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Combustion and Engine-Core Noise

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Abstract

The implementation of advanced low-emission aircraft engine technologies and the reduction of noise from airframe, fan, and jet exhaust have made noise contributions from an engine core increasingly important. Therefore, meeting future ambitious noise-reduction goals requires the consideration of engine-core noise. This article reviews progress on the fundamental understanding, experimental analysis, and modeling of engine-core noise; addresses limitations of current techniques; and identifies opportunities for future research. After identifying core-noise contributions from the combustor, turbomachinery, nozzles, and jet exhaust, they are examined in detail. Contributions from direct combustion noise, originating from unsteady combustion, and indirect combustion noise, resulting from the interaction of flow-field perturbations with mean-flow variations in turbine stages and nozzles, are analyzed. A new indirect noise-source contribution arising from mixture inhomogeneities is identified by extending the theory. Although typically omitted in core-noise analysis, the impact of mean-flow variations and nozzle-upstream perturbations on the jet-noise modulation is examined, providing potential avenues for future core-noise mitigation.

1. INTRODUCTION

According to the global air transportation outlook by the International Civil Aviation Organization (ICAO 2013), civil air traffic is projected to grow at an annual rate of 4.6 %, resulting in a doubling of air transportation over the next 15 years. Although the increasing demand in passenger and cargo transportation is a consequence of economic growth, technological advancements, and market liberation, constraining factors are operating cost, airport capacities, and environmental pollution and noise. Aircraft noise is of major concern because it adversely affects the quality of life, health, and property value of communities in proximity to airports and main flight corridors. A balanced approach is pursued to mitigate noise exposure by considering residential and land-use planning, operational procedures, operating restrictions, and noise reduction at the source. Whereas the first three strategies are concerned with regulatory noise-mitigation measures, the last strategy is potentially most effective as it aims to reduce noise at its inception.

Main contributors to noise emission from aircraft are the airframe and engines. Airframe noise, typically broadbanded with tonal components, is generated by the fuselage, nacelle, landing gear, wings, and other lifting surfaces. Engine noise has contributions from the fan, compressor, combustor, turbine, and jet exhaust and is examined in this article.

Since the introduction of the jet engine, substantial reductions in engine noise have been achieved. **Figure 1** shows the evolution of the cumulative noise emission of civil aircraft since 1960. Results are obtained from the ICAO noise database (ICAO 2014) and are presented relative to Chapter 3 of the ICAO standard.

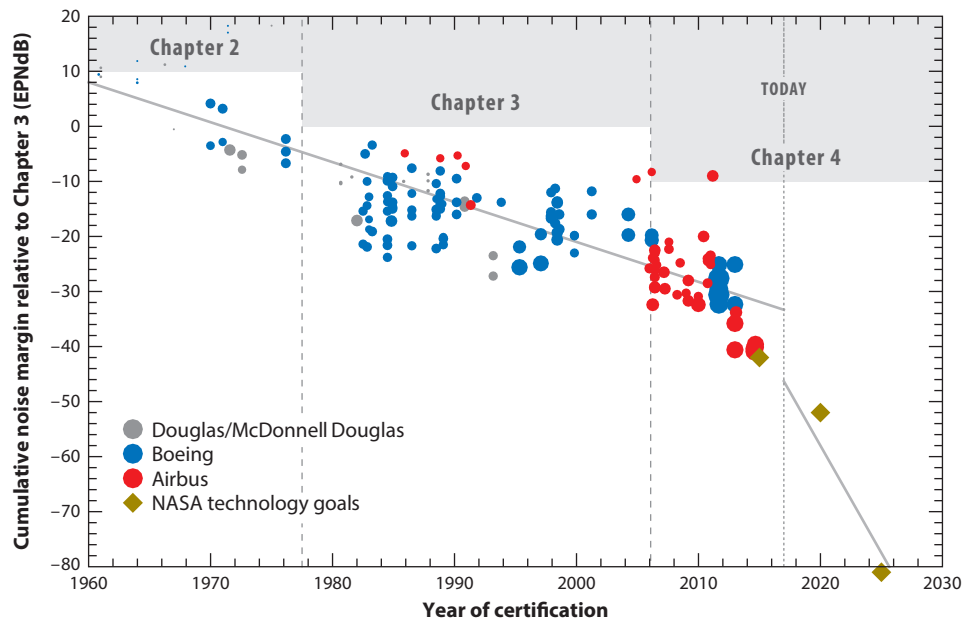


Figure 1

Evolution of cumulative aircraft noise reduction at the source (relative to Chapter 3 of International Civil Aviation Organization standard) and future technology goals for commercial aircraft. The symbol size is scaled by the engine bypass ratio, emphasizing the correlation with the noise reduction. Linear regression results are shown by solid gray lines. Data are taken from the ICAO (2014) database. Abbreviation: EPNdB, effective perceived noise level in decibels.

These improvements were primarily achieved through the introduction of the turbofan engine and the successive increase in bypass ratio, thereby reducing contributions from jet noise as a result of the lower jet-exhaust velocity. However, with the reduction in jet noise, core noise together with fan and airframe noise became a leading contributor to overall aircraft noise. The significance of core noise has already been recognized at low-power engine conditions during landing and approach and in auxiliary power units. In addition, the introduction of advanced combustion strategies, such as lean prevaporized premixed combustion, lean direct injection, and other low-emission combustion technologies (Hultgren 2011, Chang et al. 2013), can result in increased noise emissions due to the occurrence of thermoacoustic instabilities, thermal stratification, and higher turbulence levels. Other factors include the consideration of higher overall pressure ratios, higher turbine loadings, and the introduction of engine designs with higher power densities for integration in future aircraft concepts (Mongeau et al. 2013).

The recent resurgence of interest in combustion and engine-core noise is largely stipulated by ambitious aircraft noise-reduction goals (see **Figure 1**) and is enabled by progress on advanced diagnostics, computational modeling, and improved understanding of fundamental physical processes. Progress on this subject has been reviewed previously. For example, Strahle (1978) focused on noise generation from unconfined flames; Candel et al. (2009) examined scaling, theory, and experimental progress on direct combustion noise and flame dynamics in confined geometries; and Dowling & Mahmoudi (2015) analyzed mostly indirect combustion noise.

By complementing these previous reviews, here I provide a holistic discussion of noise-generating processes induced by unsteady combustion in the engine core. This article expands on the subject of direct and indirect combustion noise by also considering their impact on jet noise and far-field radiation. To this end, Section 2 provides a general overview of core-noise mechanisms and their coupling with external jet noise. This overview serves as a road map for the subsequent component-noise analysis from combustor, nozzle, turbine, and jet exhaust. With relevance to direct combustion noise, Section 3 examines experimental and modeling aspects of noise generation from unsteady combustion. The transmission and generation of noise by temperature, velocity, and mixture inhomogeneities in confined flow paths are discussed in Section 4. Section 5 considers the effects of perturbations generated upstream of the nozzle exit on the modulation of jet noise. Section 6 discusses progress on the system modeling of engine-core noise by integrating findings from component-noise analysis. The article closes by identifying research opportunities to meet future noise-reduction goals.

2. CORE-NOISE MECHANISMS AND COUPLING PROCESSES

Contributions to engine noise that are directly or indirectly generated upstream of the nozzle exit are commonly referred to as core noise. The importance of core noise was first recognized when reconciling excess levels of low-frequency broadband noise at low jet-exhaust velocities that substantially deviate from Lighthill's quadrupole jet-noise scaling (Hubbard & Lassiter 1953). Through experimental investigations, Bushell (1971) attributed this excess noise to mechanisms in the engine core, which are not present in jet-exhaust flows that are free of upstream disturbances.

Over recent years, different core-noise contributions have been identified (Strahle 1978, Mahan & Karchmer 1995). These distinctions were mainly based on the relative importance of certain noise-generating engine components, mutual interaction between different sources, or noise-source separation based on spectral content. Because core-noise mechanisms are engine specific, the introduction of such prior specifications can have substantial ramifications for the accurate description of core-noise radiation. Therefore, it appears prudent to follow a general approach

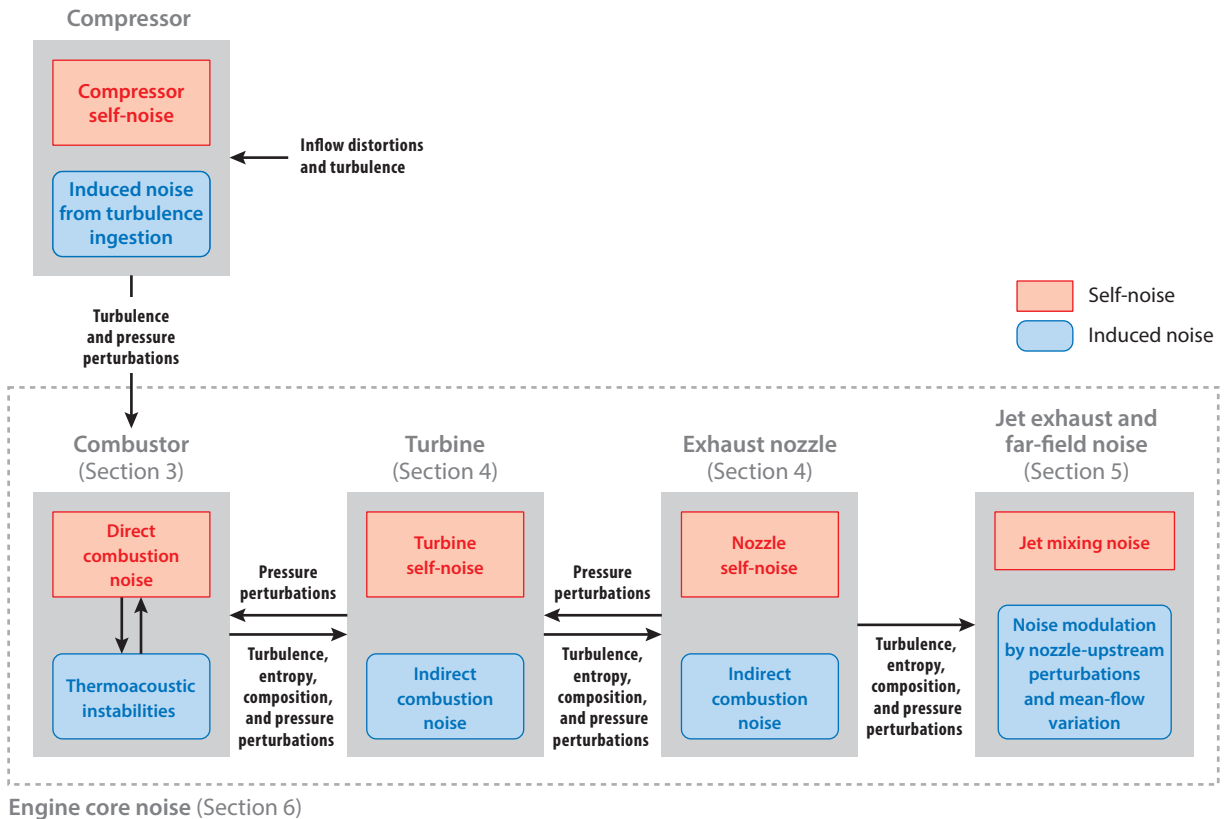


Figure 2

Core-noise mechanisms, showing the interaction between different noise-source mechanisms as indicated by the arrows.

by including all noise-source components, arising from the compressor, combustor, turbine, and nozzle, and their impact on jet-exhaust noise.

In describing relevant core-noise contributions, it is instructive to distinguish between mechanisms associated with self-noise and those associated with induced noise. **Figure 2** shows engine-core components and corresponding noise mechanisms.

Here we define self-noise as the direct generation of acoustic pressure fluctuations from an engine component. The noise-generating sources are located within the engine component, and noise is generated in the absence of the coupling with other upstream or downstream engine components through inflow perturbations or acoustic reflections. Relevant self-noise contributions arise from the compressor, combustor, turbine, and nozzle. The primary noise emission from rotating turbomachinery, which includes the compressor and turbine, is tonal noise from the rotor at harmonics of the blade-passing frequency. The interaction of the rotor wake with the stator introduces tonal noise at multiples of the blade-passing frequency with azimuthal order corresponding to the Tyler-Sofrin modes (Tyler & Sofrin 1962). Multiple interaction tones are generated from the interference of the fan wake with the first compressor stator and at internal stages due to wake interference and flow interaction at the rotor-stator separation. Superimposed to the tonal noise is broadband noise, which is generated primarily by the scattering of boundary-layer turbulence at the trailing edge of blades, the impinging of the turbulent rotor wake with the

downstream stator, the separation of the turbulent boundary layer at the blade, and through the interaction of the rotating blade with the turbulent boundary layer. Another contribution, called haystacking, results from the broadening of tonal energy through the interaction with turbulence in engine-external shear layers. The investigation of turbomachinery noise has been the subject of extensive research, as reviewed by Peake & Parry (2012).

