonardo's texts. In 1952, the summary of a letter purported to be from Leonardo to Sultan Beyazid II, dated from July 3, 1503, was found in the State Archives in Istanbul (Babinger 1952). His letter offered a suite of new technologies, informing the sultan, for example, that "God, may He be praised, has given me a knack of pulling water from ships without ropes or cables, using a machine that turns on its own" (Babinger 1952, p. 4). Leonardo's design for the Architronito, a steam-powered cannon made of copper, an adaption of a design he attributed to Archimedes, involved water being poured into the heated breach, resulting in steam pressure shooting out the cannon ball. In something of a drafted sales pitch for his invention, he remarked that "it throws iron balls with great noise and fury" and "to see the fury and hear the roar will seem like a miracle" (Paris Ms. B, f. 33r).

A careful study of the full range of Leonardo's texts reminds us that even a man who expressed at best ambiguous thoughts about war still needed to make a living, and indeed to have the money to conduct the experiments that were so fundamental to his sense of self and identity.

13. CONCLUSIONS AND SUMMARY

Leonardo's curiosity for fluid flow in the natural world, perhaps most particularly in water, was lifelong. It was a topic to which he returned time and again and in the many different forms—visual, textual, and material—that his thought was expressed. His considerations regarding flow systems were, moreover, profound, ranging from attempts to understand elemental aspects to whole-of-system analyses. Furthermore, as this review has demonstrated, we can trace his engagement with fluid flow from abstract theoretical conceptualizations to the many potential applications that his works proposed. Finally, even if all the many complexities of fluid flow remained beyond his reach, Leonardo nonetheless knew how to work with it, both harnessing flow forces for their utility to humans and making their beauty a distinctive feature of his artistic creation.

SUMMARY POINTS

1. Leonardo da Vinci's work and thought related to fluid mechanics are present across his notebooks, letters, and artwork.
2. Leonardo's remaining works offer no straightforward compilation of his knowledge, but rather a complicated picture of unfinished, scattered, and frequently revisited hypotheses and conclusions across his lifetime.
3. Key terms in fluid mechanics, such as turbulence, hold multiple and ambiguous meanings in his works.
4. Experimentation formed an important mechanism for Leonardo's thought about natural fluid flows, which was an innovation to the scientific thinking of his day, but which did not always lead him to the conclusions of modern fluid mechanics.
5. Leonardo's thinking was suggestive of such modern concepts as the no-slip condition, hydraulic jump, cardiovascular vortices, conservation of volume, and the distinctive path of ascending bubbles we now term Leonardo's paradox.
6. Leonardo thought through analogies, building-block flow patterns, and synthesis, leading both to successes—especially in the management of water—and to failures, perhaps most obviously in his pursuit of human flight.

DISCLOSURE STATEMENT

The authors are not aware of any biases that might be perceived as affecting the objectivity of this review.

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