The self-noise generated inside the combustor arises from direct combustion noise, which is associated with acoustic perturbations generated by unsteady heat release. Combustion-generated noise is broadband, having random phase and peak frequencies typically below 500 Hz (Hassan 1974). Although driven by unsteady heat release, these acoustic pressure perturbations are thought to interact only weakly with the turbulent combustion process (Muthukrishnan et al. 1978). In contrast, thermoacoustic instabilities are large-scale coherent pressure fluctuations (Candel 2002, Dowling & Stow 2003, Culick 2006, Lieuwen 2012), having substantially higher acoustic intensities than combustion noise. These instabilities are caused by the coupling between pressure oscillations, entropy and velocity perturbations, local fluctuations in mixture composition, and unsteady heat release. Thermoacoustic instabilities rely on feedback between the flame and acoustics and are therefore regarded as an induced noise mechanism.

Induced noise is generated by inhomogeneities in the velocity, pressure, temperature, and mixture composition that enter the system or are reflected at downstream engine components. These inhomogeneities trigger instabilities or interact with the mean flow, inducing noise that would otherwise be absent for clean inflow and undisturbed exit conditions. Turbulence and flow-field distortions ingested through the engine intake, or the interaction of the fan wake with the booster stage, are indirect noise source contributions in the compressor section. However, owing to high solidity and multiple stages, the propagation of noise through the compressor has substantial transmission losses. Therefore, only noise and perturbations generated at the last compressor stages warrant consideration as potential noise-inducing mechanisms in the combustor. Perturbations leaving the compressor are further attenuated and dispersed as they propagate through the diffuser and enter the combustor. Although signatures of compressor tones have been observed in combustors operating at high power (Reshotko & Karchmer 1980), broadband perturbations entering the combustor are typically outweighed by turbulent mixing and combustion processes inside the combustor. However, if sufficiently large in amplitude, these external perturbations can trigger instabilities in the combustor and undergo subcritical bifurcation (Burnley & Culick 2000). Furthermore, the conversion of entropy waves into pressure fluctuations downstream of the combustor and the subsequent reflection can couple with the combustor acoustics. This mode conversion was found to contribute to low-frequency thermoacoustic oscillations (i.e., rumble) at low-engine-power settings for gas-turbine combustors with spray atomization (Polifke et al. 2001, Zhu et al. 2001). As such, both mechanisms represent routes for induced-noise modulation and interference of broadband noise and coherent pressure oscillations.

Indirect combustion noise is generated by the interaction of inhomogeneities in velocity, temperature, and possibly also mixture composition with mean-flow and pressure gradients in the turbine and nozzle (Cumpsty & Marble 1977a,b; Marble & Candel 1977). Through this interaction, convective entropy and vorticity modes are converted into acoustic pressure waves that are partially reflected as well as transmitted, and ultimately propagate to the far field.

Apart from the direct transmission of the sound generated in the engine core, perturbations and mean-flow distortions induced by unsteady combustion and confinements can impact flow-field conditions at the nozzle exit, thereby indirectly affecting jet-mixing noise. Specifically, it has been recognized that the nozzle geometry, boundary layers, and turbulent, thermal, and acoustic fluctuations can alter jet-noise radiation (Viswanathan 2004, Zaman 2012). Furthermore, jet noise can be modulated by acoustic excitations (Crighton 1981). As such, perturbations originating

from unsteady combustion and tonal noise generated by turbine stages or thermoacoustic instabilities can modulate broadband jet noise, thus indirectly contributing to far-field radiation.

3. DIRECT COMBUSTION NOISE

This section examines the generation of noise from unsteady combustion. It begins with a physical description and then reviews progress on mathematical modeling that includes the characterization of the turbulent flow field, combustion processes, acoustic source term representation, and resulting noise radiation. This is followed by a discussion of challenges and research opportunities.

3.1. Physical Description of Direct Combustion Noise

The direct generation and transmission of combustion noise have been studied experimentally in canonical open-flame configurations operated under premixed and nonpremixed conditions. In foundational experiments on turbulent premixed flames, Smith & Kilham (1963) examined the principal dependence of noise emission on burner geometry, mass flow rate, and fuel mixture. By relating the acoustic power, P_{ac} , to flame speed, S_L , nozzle diameter D_J , and jet exit velocity U_J , they observed a monopole-like scaling of the form

$$P_{ac} \propto \rho S_L^l D_J^m U_J^n, \quad (1)$$

with $l = 2$, $m = 2$, and $n = 2$, whereas dimensional arguments dictate $(l + n) = 3$ and $m = 2$. This was explained by representing the flame as an uncorrelated distribution of acoustic sources that oscillate with intensity and frequency related to the local heat release and turbulence fluctuations. Bragg (1963) provided a theoretical explanation for this scaling by relating the volumetric expansion of individual source elements with characteristic dimension proportional to the flame thickness.

Thomas & Williams (1966) experimentally examined the monopole noise characteristics further and measured the noise emission from the combustion of a homogeneous spherical premixed mixture. Their analysis showed a quartic dependence of the acoustic power on the burning velocity, and they attributed the difference from the results of Smith & Kilham (1963) to the interference of acoustic sources in a turbulent flame.

Hurle et al. (1968) recognized that the rate of volume variation can be related to the consumption rate of the reactants, which they estimated from narrowband chemiluminescence measurements of excited radicals of C_2 and CH . These free radicals are formed in the reaction zone and are markers for the heat-release rate. This study considered piloted turbulent premixed ethylene-air flames with different burner diameters. Price et al. (1969) extended these measurements to nonpremixed and spray flames with different fuels. Results supported the monopole noise-source character of jet flames, and good agreement between far-field pressure measurements and calculations from emission intensities was obtained. Shivashankara et al. (1974) performed similar emission measurements on a premixed flame and performed cross-correlation analysis that confirmed the direct dependence of the acoustic pressure on the emission intensity.

Kumar (1976) reported substantial differences in the noise characteristics between premixed and nonpremixed flames, which were attributed to differences in the flame structure, flame length, and turbulence. Specifically, this study showed that noise sources in premixed flames are associated with unsteady fluctuations in the flame surface area, which modifies the heat release in the corrugated and thin-reaction zone regimes. In contrast, the main noise sources in nonpremixed flames are associated with dilatational effects generated by mixing and turbulence-induced strain-rate variations.

Strahle (1971, 1973) used the acoustic analogy of Lighthill (1952) to develop scaling relations for premixed and nonpremixed flames, confirming the general monopole behavior and low-frequency combustion-noise characteristics. By considering wrinkled flamelet and distributed reaction zone regimes, he found that Equation 1 with exponents $l = 3$, $m = 3$, and $n = 2$ is in better accordance with acoustic power measurements. Scaling relations for combustion noise from turbulent nonpremixed flames were derived by considering the small-turbulent Damköhler limit. However, this limit was obtained from premixed flame arguments (Peters 2000), and a mixing-controlled high-Damköhler limit appears to be more appropriate. The limit of infinitely fast chemistry was considered by Klein & Kok (1999), who developed an integral equation for sound radiation from nonpremixed flames. The sound spectrum was expressed in terms of the turbulence spectrum of the mixture fraction, and numerical simulations of the reacting flow field were used to infer the shape of the acoustic spectrum.

Kotake & Takamoto (1987) examined the effects of turbulence, nozzle geometry, and variations in the equivalence ratio on the acoustic radiation in premixed flames. From correlation measurements between acoustic pressure fluctuations and the time derivative of the light-emission intensity, they found that noise sources in lean premixed flames are mostly distributed in the upper part of the flame, and the dominant noise sources in fuel-rich flames are located in the region below the maximum flame temperature. Although they observed that the acoustic power is insensitive to the level of inflow turbulence at fuel-rich conditions, they showed that higher turbulence levels can lead to a significant increase in acoustic radiation at fuel-lean conditions. These findings are consistent with measurements by Kilham & Kirmani (1979), who rationalized higher noise emissions at lean and stoichiometric conditions with higher turbulent burning velocities.

Rajaram & Lieuwen (2003, 2009) measured the acoustic spectra of turbulent premixed flames and related it to the heat-release spectrum and transfer function. Multiple burner diameters, jet Reynolds numbers, fuels, and equivalence ratios were considered. Using chemiluminescence emission measurements, they observed that the acoustic source is strongest in the upper part of the flame, and its axial extent was correlated to the peak frequency.

Singh et al. (2004, 2005) performed spectral measurements in nonpremixed and partially premixed flames. The hydrodynamic and thermochemical fields in these flames were previously examined (Meier et al. 2000, Schneider et al. 2003) to provide a comprehensive database for model validation. Comparisons with measurements of nonreacting jets at comparable operating conditions showed that premixed flames exhibit higher noise emissions. With an increasing level of partial premixing, the sound spectrum increases at all frequencies, and the velocity scaling exponent of the overall sound pressure level has a strong dependence on the equivalence ratio.

3.2. Mathematical Description of Direct Combustion Noise

The quantitative description of combustion noise requires the consideration of unsteady combustion, the turbulent flow field, acoustic sources, and the transmission of the sound. The scales of these processes are the characteristic acoustic wavelength λ , characteristic flame length L , turbulence integral length scale l_t , Kolmogorov length scale η , and flame thickness δ_F (see **Figure 3**). Application of scaling arguments (Crighton 1992, Peters 2000, Veynante & Vervisch 2002) leads to

$$\text{Ma}\lambda \sim \xi L \sim l_t \sim \text{Re}_t^{3/4} \eta \sim \text{Re}_t^{3/4} \text{Ka}^{-1/2} \delta_F, \quad (2)$$

where Ma is the Mach number, Re_t is the turbulence Reynolds number, and Ka is the turbulent Karlovitz number, which compares the characteristic chemical timescale to the Kolmogorov timescale. The scaling parameter for the flame thickness is $\xi \approx \text{Re}_t Z_{\text{st}}^{-1}$ for turbulent diffusion flames (with Z_{st} the stoichiometric mixture fraction), and $\xi \approx (\text{Re}_t \text{Ka}^2)^{1/4} (1 + \text{Re}_t^{1/2})^{-1}$ for

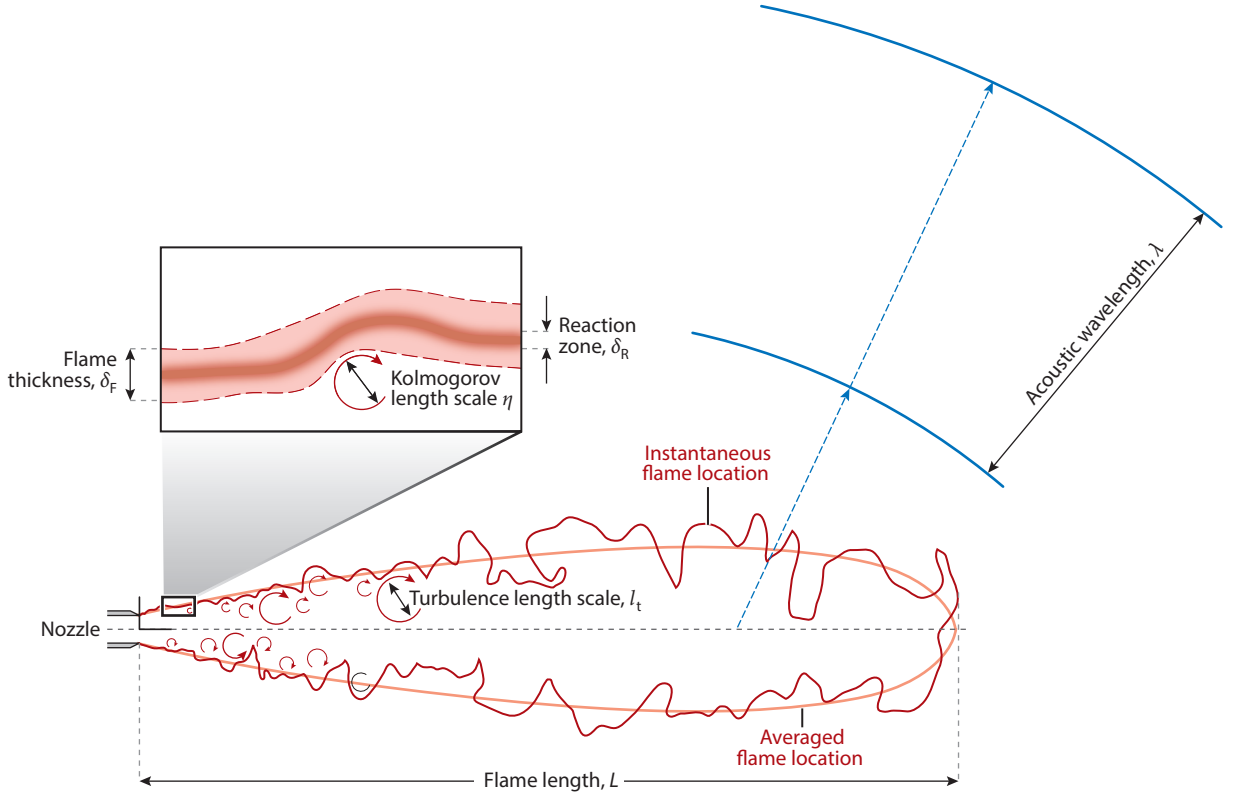


Figure 3

Characteristic scales in a turbulent jet flame.

turbulent premixed flames. Equation 2 shows that for practical applications combustion noise involves a wide range of scales. The separation of these scales depends on the spatial structure of the underlying turbulent flow field, the reaction chemistry and flame structure, and the correlation and spatial extent of the acoustic sources. This section reviews models for these physical mechanisms. Beginning with the mathematical modeling of combusting flows, the description of acoustic sources and noise radiation is then considered.

3.2.1. Mathematical representation of unsteady combusting flows. The spatiotemporal evolution of a chemically reacting flow is described by the conservation equations for mass, momentum, species, and sensible enthalpy (Williams 1985):

$$D_t \rho = -\rho \nabla \cdot \mathbf{u}, \quad (3a)$$

$$\rho D_t \mathbf{u} = -\nabla p + \nabla \cdot \boldsymbol{\sigma}, \quad (3b)$$

$$\rho D_t \mathbf{Y} = -\nabla \cdot \mathbf{j} + \rho \dot{\omega}, \quad (3c)$$

$$\rho D_t h = -\nabla \cdot \mathbf{q} + D_t p + \boldsymbol{\sigma} : \nabla \mathbf{u} + \rho \dot{\omega}_T, \quad (3d)$$

where $D_t = \partial_t + \mathbf{u} \cdot \nabla$ is the substantial derivative, ρ is the density, $\mathbf{u}$ is the velocity vector, p is the pressure, $\mathbf{Y}$ is the vector of N_s species mass fractions, $\dot{\omega}$ is the vector of species production rates, h is the sensible enthalpy, and $\dot{\omega}_T$ is the heat-release rate. The viscous stress tensor is denoted by

σ , $\mathbf{j}$ is the species diffusion flux matrix, and $\mathbf{q}$ is the energy flux. Equations 3a–d are accompanied by the ideal gas law, relating the pressure to the density, temperature, and composition:

$$p = \rho RT, \quad (4)$$

where R is the gas constant, which is a function of the molecular weight W and species composition.

Because of the large number of species, the chemical stiffness, and the scale separation, a direct numerical solution of Equation 3 becomes prohibitive for practical applications. Lower-dimensional manifold representations were developed to reduce the dimensional complexity of the chemistry. In these models, the thermochemical state space is represented in terms of a reduced set of scalars Ψ , typically consisting of some subset of mixture fraction Z , reaction progress variable C , strain rate a , and scalar dissipation χ . These manifolds are constructed from the prior solution of representative flame configurations, such as laminar counterflow diffusion flames, freely propagating premixed flames, or unsteady ignition flame elements. Examples of low-dimensional manifold models are the steady laminar flamelet formulation (Peters 1984), flame prolongation of intrinsic lower-dimensional manifold (Gicquel et al. 2000), flamelet-generated manifold method (van Oijen & de Goey 2000), and flamelet/progress variable formulation (Pierce & Moin 2004, Ihme et al. 2005). The last three models share similarities in that the flame structure is obtained from the solution of one-dimensional flamelet equations, and all thermochemical quantities, denoted by the state vector $\Phi = (\rho, \dot{\omega}, \mathbf{Y}^T, T, \dots)^T$, are parameterized by Z and C . The evolution of the reacting flow field is then described by Equations 3a and 3b and conservation equations for the mixture fraction and progress variable:

$$\rho D_t Z = -\nabla \cdot \mathbf{j}_Z, \quad (5a)$$

$$\rho D_t C = -\nabla \cdot \mathbf{j}_C + \rho \dot{\omega}_C, \quad (5b)$$

and Equation 4 is replaced by a thermochemical state relation:

$$\Phi = \Phi(Z, C). \quad (6)$$

Apart from these reaction-transport manifold approaches, other combustion models have been developed, including the conditional moment closure model (Klimenko & Bilger 1999), transported probability density function methods (Pope 1985, Haworth 2010), linear eddy model (Kerstein 1988), and thickened flame model (Colin et al. 2000). Recently, Wu et al. (2015) developed a Pareto-efficient combustion framework that integrates different combustion submodels to provide direct control over model errors and computational complexity subject to user-specific requirements.

In this context, it is important to emphasize that these combustion models rely on different model approximations, which include assumptions about the flame structure, description of scalar mixing processes, and turbulence/chemistry interaction. Although the accuracy of these combustion models has been assessed using direct numerical simulation (DNS) databases and experiments, these comparisons are largely limited to single-point statistics and conditional data of combustion properties. However, acoustic sources typically depend on two-point correlations and integral scales that are rarely considered, except in a few studies (Renfro et al. 2004, Ihme & Pitsch 2012). Because the spatiotemporal correlation of the heat-release rate is related to the acoustic emission, further research is needed to assess combustion models with regard to its accurate description of the coherent flame structure, the spectral content of the acoustic source terms, and transient extinction effects, which all contribute to broadband noise emission.

3.2.2. Mathematical modeling of direct combustion noise. Different methods can be utilized to predict noise emission from reacting flows. Several methods developed for aeroacoustics

applications can be extended to reacting flows, and these methods can be categorized into direct and hybrid techniques (Wang et al. 2006). In the following, both techniques are reviewed with relevance to applications to combustion noise.

3.2.2.1. Direct methods. Direct methods utilize the same physical model to describe the sound generation from the unsteady reacting flow and its propagation, refraction, and scattering. The main advantage of these methods is that they introduce minimal model approximations and explicitly retain the coupling between heat release, species conversion, hydrodynamics, and acoustics. However, to do this, direct methods must represent a wide range of spatial, temporal, and energetic scales, as shown in Equation 2, thus introducing challenges regarding numerical discretization, boundary conditions, spatial resolution, and representation of transport properties and reaction chemistry. Although aspects of the numerical discretization and specification of boundary conditions are common to those employed in aeroacoustics (Wang et al. 2006), others are germane to combustion noise and require special consideration. In particular, the extended combustion domain and the fact that the acoustic sources occupy an extended region of the flame introduce conflicting demands on resolving and containing the entire flame in the computational domain. This contrasts with combustion simulations, in which the spatial resolution is concentrated in regions of high shear, turbulence/chemistry interaction, and chemical nonequilibrium effects near the nozzle exit. The challenge is compounded by the low frequency of the sound, so that simulations of noise propagation over a few characteristic acoustic wavelengths can substantially increase the computational domain and mesh requirements. Furthermore, variations in thermochemical properties in the flame cause acoustic refractions and distortions (Atvars et al. 1966).

Direct computations have been performed to examine combustion-noise sources and to evaluate acoustic analogies. Zhao & Frankel (2001) conducted DNS of an axisymmetric premixed jet, showing that the acoustic source term is noncompact and that the low-frequency noise characteristic is attributed to the shift of shear-layer instabilities to lower frequencies. Talei et al. (2013) used DNS to examine the effects of thermospecies diffusion in an acoustically excited premixed jet flame, showing that annihilation by flame pinch-off and consumption of pockets of unburned mixture are the main monopole sound sources for conditions in which the Lewis number, comparing thermal to mass diffusivity, exceeds unity. This can be explained by the enhanced burning rate as a result of the compressive strain and imbalance between diffusive and thermal transport at the flame tip.

3.2.2.2. Hybrid methods. Hybrid methods employ different solution techniques to represent the unsteady flow field and acoustics. Approaches based on acoustic analogies, scale separation, and flow-field decomposition developed for aeroacoustic applications (Wang et al. 2006) can be extended to combustion noise, and Bailly et al. (2010) provided derivations of acoustic analogies applied to reacting flows.

Acoustic analogies are derived by separating the unsteady flow into a nominal acoustic source and an acoustic propagation operator. Common to all analogies is the assumption that the sound generation is a passive process, and any feedback of the acoustics on the unsteady flow field appears within the source. Different analogies have been developed depending on the decomposition and physical representation of the source and propagation operators.

Perhaps the most prominent acoustic analogy is due to Lighthill (1952), which was originally developed for aeroacoustics and later extended to combustion noise (Crighton et al. 1992, chapter 13). Lighthill's analogy is derived by rearranging the conservation equations for mass and momentum into a linear wave operator for a homogeneous, stationary medium and an equivalent acoustic source term distribution. Using pressure as an aeroacoustic variable, one obtains the

pressure form of Lighthill's analogy by performing the operation $\partial_t(\text{Equation 3a}) - \nabla \cdot (\text{Equation 3b})$, and adding the term $c_\infty^{-2} \partial_t^2 p$ to both sides of the resulting equation (Doak 1972):

$$(c_\infty^{-2} \partial_t^2 - \nabla^2) p' = \nabla \cdot \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u} - \boldsymbol{\sigma}) - \partial_t^2 (\rho' - c_\infty^{-2} p'), \quad (7)$$

where $p' = p - p_\infty$ and $\rho' = \rho - \rho_\infty$ are pressure and density fluctuations, which are evaluated with respect to the ambient conditions p_∞ and ρ_∞ , and c_∞ is the speed of sound. The first term on the right-hand side combines the unsteady momentum flux and viscous stress tensor, and the last term contains the excess density (Morfey 1973), $\rho_e = \rho' - c_\infty^{-2} p'$, which is associated with variations in entropy. Although the excess density is commonly neglected in aeroacoustic applications (Lighthill 1952), unsteady combustion processes introduce substantial entropy fluctuations so that this term becomes an important source for combustion noise. Crighton et al. (1992, chapter 13) rewrote the temporal derivative of the excess density as $\partial_t \rho_e = (\rho_\infty / \rho) D_t \rho + \nabla \cdot ([\rho_\infty - \rho] \mathbf{u}) - c_\infty^{-2} \partial_t p'$, and the Gibbs-Duhem relation was introduced to express the substantial derivative of the density in terms of spatiotemporal variations of the thermochemistry, which arises from diffusive-dissipative processes, heat release, and species conversion.

Flemming et al. (2007) used Lighthill's analogy to predict combustion noise from an open jet diffusion flame. The unsteady turbulent flow field was calculated from a large-eddy simulation (LES) using a mixture-fraction-based flamelet formulation. The acoustic field was computed from a first-order splitting of Lighthill's wave equation, and the second temporal derivative of density was considered as the primary acoustic source.

Ihme et al. (2009) directly coupled a flamelet-progress variable combustion model with Lighthill's acoustic analogy, and the state relation (Equation 6) was used to express the substantial derivative of density in terms of the mixture fraction and progress variable, $D_t \rho = \partial_Z \rho D_t Z + \partial_C \rho D_t C$, resulting in the following equation:

$$(c_\infty^{-2} \partial_t^2 - \nabla^2) p' = \nabla \cdot \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) - \partial_t \nabla \cdot ([\rho_\infty - \rho] \mathbf{u}) - \rho_\infty \partial_t (\partial_C \ln(\rho) \dot{\omega}_C) \\ - \rho_\infty \partial_t (\partial_Z \rho^{-1} \nabla \cdot \mathbf{j}_Z + \partial_C \rho^{-1} \nabla \cdot \mathbf{j}_C) - \nabla \cdot \nabla \cdot \boldsymbol{\sigma} + c_\infty^{-2} \partial_t^2 p', \quad (8)$$

where the equivalent acoustic sources on the right-hand side represent, respectively, contributions due to Reynolds stresses, the mass flux, reaction rate, diffusive transport in composition space, viscous stresses, and pressure perturbations. Ihme & Pitsch (2012) performed a scaling analysis to show that the far-field acoustic power due to the unsteady Reynolds stresses follows Lighthill's well-known Ma^5 scaling, the acoustic power by the unsteady mass flux scales with Ma^3 , and the chemical monopole source term is most prominent. The acoustic power of the chemical source term scales linearly with the Mach number and quadratically with the Damköhler number. Equation 8 was solved by including source terms due to Reynolds stresses, unsteady mass flux, and the chemical source term. These source terms were evaluated from Favre-filtered LES results, and the acoustic far field was evaluated using a free-space Green's function that was solved in Fourier space. **Figure 4** illustrates results from this hybrid simulation applied to a turbulent jet diffusion flame (Ihme et al. 2009, Ihme & Pitsch 2012). A comparison of the two acoustic source terms (**Figure 4b,c**) shows that the source due to the Reynolds stresses is mainly confined to the shear-layer region in the lower part of the flame. In contrast, the source arising from the chemical reaction is aligned with the location of the stoichiometric mixture, extending to the upper part of the flame. Comparisons of the sound pressure level and directivity (**Figure 4d,e**) confirm that the chemical source term is the dominant source, exhibiting only mild directivity.

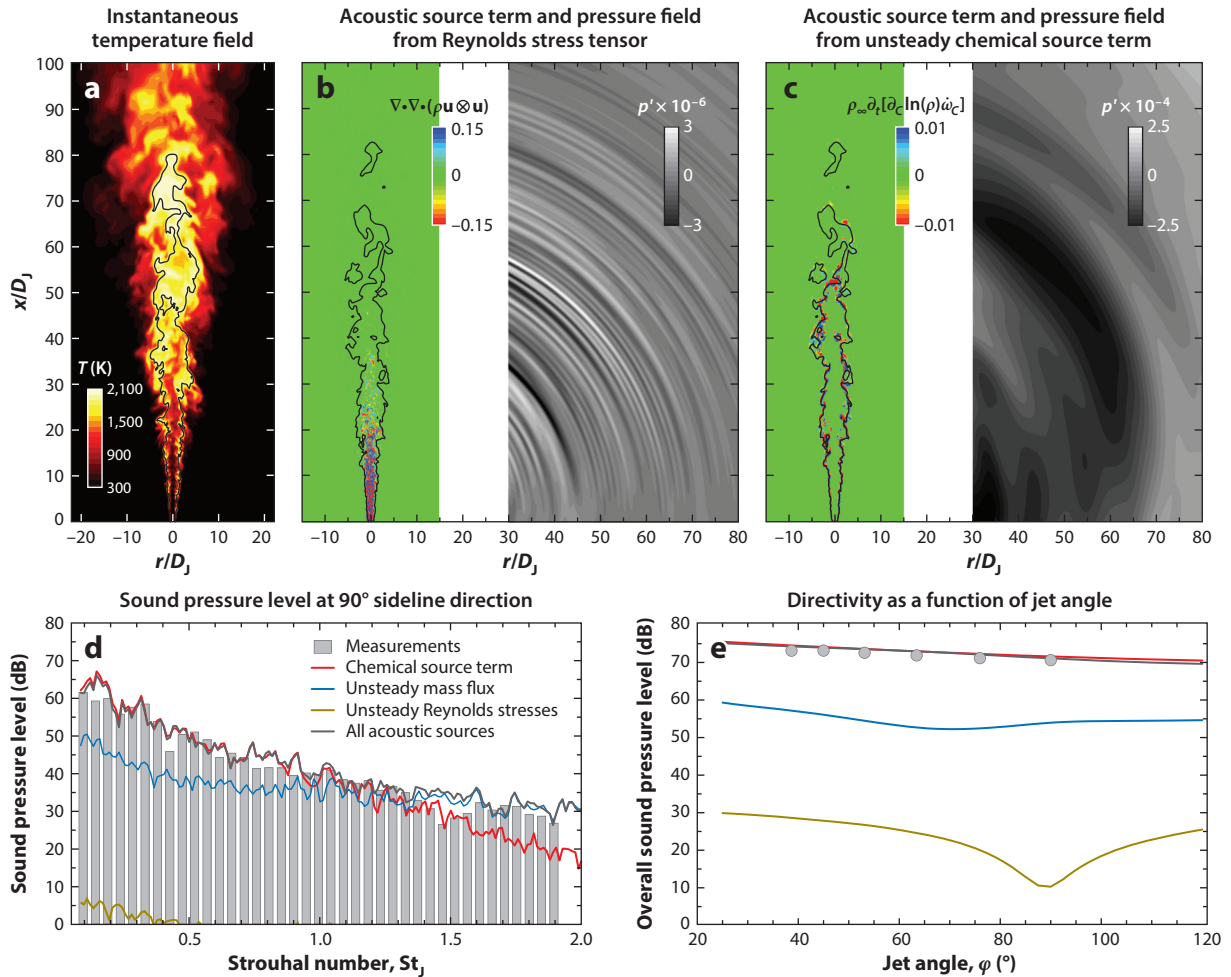


Figure 4

Hybrid simulation of combustion noise from a turbulent jet diffusion flame, showing (a) the instantaneous temperature field; (b) normalized acoustic source term (color) and pressure field (gray) from unsteady Reynolds stresses, and (c) unsteady reaction rate. Source-term contributions to the acoustic far-field pressure are shown for the sound pressure level at (d) the 90° sideline and (e) directivity. The temperature and acoustic source terms are computed from a large-eddy simulation, and the pressure field is obtained from the integral form of Equation 8. The black lines in panels *a–c* denote the isocontour of the stoichiometric mixture. Acoustic source terms and pressure in panels *b* and *c* are nondimensionalized by reference quantities and hydrodynamic scales. Figure adapted from Ihme et al. (2009) with permission from Elsevier.

Lighthill's wave operator does not include the interaction of the sound by flow-field convection and refraction due to local variations in the speed of sound (Goldstein 1976). Phillips (1960) derived an acoustic analogy accounting for effects of convection and refraction by the flow field and inhomogeneities in the media, which was later extended to combustion noise by Chiu & Summerfield (1974) and Kotake (1975). Phillips's analogy is obtained by combining Equations 3a and 3b and differentiating Equation 4 to express the sensible enthalpy in terms of pressure and density. The resulting inhomogeneous-media wave equation takes the following

form:

$$D_t \left(\frac{1}{\gamma} D_t \ln p \right) - \nabla \cdot \left(\frac{c^2}{\gamma} \nabla \ln p \right) = \nabla \mathbf{u} : \nabla \mathbf{u} + D_t \left(\frac{\gamma - 1}{c^2} \dot{\omega}_T \right) + D_t^2 (\ln R) \\ - D_t \left(\frac{\gamma - 1}{\rho c^2} [\nabla \cdot \mathbf{q} - \boldsymbol{\sigma} : \nabla \mathbf{u}] \right) - \nabla \cdot (\rho^{-1} \nabla \cdot \boldsymbol{\sigma}), \quad (9)$$

where the left-hand side represents an approximate moving-media wave operator, accounting for effects of convection and refraction of the sound by mean flow and variations of the speed of sound in the source region. However, this incomplete wave operator does not consider effects of the shear on sound propagation (Pridmore-Brown 1958, Doak 1972, Lilley 1974). This shear-refraction term appears as an apparent source as the first term on the right-hand side of Equation 9. Equivalent acoustic source terms on the right-hand side consist of the convective derivative of the heat-release rate, variations in the molecular weight of the gas mixture, and thermoviscous-dissipative effects by energy flux, viscous dissipation, and viscous stresses. Simplifying assumptions commonly invoked to make Phillips's analogy tractable include the consideration of low-speed flows so that substantial derivatives reduce to temporal derivatives, the assumption of a constant specific heat ratio, the consideration of small pressure perturbations, and the omission of viscous-diffusive source terms. In a hybrid simulation, Ihme et al. (2006) employed Phillips's analogy with these approximations to examine effects of the inhomogeneous medium on the directivity pattern from a turbulent jet flame.

The analogy due to Lilley (1974) includes additional propagation effects into the wave operator. This third-order wave equation is obtained by taking the substantial derivative of Equation 9, and introducing the momentum equation to rearrange terms associated with pressure perturbations into the wave propagation operator. Although Lilley's equation has been examined in the aeroacoustics community, it has not yet been applied to combustion noise.

Alternatives to acoustic analogies are hydrodynamic/acoustic splitting techniques (Hardin & Pope 1994) by which the flow field is separated into an incompressible, nonradiating solution that describes the source evolution, and the acoustic field is obtained as a solution to an unsteady compressible perturbation equation. Goldstein (2003) formulated a general splitting theory by separating the flow field into a base flow and a residual flow, resulting in a set of linearized Navier-Stokes equations with a generalized source term. This approach is different from previously discussed analogies that describe the noise radiation by a scalar wave equation for an acoustic variable (pressure or density) and an equivalent source term. Goldstein's formalism was later extended by Seo & Moon (2006) to the linearized perturbed compressible equations to address stability issues, associated with the under-resolved vorticity field.

Ewert & Schröder (2003) considered a different splitting approach and developed the acoustic perturbation equations, and this approach was extended by Bui et al. (2009) to reacting flows. Validation was performed in a jet flame for which the base flow was obtained from a time-averaged LES solution. In this study, the substantial derivative of density was identified as the primary acoustic source term.

3.3. Discussion and Research Needs

Although recent efforts have improved our understanding of direct combustion noise, aspects of source-term identification, mathematical modeling, and applicability of acoustic analogies for predicting combustion noise from confined flames remain open.

With regard to physical understanding, a main ambiguity is the characterization of acoustic source mechanisms from premixed, diffusion, and partially premixed flames. A hypothesis for the

significantly higher noise emission observed in partially premixed flames (Singh et al. 2005) is the presence of a double reaction layer, consisting of a rich premixed zone and a diffusion zone. Another issue is assessing the importance of local extinction and reignition on the acoustic radiation. The frequency of these local quenching and extinction events is related to the excursion of the dissipation rate from its critical value and can contribute to high-frequency noise emissions. Other fundamental aspects are the examination of the effects of refraction and scattering by velocity and temperature gradients in the flames. Although viscous-diffusive contributions are commonly regarded as acoustically inefficient (Obermeier 1985), they can potentially become important in reacting flows (Morfey 2003); this issue has not yet been fully addressed and requires further analysis.

In the context of turbulent combustion modeling, contributions arising from combustion-model approximations, subgrid-scale models, and turbulent/chemistry closures to noise radiation have not been fully examined. In simulations of practical configurations, the flame structure is not fully resolved over the extended source domain so that closures for nonlinear acoustic sources and turbulence/chemistry interaction may require consideration for the accurate description of far-field radiation. DNS can assist in assessing the importance of these sound source contributions and developing combustion-noise closure models.

Although acoustic analogies and hybrid methods have been successfully applied to combustion noise in open flames, the one-way coupling introduces limitations in applications to direct combustion noise of confined flames. Specifically, the interaction of the sound with the hydrodynamic flow through the scattering and reflection of the pressure at walls and nozzles requires consideration to describe precursors that trigger thermoacoustic instabilities. Multiple-scale techniques, recently developed for the two-way linear coupling of reactive hydrodynamics and acoustics (Magri et al. 2017), represent a promising avenue for application to combustion noise. A higher level of fidelity is achievable by solving the compressible reacting flow equations through LES in conjunction with suitable combustion models. Although this approach represents the nonlinear coupling, the low-Mach number operating condition in gas-turbine combustors imposes severe time-step constraints.

4. GENERATION AND TRANSMISSION OF NOISE IN NOZZLES AND TURBINE STAGES

Whereas noise directly generated inside the combustion chamber propagates through the combustor-downstream engine components before radiating to the far field, nonacoustic perturbations exiting the combustor represent a potent source of indirect sound. Specifically, turbulent mixing, unsteady combustion, and dilution introduce inhomogeneities in the velocity, temperature, and mixture composition that exit the combustor. Combustors are designed to ensure that fuel conversion is completed inside the combustion chamber to maximize fuel utilization and thermal efficiency. Consequently, the direct noise source is no longer active downstream of the combustor, and the downstream flow can—to a good approximation—be represented as chemically frozen or equilibrated. The interaction of these inhomogeneities with mean-flow gradients encountered in downstream nozzle and turbine stages generates acoustic pressure fluctuations. These noise-generating mechanisms are collectively referred to as indirect combustion noise. Contributions arising from vorticity fluctuations are associated with vortex noise, and indirect noise generation by temperature fluctuations in the form of hot or cold spots exiting the combustor are referred to as entropy noise. The role of entropy variations as indirect noise contributors was first identified theoretically in the seminal work of Candel (1972) and has been further investigated experimentally and computationally.

This section examines indirect combustion noise. Theoretical analysis tools are first reviewed and then extended to include compositional variations, thereby introducing composition noise

as an additional source of indirect combustion noise. Subsequently, these theoretical results are connected to experimental analysis and future research needs are discussed.

4.1. Mathematical Description

The interaction of acoustic and entropy waves with a nonuniform mean flow, encountered in diffusers, nozzles, and turbine stages, generates additional sound that contributes to engine-core noise. Different methods to describe the acoustic transmission and reflection in nozzles and turbines have been developed, including compact nozzle theory (Tsien 1952; Candel 1972; Cumpsty & Marble 1977a,b; Marble & Candel 1977; Dowling 1995), the effective nozzle length method (Stow et al. 2002, Goh & Morgans 2011), the linear nozzle element technique (Moase et al. 2007, Giauque et al. 2012), expansion methods (Mani 1981, Duran & Moreau 2013, Duran & Morgans 2015), nonlinear analyses (Bloy 1979, Huet & Giauque 2013), and numerical simulations (Mühlbauer et al. 2009, Leyko et al. 2011, Palies et al. 2011, Mishra & Bodony 2013, Papadogiannis et al. 2015). Common to all methods is the consideration of a homogeneous gas mixture that is described by conservation of mass, momentum, energy, and entropy.

Although interactions among entropy, acoustic, and vorticity perturbations have been examined (Chu & Kovásznyai 1958), the relevance of mixture inhomogeneities as a contributor to indirect combustion noise has not yet been considered. To address this, we examine whether compositional inhomogeneities exiting the combustor as a result of incomplete mixing, mixture stratification, or dilution represent an additional contribution to indirect combustion noise. These mixture inhomogeneities can become increasingly important with the implementation of compact burners, high-power-density engine cores, and advanced low-emission combustors (Hultgren 2011, Chang et al. 2013).

Candel (1972) and Marble & Candel (1977) used compact nozzle theory to examine the transmission and reflection of acoustic and entropy waves in a one-dimensional nozzle. This theory is valid in the limit of small Helmholtz numbers, $He = l\omega/c$ (with l the characteristic length of the nozzle; the speed of sound c is commonly evaluated at the nozzle throat), so that perturbations are assumed to propagate quasi-steadily through the nozzle. Using this theory, they derived transfer functions relating acoustic and entropy perturbations between the inlet and outlet of the nozzle. Here, this theory is extended to examine the effects of mixture fluctuations on the transfer matrix. For this, a chemically frozen flow with frozen internal energy modes is considered, so that all species are uniquely represented in terms of the mixture fraction, $\mathbf{Y} = \mathbf{Y}(Z)$.

Governing equations for perturbations of the mass flow rate, total sensible enthalpy, entropy, and mixture fraction (neglecting vortical disturbances) provide jump conditions across the nozzle:

$$[\![dm]\!]_a^b = 0, \quad [\![db_t]\!]_a^b = 0, \quad [\![ds]\!]_a^b = 0, \quad [\![dZ]\!]_a^b = 0, \quad (10)$$

where the indices a and b denote the conditions at the inlet and at the exit of the nozzle, respectively (see **Figure 5**).

The jump conditions for the conserved quantities can be related to the primitive state variables by recalling the differentials of the total sensible enthalpy, state equation, and Gibbs equation for a multicomponent gas mixture:

$$db_t = db + udu, \quad (11a)$$

$$dp = RT d\rho + \rho T \frac{dR}{dZ} dZ + \rho R dT, \quad (11b)$$

$$ds = \frac{1}{T} db - \frac{R}{p} dp - \frac{1}{T} \sum_{i=1}^{N_s} \frac{\mu_i}{W_i} \frac{dY_i}{dZ} dZ, \quad (11c)$$

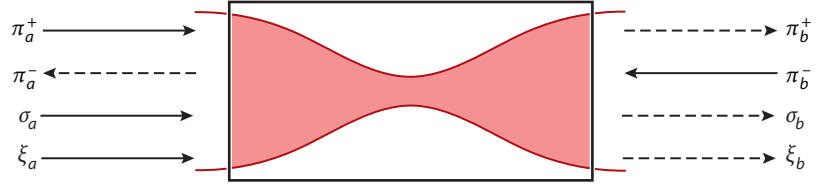


Figure 5

Schematic illustration of a compact nozzle, showing acoustic, entropy, and compositional waves at subsonic conditions. The indices a and b denote the conditions at the inlet and at the exit of the nozzle, respectively.

where $db = c_p dT + \sum_{i=1}^{N_s} b_i (dY_i/dZ) dZ$ and $\mu_i = \mu_i^0 + \mathcal{R}T \ln(p_i/p_0)$ is the chemical potential of species i , which is a function of pressure and temperature (Job & Herrmann 2006).

By considering a subcritical nozzle, in which the flow remains subsonic, one can identify four invariants at each side of the nozzle (see **Figure 5**), corresponding to the downstream and upstream propagating acoustic waves, convective entropy wave, and convective compositional wave:

$$\pi^\pm = \frac{1}{2} \left(\frac{p'}{\gamma \bar{p}} \pm \frac{u'}{\bar{c}} \right), \quad \sigma = \frac{s'}{c_p}, \quad \xi = \frac{1}{R} \frac{dR}{dZ} Z'. \quad (12)$$

Expressed in terms of these characteristic variables, the dimensionless form of the conserved quantities can be written as

$$\frac{\dot{m}'}{\bar{m}} = \left(1 + \frac{1}{\text{Ma}} \right) \pi^+ + \left(1 - \frac{1}{\text{Ma}} \right) \pi^- - \sigma - (\Phi - \Psi + 1)\xi, \quad (13a)$$

$$\frac{b'_t}{\bar{b}_t} = \frac{2(\gamma - 1)}{2 + (\gamma - 1)\text{Ma}^2} \left((1 + \text{Ma})\pi^+ + (1 - \text{Ma})\pi^- + \frac{\sigma + \Phi\xi}{\gamma - 1} \right), \quad (13b)$$

$$\frac{s'}{c_p} = \sigma, \quad (13c)$$

$$\frac{1}{R} \frac{dR}{dZ} Z' = \xi, \quad (13d)$$

where $\Phi = 1/(c_p T) \sum_{i=1}^{N_s} (\mu_i/W_i) (dY_i/dZ) [R/(dR/dZ)]$ and $\Psi = \sum_{i=1}^{N_s} [b_i/(c_p T)] (dY_i/dZ) [R/(dR/dZ)]$. Upon inserting Equations 13a–d into Equation 10, one obtains a linear set of algebraic equations that relates the four incoming waves (π_a^+ , σ_a , ξ_a , and π_b^-) to the four outgoing waves (π_b^+ , σ_b , ξ_b , and π_a^+). By setting π_b^- to zero, one can derive transfer functions for the acoustic response at the nozzle exit to inlet excitations by acoustic, entropy, and compositional perturbations. These relations are functions of the Mach number and the chemical potential at the inlet and exit of the nozzle, taking the following form:

$$\frac{\pi_b^+}{\pi_a^+} = \frac{2(1 + \text{Ma}_a)\text{Ma}_b}{(1 + \text{Ma}_b)(\text{Ma}_a + \text{Ma}_b)} \frac{(1 + \frac{\gamma-1}{2}\text{Ma}_b^2)}{(1 + \frac{\gamma-1}{2}\text{Ma}_a\text{Ma}_b)}, \quad (14a)$$

$$\frac{\pi_b^+}{\sigma_a} = \frac{(\text{Ma}_b - \text{Ma}_a)\text{Ma}_b}{2(1 + \text{Ma}_b)(1 + \frac{\gamma-1}{2}\text{Ma}_a\text{Ma}_b)}, \quad (14b)$$

$$\begin{aligned} \frac{\pi_b^+}{\xi_a} &= \frac{(\gamma - 1)(\Phi_b - \Phi_a)(1 + \frac{\gamma-1}{2}\text{Ma}_b^2) \text{Ma}_a\text{Ma}_b}{(\gamma - 1)(1 + \text{Ma}_b)(\text{Ma}_a + \text{Ma}_b)(1 + \frac{\gamma-1}{2}\text{Ma}_a\text{Ma}_b)} \\ &\quad + \frac{\text{Ma}_b \left[(\Phi_a - \Phi_b) + \frac{(\gamma-1)}{2}(\Phi_a\text{Ma}_b^2 - \Phi_b\text{Ma}_a^2) \right]}{(\gamma - 1)(1 + \text{Ma}_b)(\text{Ma}_a + \text{Ma}_b)(1 + \frac{\gamma-1}{2}\text{Ma}_a\text{Ma}_b)}, \end{aligned} \quad (14c)$$

and similar relations for the reflected waves π_a^- can be derived. The transfer functions given in Equations 14a and 14b were derived by Marble & Candel (1977), and Equation 14c introduces an additional indirect noise contribution that is caused by nozzle-upstream perturbations in the mixture composition. With the assumption of equal chemical potential and thermochemical state at both sides of the nozzle, $\Phi_a = \Phi_b$, the ratio between the indirect mixing noise and entropy noise can be computed as

$$\frac{\pi_b^+}{\xi_a} \frac{\sigma_a}{\pi_b^+} = \Phi, \quad (15)$$

which depends on the chemical potential of the mixture composition. A similar analysis can be performed for choked and supersonic nozzles, where additional conditions for the critical mass flow and shock relations are required to close the system of linear equations.

Several assumptions on the state of the mixture were introduced in the above derivation to obtain closed-form transfer functions. Further analysis is necessary to quantify the significance of the herein identified mixing-induced indirect combustion noise.

Owing to intrinsic assumptions, the application of compact nozzle theory is restricted to conditions that are described by small Helmholtz numbers, which are often adequate for providing reasonable estimations of indirect combustion noise (Stow et al. 2002, Leyko et al. 2009). However, extensions of this theory are necessary to consider effects of nozzle geometry and finite disturbances. In fact, Marble & Candel (1977) considered the transmission and reflection in a choked nozzle of finite length. For this, they assumed a linear velocity profile through the nozzle, resulting in a hypergeometric differential equation. From the solution, they showed that the amplitude and phase of the transmitted acoustic waves exhibit a pronounced dependence on the Helmholtz number and nozzle-exit Mach number. By approximating the nozzle by elements with piecewise linear velocity profiles and appropriate matching conditions, one can further extend this theory to represent more complex geometries. This was done by Moase et al. (2007), who considered the forced frequency response of choked nozzles and supersonic diffusers of arbitrary shape. Through this analysis, the nonlinear nozzle response was shown to become important with increasing Mach number and forcing amplitude. Later, Giaque et al. (2012) applied this linear nozzle-element model to subcritical nozzles to examine the influence of nozzle geometry on the transfer function.

Instead of representing the nozzle by elements with linear velocity profiles, Stow et al. (2002) considered a different approach in which the perturbed flow solution was expanded for low frequencies: $\phi'(x, \text{He}) = \phi'_0(x) + \text{He}\phi'_1(x) + \mathcal{O}(\text{He}^2)$. The method was applied to annular flows in choked nozzles. The zeroth-order term recovered the reflection coefficients for compact choked nozzles, and the first-order correction introduces velocity integrals, which were evaluated by approximating the nozzle by a straight duct with an effective length represented by the inlet mean velocity and the convective time through the throat. This first-order term provides a phase correction to the reflection coefficient. Goh & Morgans (2011) extended this analysis to examine the response of supercritical nozzles to acoustic and entropy disturbances.

Duran & Moreau (2013) studied transfer functions of subsonic and choked nozzles by formulating the linearized Euler equations in terms of the invariants given in Equations 13a–c, resulting in a system of variable-coefficient linear differential equations. Instead of directly expanding the solution vector, they used the Magnus expansion (Blanes et al. 2009) to expand the coefficient matrix. This approach allows the consideration of general continuous mean-flow profiles and nozzle geometries. The method was applied to compute reflection and transmission coefficients of subsonic and choked nozzles and was later extended by Duran & Morgans (2015) to examine azimuthal waves in annular nozzles. Results obtained with this method support previous findings

that transfer functions exhibit a strong dependence on perturbation frequency and nozzle geometry. In addition to extending the analysis to finite frequency response, the nonlinear perturbation response was addressed by Huet & Giauque (2013). They considered a quasi-steady flow evolution in order to obtain an analytical evaluation of the Riemann invariants in the presence of large-amplitude entropy excitations.

The generation of indirect noise by entropy disturbances in a turbine cascade was first studied by Cumpsty & Marble (1977a,b). In this analysis, they assumed that the blade row was compact and so could be represented by an infinitely thin disk. Application to isolated blade rows and compressor stages showed that entropy noise increases with increasing Mach number, pressure ratio, and blade loading, which provides an explanation for observed excess noise from gas turbines. Compared to recent advances on the formulation of analytic methods for nozzle flows, extensions to finite-frequency response and the description of stage geometries have received only little attention; further research is therefore needed on developing transfer functions with higher physical fidelity for rotating turbomachinery.

Numerical simulations were performed to evaluate low-order models and examine transfer mechanisms. Leyko et al. (2010) and Mishra & Bodony (2013) evaluated the accuracy of actuator disk theory by Cumpsty & Marble (1977b) through two-dimensional numerical simulations. Both simulations considered an isolated blade row. Results from these investigations suggest that actuator disk theory provides adequate predictions for conditions with $He < 0.6$ but becomes inaccurate when describing evanescent modes at the leading and trailing edges and mode distortion of initially planar waves by the mean flow. The advection, dissipation, and distortion of entropy modes were further examined through three-dimensional simulations in a fully developed turbulent channel flow by Morgans et al. (2013) and a transonic turbine stage by Papadogiannis et al. (2015). These simulations emphasized the importance of three-dimensional effects for the transmission of high-frequency temperature perturbations, indicating that low-order models and probably also two-dimensional calculations most likely overpredict the indirect noise generation in turbine stages with multidimensional blade geometries.

4.2. Experimental Analysis

Some of the first experiments to investigate entropy noise in supersonic and subsonic nozzles were performed by Zukoski & Auerbach (1976) and Bohn (1976). In these experiments, electric heaters were installed in the flow path upstream of the nozzle to periodically heat the gas. The induced temperature variations were limited to a few Kelvins. Because the heating introduces acoustic fluctuations due to density variations in addition to entropy perturbations, the interpretation of the measurements was complicated, and compensation techniques were developed in an attempt to separate both contributions.

Bake et al. (2008, 2009b) undertook a comprehensive measurement campaign to systematically investigate entropy noise mechanisms in a canonical flow configuration. To this end, an entropy wave generator facility was constructed. The entropy wave generator was instrumented with thermocouples, a vibrometer, and microphones, and the impedance at the outlet was characterized to establish a data set for analysis and model evaluation. Parametric investigations included the consideration of variations in the mass flow rate, nozzle Mach number, modulation of the heating power, and nozzle geometry. These measurements showed that the pressure fluctuations in the nozzle throat increase monotonically up to $Ma \simeq 0.6$ and decrease for higher Mach numbers. Howe (2010) suggested that this reduction in the overall sound generation can be explained by combined effects of the cancellation of entropy noise by vortex sound due to flow separation and the distortion of temperature inhomogeneities in the nozzle. Analysis by Leyko et al. (2011) suggested

that the measured pressure signal contained contributions from acoustic reflections at the exhaust system. Other source mechanisms that contribute to indirect broadband-noise generation have recently been examined by considering the acceleration of vorticity fluctuations (Kings & Bake 2010) and the acceleration of cold spots through a nozzle in a hot acoustic test rig (Knobloch et al. 2015).

Apart from fundamental investigations of individual excitation mechanisms by entropy and vorticity, experiments on generic combustors were conducted to examine the effects of unsteady turbulent combustion on direct and indirect noise emission (Eckstein et al. 2006, Bake et al. 2009a). These generic combustor designs retain essential features of fuel injection, mixing, flame stabilization, and operating conditions that are representative of realistic gas-turbine combustors, yet experimental instrumentation and modular geometry enable independent model evaluation in numerical simulations. Although substantially more challenging, these experiments often reveal physical mechanisms that are not contained in fundamental experiments. For example, Bake et al. (2009a) observed an augmented broadband noise-generation mechanism at frequencies above 1 kHz, which they attributed to the interaction of entropy and vorticity fluctuations with shear layers and turbulent nozzle flow, as well as the strong swirl component introduced by the fuel injector. Eckstein et al. (2006) focused on the generation and feedback of entropy waves with self-excited flames in a generic liquid-fuel combustor. With relevance to the acoustic nozzle reflection and indirect noise generation, they found that the reflected acoustic waves, π_a^-/ξ_a , are insufficiently strong to alter the thermoacoustic mode. These augmented mechanisms and secondary interactions are typically not considered in idealized configurations or included in theoretical models.

4.3. Discussion and Research Needs

Many of the theoretical models reviewed in previous sections rely on simplifications that limit the consideration of viscous-dissipative effects, boundary layers, flow separation, and the nonlinear interaction of perturbations with turbulence and the mean flow. Global stability analysis and linearized multidimensional Navier-Stokes simulations represent attractive techniques to examine the impact of these effects, thereby bridging the gap between low-order analytic methods and computationally expensive high-fidelity simulations.

The extended theory derived in Section 4.1 suggests that inhomogeneities in the mixture composition represent an additional mechanism for indirect combustion noise. This mechanism exhibits a direct dependence on the combustion regime, operating conditions, and product-gas composition; theoretical and experimental investigations can assist in examining its importance to the overall noise emission.

There is a need for the systematic examination of coupling effects between the interaction of the combustor and downstream engine components. Results from these investigations will guide modeling techniques that often rely on the one-way coupling between engine subcomponents, the specification of boundary conditions, and model selection for predicting unsteady combustion processes. Although the role of entropy noise as a feedback mechanism for thermoacoustic instabilities has been recognized, further experimental studies are needed to quantify the interaction mechanisms of reflected pressure waves and unsteady combustion at conditions relevant for gas-turbine engines. Furthermore, we need to address the lack of detailed multidimensional measurements of noise generation from engine flow paths that include the combustor, turbine, nozzle, jet exhaust, and acoustic far field. Making these experiments viable for model evaluation requires the characterization of boundary conditions, statistical information about velocity and scalar combustion fields, and spectral data to establish causalities between far-field acoustic radiation and noise-generating mechanisms inside the flow path.

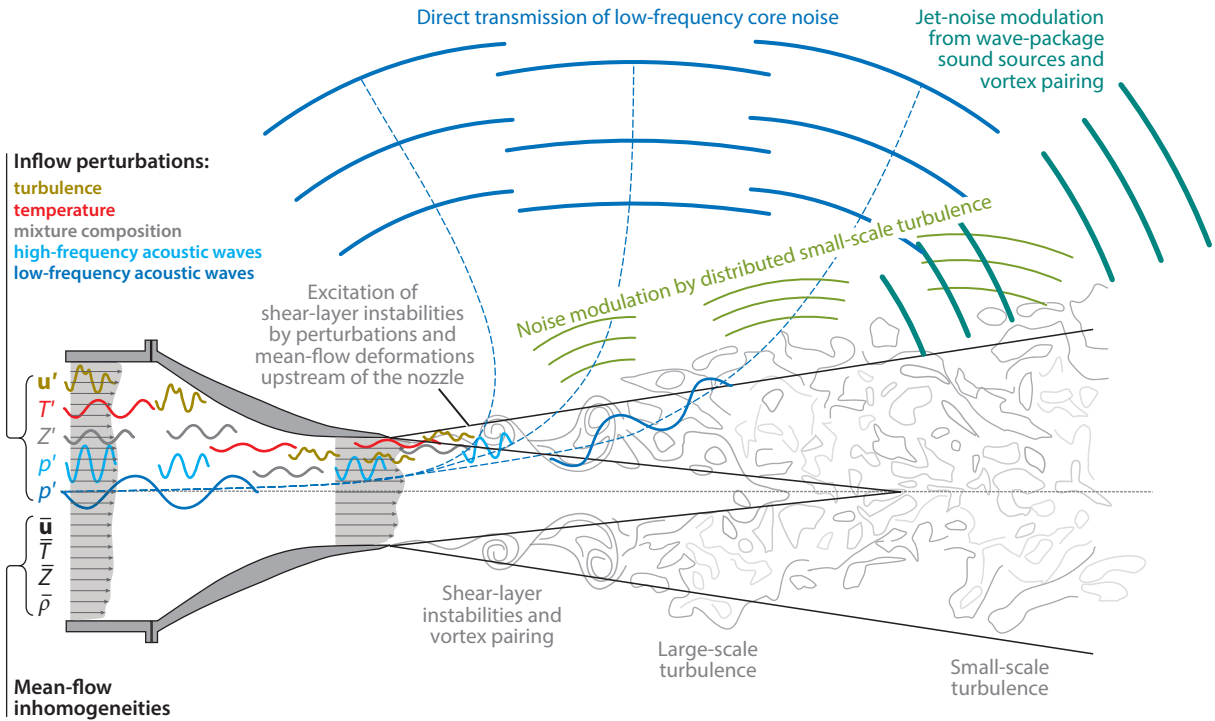


Figure 6

Schematic illustration of jet-noise modulation through direct combustion noise and excitation of the shear layer by mean-flow inhomogeneities and perturbations generated upstream of the nozzle.

5. JET-EXHAUST NOISE AND JET-NOISE MODULATIONS

The analysis of core noise is commonly restricted to direct and indirect combustion noise mechanisms that originate upstream of the exhaust nozzle. Previous studies on combustion-generated noise therefore stopped at the nozzle exit, implicitly assuming that nozzle-upstream generated noise by unsteady combustion and internal flow-path interactions directly propagates to the far field. This approach, however, neglects secondary noise mechanisms induced through modifications of the jet-noise sources by mean-flow deformation and perturbations exiting the engine core. In particular, it is known that disturbances, boundary layers, and operating conditions impact the spatiotemporal evolution of the shear layer, potential core, and turbulent flow-field structure of the exhaust jet and acoustic sound field. In addition to core-noise contributions, it is therefore pertinent to also consider these induced jet-noise mechanisms in the analysis of engine-core noise.

5.1. Jet-Noise Modulation

Three main mechanisms can be identified as affecting the flow field and noise generation of mixing layers and jets: (a) nozzle exit conditions, (b) external excitations, and (c) operating conditions (Figure 6). Significant efforts have been undertaken to investigate the role of initial conditions and forcing on the jet noise (Crighton 1981, Ho & Huerre 1984). It is therefore instructive to examine essential physical processes in an attempt to link these to conditions relevant to engine-core noise.

Pioneering work by Crow & Champagne (1971) on isothermal subsonic jets has shown that the initial thickness and state (laminar/turbulent) of the boundary layer and the fluctuating intensity at the nozzle exit control the development of the flow field and transition. The evolution of the shear layer in jets emanating from nozzles with weakly disturbed exit conditions is governed by linear instability waves (Hussain & Zedan 1978). Initially, axisymmetric large-scale vortex pairing is observed near the nozzle exit, which transitions to jet-column instabilities with azimuthal variations. The rapid growth of the shear layer reduces the length of the potential core, and velocity fluctuations peak at the location of the first vortex merging, after which they decay to reach a steady level. The pairing of shear-layer vortices introduces an additional sound-producing mechanism that is absent for jets with a turbulent inlet state (Bridges & Hussain 1984). A different picture emerges for initially turbulent nozzle exit conditions. This state is common for practical jet flows and is achieved experimentally, for instance, by tripping the boundary layer in the nozzle. For highly disturbed and turbulent initial states, the transition length is reduced, and the shear layer grows at a lower rate. The evolution of the shear layer is controlled by three-dimensional interactions, and the axisymmetric vortex-pairing mode becomes insignificant. Additionally, there is an extension of the potential core and lower peak turbulence levels that approach a quasi-self-preserving turbulent state with increasing initial disturbance intensity.

Affected by the nozzle-exit conditions, the evolution of the shear layer and turbulence generation in the jet induces different acoustic sources (Mollo-Christensen et al. 1964, Bridges & Hussain 1984, Harper-Bourne 2010). From an analysis of jet-noise data, Tam et al. (1996, 2008) deduced two empirical spectra that match the radiation characteristics in downstream and sideline directions. Wave-packet analysis, recently reviewed by Jordan & Colonius (2013), provides a physics-based description of jet-noise mechanisms. In this analysis, the coherent portion of the velocity field of a turbulent jet is represented by a wave packet, which is characterized by a constant convection velocity, low azimuthal wave number, and spatial coherence over scales larger than the integral scales. Because of this extended coherence, this wave packet is acoustically more efficient and is linked to the sound radiation through the induced near-field pressure.

Because vortex-pairing sound and higher turbulence levels are observed in jets with laminar or weakly disturbed inlet conditions, they are significantly louder than jets in which this large-scale transition route is bypassed by highly disturbed nozzle-exit conditions. Experiments on jets at low and high subsonic flow conditions by Zaman (2012) confirmed this behavior, showing that a jet with a tripped boundary layer emits less noise. These measurements, however, also indicated that a higher turbulence intensity in the nozzle core increases the noise radiation. Fontaine et al. (2015) performed experiments to examine the effect of the turbulent boundary-layer thickness on the jet noise, showing that the downstream radiation of the high-frequency sound scales with the momentum thickness at the nozzle exit, while the spectral peak was less affected by the boundary-layer state. In contrast, the frequency of the sideline radiation showed better collapse when scaling with the nozzle diameter, suggesting that distributed small-scale turbulence in the jet is the prevailing noise source.

Bogey et al. (2012) and Bogey & Marsden (2013) performed LES calculations in which the initial boundary-layer state was systematically modulated, showing a dramatic reduction of the overall sound pressure level, in excess of 12 dB, at the 90°-sideline angle between untripped and tripped boundary layers at peak turbulence levels of 12%. This difference decreases slightly with increasing jet-forward direction. The persistence of weak vortex-pairing noise was attributed to moderate-Reynolds number conditions and shear-layer thickness, suggesting that large-scale structures remain present even at highly turbulent nozzle-exit conditions.

The noise radiation from heated jets was first investigated by Hoch et al. (1973) and Tanna (1977), and the work of Viswanathan (2004) provides an excellent overview of current knowledge.

Although several questions regarding density effects and noise-source mechanisms in heated jets remain open, it is now understood that the observed change in the sound spectrum and the occurrence of a hump near the spectral peak at increasing temperature are direct results of the reduced Reynolds number. Previously reported shifts in the peak frequency and increased noise levels at higher frequencies are now linked to parasitic noise originating from the test rig. This suggests that these spurious jet-noise contaminations can become increasingly important for engines with low exit Mach number (due to increasing bypass ratio) and significant exhaust enthalpy.

Apart from the linear propagation of internally generated noise to the far field, acoustic fluctuations contribute to the broadband amplification of jet noise through tonal excitation. Bechert & Pfizenmaier (1975, 1977) and Moore (1977) were the first to demonstrate that tonal excitation can lead to a substantial increase in the sound pressure, reaching values in excess of 7 dB. Moore (1977) showed that broadband noise amplification depends on the level and amplitude of the tonal excitation and jet Mach number. The amplification reduces with increasing Mach number, and the directivity remains largely unaffected by the forcing. A maximum level of amplification is achieved at forcing frequencies around the preferred Strouhal number, and substantial amplification can also be achieved through broadband forcing. This was attributed to the excitation of preferred-mode instabilities. Excitation at low-amplitude levels resulted in locking with the linear modes, and a nonlinear response in conjunction with modifications of the downstream development of turbulence was observed at higher excitation levels.

In contrast to these investigations, measurements by Kibens (1980), Laufer & Yen (1983), and Hussain & Hasan (1985) on jets at low-Mach number and low-Reynolds number conditions showed a reduction of the broadband noise and excitation of subharmonics when forcing at the shear-layer instability frequency. At these conditions, the harmonic forcing alters the large-scale shear-layer structure, and the organized vortex pairing introduces a stationary sound source with strong directivity. Although these forcing frequencies belong to the upper range of core noise, it is important to recognize that these operating conditions are quite relevant for turboshaft engines in auxiliary power units or helicopters and compact high-power-density engine cores.

Few studies have examined the effect of excitation on heated jets. Compared to unheated jets, Jubelin (1980) reported that broadband amplification in heated jets is reduced and substantially more localized around the excitation frequency. The amplification is less uniform and most prominent toward the forward arc. Furthermore, heated jets are substantially more receptive to upstream excitations, but their receptivity decreases with increasing temperature, pointing at effects of higher dissipation and shear-layer laminarization due to increased temperature-dependent viscosity. O'Brien et al. (2016) performed simulations of a representative engine-core flow path, consisting of independently verifiable components, by using a coupled modeling approach to examine effects of nozzle-upstream perturbations on far-field noise. In this approach, they simulated a dual-swirl gas-turbine combustor using compressible reacting LES; represented the transfer function of a single-stage turbine by actuator disk theory; obtained the flow through the nozzle and in the jet from compressible LES, matching test point 49 of Tanna (1977) with a jet-exit Reynolds number of $Re_j = 2.3 \times 10^5$ and temperature ratio of $T_j/T_\infty = 2.857$; and evaluated the acoustic far field using the method due to Ffowcs Williams & Hawkins (1969). Compressible reacting LES are necessary to represent pressure fluctuations inside the combustion chamber. Although these calculations are computationally expensive, advances in high-performance computing and computational resources have made such calculations feasible. In addition, progress in the formulation of predictive combustion models, boundary conditions, and wall models and improved resolution of the turbulent flow field in the combustor, nozzle, and jet exhaust allow one to represent acoustic spectra and flow dynamics over the frequency range of practical relevance.

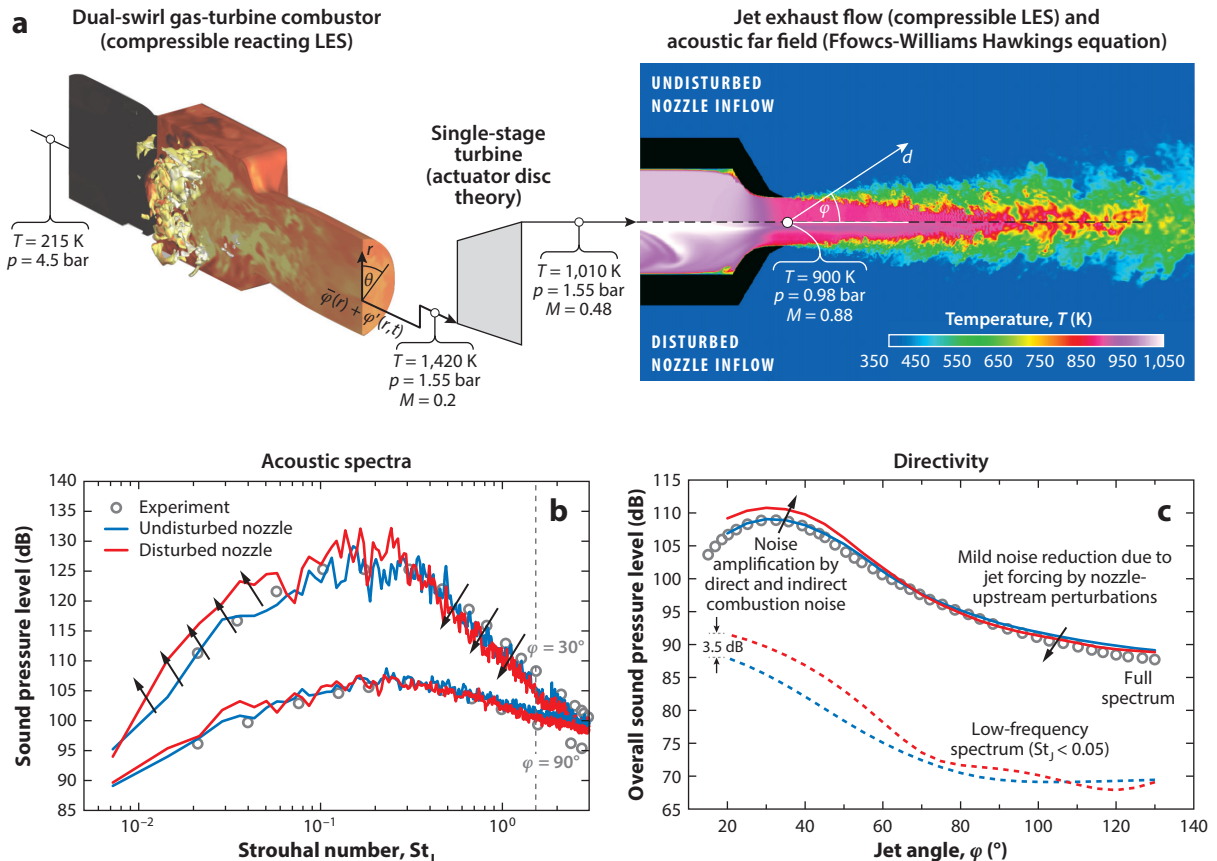


Figure 7

(a) Hybrid simulation of a representative flow path, consisting of independently verifiable engine components, combining compressible reacting large-eddy simulation (LES) of a dual-swirl gas-turbine combustor, actuator disk theory of a single-stage turbine, compressible LES of nozzle and turbulent jet flow, and Ffowcs Williams & Hawkins's (1969) method for the far-field noise radiation. (b) Acoustic spectra and (c) directivity at a distance of $d/D_j = 72$.

Figure 7 shows results for the flow path components of the combustor, turbine, nozzle, and jet exhaust and for the acoustic far field. Predictions of the acoustic spectra and directivity show that far-field noise amplification by direct and indirect combustion noise is confined to downstream radiation angles of $\phi \leq 70^\circ$ with a maximum noise amplification of 3 dB compared to undisturbed (clean) nozzle flow. A mild attenuation of far-field noise is observed at sideline and upstream radiation angles, which is attributed to noise reduction at Strouhal numbers above 0.2 and reduced contributions from core noise at these radiation angles. These simulation results indicate that the nozzle and jet-exhaust flow require consideration in the analysis of engine-core noise.

5.2. Discussion and Research Needs

The following physical description of the effects of nozzle-upstream perturbations and mean-flow inhomogeneities on the far-field noise radiation emerges (see **Figure 6**). Owing to scale separation, low-frequency acoustic waves (with $St_j \lesssim 0.1$) generated upstream of the nozzle are directly

transmitted to the far field without significant interaction with the shear layer or turbulent flow field. In contrast, tonal and broadband acoustic and velocity perturbations at higher frequencies ($St_j \gtrsim 0.2$) can couple with the initial shear layer and jet-column instability. Hydrodynamic disturbances from turbulence, thermal, and compositional fluctuations convected by the mean flow are additional sources for triggering instabilities. Depending on the flow conditions and initial state of the shear layer, the linear response to small perturbations at $Re_j \lesssim 10^5$ suppresses broadband noise by containing the energy in large-scale structures. A nonlinear response by large perturbations at $Re_j \gtrsim 10^5$ causes a faster shear-layer transition and higher levels of small-scale turbulence, leading to nearly uniform broadband noise amplification.

In summary, hydrodynamic fluctuations and mean-flow deformations, which are generated upstream of the nozzle, represent potentially equally effective mechanisms at modifying the acoustic source-term distribution in the jet. As such, far-field noise emission can be regarded as a combination of low-frequency core noise and jet noise from fine-scale and large-scale turbulence, which is modulated by perturbations and mean-flow inhomogeneities upstream of the nozzle. Although low-order methods and compact theories can be utilized to reasonably estimate low-frequency contributions, nonlinear techniques or compressible LES are necessary to represent nozzle-upstream flow-field features that are effective in triggering shear-layer and column-mode instabilities.

Recognizing that nozzle-upstream excitations can effectively attenuate broadband noise, opportunities arise to exploit this mechanism to counterbalance low-frequency amplification by combustion noise. Recent progress on the development of adjoint methods (Kim et al. 2014), resolvent norm analysis (Garnaud et al. 2013), and optimal mode-shape and base-flow modifications (Brandt et al. 2011, Nichols & Lele 2011) represents interesting avenues for application to jet-noise reduction.

6. ENGINE-CORE NOISE

The principal understanding, analysis, and modeling of noise contributions from direct and indirect combustion noise, and the transmission and modulation of jet noise, has guided the investigation of engine-core noise in full-scale engines, auxiliary power units, helicopter engines, and combustor component rigs (Reshotko & Karchmer 1980, Hultgren & Miles 2009, Gordon 2015, Schuster et al. 2015, Stout et al. 2015, Tam & Parrish 2015, Livebardon et al. 2016, O'Brien et al. 2016). However, the detailed spatiotemporal characterization of noise-source mechanisms and acoustic transmission is limited, given restricted experimental access, hostile operating conditions, combustion-physical coupling mechanisms, different far-field noise-source contributions, and geometric complexities. Instead, temperature sensors and microphone probes are commonly employed.

In addition to commonly used spectral analysis of single-point pressure measurements, coherence techniques and modal analyses have been employed to identify far-field noise contributions, source locations, and propagation characteristics of combustion noise. These techniques were successfully applied to the analysis of multiple microphone measurements from different turbofan engines—e.g., AVCO-Lycoming YF-102 (Karchmer & Reshotko 1976), Pratt & Whitney JT15D (Reshotko & Karchmer 1980), General Electric CF6-50 (Doyle & Moore 1980), and Honeywell TECH977 (Miles 2009, 2010; Schuster et al. 2015)—separating indirect and direct combustion noise contributions and determining peak frequencies and spectral shapes of core-noise components. Results from these static engine measurements revealed differences in the core-noise peak frequency, varying between 250 and 400 Hz, and the relative spectral contribution of direct over indirect noise. However, given the superposition of different noise contributions,

these analysis tools have limitations in separating noise sources, acoustic and hydrodynamic modes, and low-pass filtering can lead to attenuations of certain noise contributions in cross-correlation analysis (Miles 2010).

Schuster et al. (2015) and Gordon (2015) conducted measurements on an extensively instrumented dual-spool turbofan engine to obtain simultaneous temperature and pressure data in the combustor, interturbine duct, and mixer. Modal analysis in the combustor identified frequency-dependent acoustic mode shapes, with a dominating planar mode at frequencies below 500 Hz, and counter-rotating helical and higher-order modes observed at successively higher frequencies. Interestingly, the temperature measurements showed fluctuations with a magnitude in excess of 500 K in the combustor, which decayed by an order of magnitude in the interturbine duct. The cross-spectral analysis showed weak spatial coherence of the temperature signal in the combustion chamber, which was attributed to the inhomogeneous combustion and mixing field, and significantly higher coherence in the interturbine duct.

Miles (2009) identified significant differences in the coherence characteristics between real engines and generic single-cup combustor configurations. This was attributed to the presence of azimuthal modes in annular combustors, higher power settings, and the influence of sound sources from the fan, jet, and combustor-turbine interaction. Such effects require consideration in modeling and in relating measurements on idealized configurations to real engine tests. As such, full-scale engine tests fill an important gap in analyzing noise-source mechanisms, calibrating low-order models, and identifying global performance quantities.

Tam & Parrish (2015) used measurements from an F-22A tactical military aircraft to discriminate noise components that are unique to full-scale engines. Comparisons against similarity spectra indicate the presence of a dominant noise component with directivity along shallow jet angles and dependence on engine-power setting. This source was attributed to possible contributions from indirect combustion noise; however, its persistence at high frequencies is different from the results of Miles (2010), who associated indirect combustion noise to frequencies below approximately 500 Hz. Stout et al. (2015) undertook detailed investigations on the same aircraft by using a four-microphone intensity probe. Results from this analysis showed that the jet-source region contributing to the maximum acoustic intensity moves upstream with increasing frequency, and the acoustic source location moves downstream and broadens for conditions in which the afterburner is employed.

The development and selection of models for the prediction of engine-core noise are dictated by user-specific requirements about accuracy, fidelity, and computational cost. Different approaches have been pursued. Modeling approaches for predicting noise and flow-field dynamics in the combustor, turbine/nozzle, and jet can be differentiated into low-order approaches and multidimensional numerical simulations.

At the lowest level of computational complexity are low-order empirical combustion-noise models for far-field directivity and spectral distribution. These models often rely on semi-empirical information for total acoustic power that are developed with information from engine-test data. They are calibrated by engine manufactures for specific engine types (Stone et al. 2011, Hultgren 2012) and take into account information about combustor geometry, operating conditions, combustion environments, and turbine losses. These semi-empirical combustion-noise models are utilized in aircraft noise prediction programs (ANOPP) (Zorumski 1982). Representative results are shown in **Figure 8a**, comparing component noise predictions and far-field measurements of a TECH977 turbofan engine at 60% corrected fan speed at two polar radiation angles measured with respect to the downstream direction (Royalty & Schuster 2008). These models provide adequate descriptions of noise levels for engine classes used in their development. However, their applicability to new engine designs as well as outside of their calibration range is uncertain.

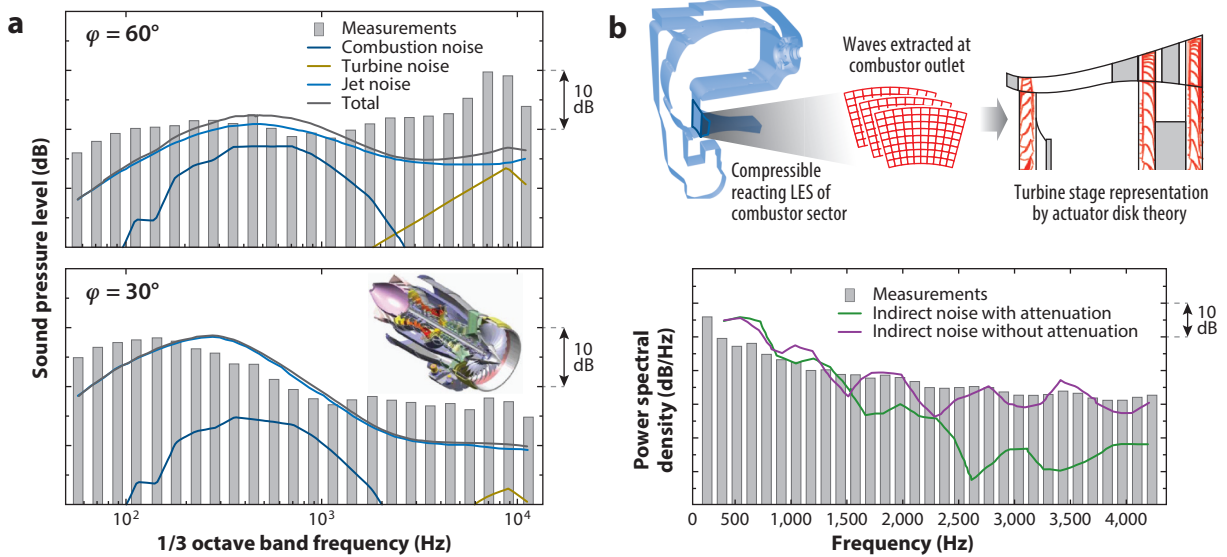


Figure 8

Comparison of different modeling approaches for the prediction of engine-core noise. (a) Results from a low-order aircraft noise prediction program (ANOPP) for a TECH977 dual-spool high-bypass-ratio turbofan. (b) Hybrid simulations of an engine sector of a turboshaft helicopter engine combining large-eddy simulation (LES) for the combustor simulation and actuator disk theory for the turbine. Results adapted from Royalty & Schuster (2008) and Livebardon et al. (2016) with permission from the American Institute of Aeronautics and Astronautics and Elsevier.

In contrast to low-order models, unsteady three-dimensional simulations represent the other spectrum of core-noise predictions. In particular, LES methods are particularly attractive for accurately describing turbulent flows and unsteady source mechanisms that contribute to the generation and transmission of engine-core noise. As such, LES techniques have been employed successfully to predict turbulent combustion in generic and realistic gas-turbine combustors and jet-exhaust flows. However, because of the computational complexity, consideration of moving geometries, and disparity of timescales associated with subsonic combustion and transonic turbine and jet-exhaust flows, the monolithic simulation of the entire engine core using a single solver has not yet been considered.

Coupled methods have been developed for the computationally efficient prediction of engine flow paths (Schlüter et al. 2005). In these methods, multiple specialized flow solvers are combined to enable accurate and efficient predictions of integrated engine components. An example of a coupled method applied to a representative engine flow path is illustrated in **Figure 7**, and other techniques can be considered (Tyacke & Tucker 2015). The main distinguishing features are in the selection of submodels employed for simulating individual engine components, coupling and exchange of information between solvers, and synchronization of different solvers.

LES is an attractive method for simulating flows in the combustor because it facilitates the consideration of multicomponent evaporation, mixing, turbulence, chemical reaction, heat release, the generation of direct combustion noise, and potentially also the occurrence of thermoacoustic instabilities. The representation of the flow field and sound in the turbine allows for different techniques. Analytic methods, discussed in Section 4, provide estimates for the acoustic transfer matrix across turbine stages. However, these simplified methods rely on assumptions about the

geometry and mean-flow representation that limit their predictive characteristics. Multidimensional simulations overcome these issues, and Reynolds-averaged Navier-Stokes (RANS) methods are commonly employed to represent the ensemble-averaged turbulent flow field, boundary layers, and rotor/stator wake structures. Solutions from these RANS calculations can then be used as input to acoustic analogies, multiple-scale techniques, or linear methods for the computation of the acoustic field. Alternatively, compressible nonreacting LES can be employed to obtain the unsteady flow field through the turbine. However, the high Reynolds numbers and requirements for resolving the boundary layer, tip gaps, and rotating geometry introduce substantial resource requirements that impose limitations for implementation in routine applications. To accurately capture the interaction of different engine components, such as hot-streak propagation, acoustic reflections, and instabilities, one must couple different solution techniques. Because different solution methods solve for different flow-field quantities (primitive, conserved, and characteristic quantities), employ different decomposition methods (filtering, ensemble averaging, harmonic methods), and use different spatiotemporal resolutions, prolongation and restriction operators are necessary to consistently exchange information across interfaces. A mathematically stable and physically consistent interface is particularly important for core-noise simulations to eliminate spurious acoustic reflections or dispersion of acoustic waves.

Livebardon et al. (2016) used a coupled method combining compressible reacting LES for a sector of the combustor and actuator disk theory for the turbine to simulate combustion noise of a helicopter engine. The propagation of the core noise to the far field and coupling with the exhaust jet were not included. This investigation focused on examining contributions of direct and indirect combustion noise. Results from this study are illustrated in **Figure 8b**, showing comparisons of measured power spectral density and computations of indirect combustion noise at the turbine exit. Interestingly, these simulations showed that indirect noise is the main contributor to turbine-exit noise, and the low level of direct noise was attributed to the consideration of a single-sector combustion chamber.

FUTURE ISSUES

1. The physical description and computational modeling techniques for the acoustic sources from unsteady turbulent combustion are not complete. In particular, the impact of localized and stochastic combustion events associated with ignition, extinction, and transient blowout dynamics on noise radiation and the generation of acoustic precursor events requires further analysis. Understanding these events becomes increasingly important with the implementation of compact combustion chambers, premixed combustion strategies, and high-power engine cores. In addition, advances in the development of high-fidelity combustion models and subgrid closures are necessary to accurately predict these unsteady combustion events and acoustic sources. DNS and high-speed diagnostics can have a significant impact in addressing these issues.
2. Further fundamental examination of the coupling of engine components and the characterization of the impact of acoustic reflection on the onset of instabilities are needed. This has direct consequences on the modeling of thermoacoustic precursor events, the formulation of boundary conditions in numerical simulations, and the integration of coupled engine-core noise models.

3. The relative contribution of direct and indirect combustion noise is strongly dependent on the engine type, operating conditions, and interaction with other noise-source mechanisms. Further research is therefore needed to describe these contributions in integrated engine flow paths under the consideration of interaction processes between the combustor, turbine, nozzle, and jet exhaust.
4. Other noise sources, such as those arising from jet-noise modulation, turbine tones, and thermoacoustic instabilities, need further study to ensure that noise goals are met.
5. The description of the generation of indirect noise and the transmission and attenuation of direct combustion noise in engine cores largely relies on low-order models. Further progress is needed to improve model fidelity to include geometric complexities, realistic flow-field descriptions, and additional contributions from hydrodynamic and thermochemical perturbations through nonlinear coupling between turbulence, boundary layers, and mean flow.
6. The need for full-scale engine tests is recognized for evaluating design modifications and validating noise-prediction models. Multidimensional flow-field measurements in the combustor and turbine and the characterization of boundary conditions are desirable to effectively use these measurements to validate high-fidelity models.
7. To address the pressing need for meeting ambitious noise-reduction goals, opportunities arise for suppressing combustion noise at the source through combustor-design modifications. In addition, active and passive control of nozzle-upstream perturbations and the mean flow are attractive pathways for far-field noise reduction by jet-noise modulations. Adjoint methods and base-flow modifications represent interesting techniques to guide the development of such noise-mitigation strategies.

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The author is not aware of any biases that might be perceived as affecting the objectivity of this review.

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