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Distributivity in Formal Semantics

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Abstract

Distributivity in natural language occurs in sentences such as *John and Mary (each) took a deep breath*, when a predicate that is combined with a plurality-denoting expression is understood as holding of each of the members of that plurality. Language provides ways to express distributivity overtly, with words such as English *each*, but also covertly, when no one word can be regarded as contributing it. Both overt and covert distributivity occur in a wide variety of constructions. This article reviews and synthesizes influential approaches to distributivity in formal semantics and includes pointers to some more recent approaches. Theories of distributivity can be distinguished on the basis of how they answer a number of interrelated questions: To what extent can distributivity be attributed to what we know about the world, as opposed to the meanings of words or silent operators? What is the relationship between distributivity and plurality? Does distributivity always reach down to the singular individuals in a plurality? If not, under what circumstances is distributivity over subgroups possible, and what is its relation to distributivity over individuals?

1. INTRODUCTION: SOME KEY PHENOMENA AND QUESTIONS

The term *distributivity* has been applied to a variety of phenomena in natural language. All of them have in common that a predicate is applied to the members or subset of a group or set, or to the parts of a plurality. Following Choe (1987) and others, I refer to the predicate as the Share and to the plurality as the Key. For example, in sentence 1*a*, the subject *the girls* is the Key, and the verb phrase (VP) *smiled* is the Share. Likewise, in sentence 1*b*, the subject is the Key, and the VP *are wearing a dress* is the Share:

(1a) The girls smiled.

(1b) The girls are wearing a dress.

Sentence 2 can be understood either distributively or nondistributively, depending on how many sand castles were built:

(2) The children built a sand castle.

The semantic contribution of indefinite noun phrases, such as *a dress* in sentence 1*b* and *a sand castle* in sentence 2, is often captured by an existential quantifier. One of the tasks for formal theories of distributivity is to explain what gives this existential quantifier the ability to range over multiple entities in distributive readings even though it corresponds to a morphologically singular indefinite.

The lack of a nondistributive interpretation in sentences 1*a* and 1*b* is clearly connected to what we know about the world (for example, we know that multiple people cannot wear the same dress simultaneously). In other cases, it can be attributed to the presence of a distributive marker, such as the word *each*:

(3) Each of the children built a sand castle.

Distributive markers can belong to multiple syntactic categories. In sentence 3, it is a determiner; in sentence 4*a*, it is an adverb; and in sentence 4*b*, it is an adnominal modifier:

(4a) The children each built a sand castle.

(4b) The children built one sand castle each.

In all these cases, the marker has the same form, and one feels that it also has the same meaning. In examples such as sentences 1 and 2, there is no overt marker of distributivity. Depending on whether distributivity can be traced to the presence of an overt marker, we can distinguish between overt and covert distributivity.

Not all Keys are denoted by subjects. In sentence 5*b*, on its distributive reading, the Key is the indirect object *two girls*, not the subject. In sentence 5*a*, the Key can be either *the girls* or *the boys*, depending on whether there were two stories per girl or per boy:

(5a) The girls told the boys two stories each.

(5b) John gave a pumpkin pie to two girls.

In all the examples provided so far, distributivity always reaches down to singular individuals in the Key. But distributivity down to subgroups has been argued to be possible as well, with important implications for the analysis of distributivity as a whole. For example, sentence 6, adapted from Gillon (1987), arguably involves distribution over pluralities of men, at least on the reading on which it is true in the actual world (given that Gilbert and Sullivan wrote operas together but, unlike Mozart and Handel, not individually):

(6) Mozart and Handel and Gilbert and Sullivan wrote operas.

Likewise, sentence 7, from Lasersohn (1998), can be understood as describing a collection of pairs of shoes such that each pair costs \$50:

(7) The shoes cost \$50.

One important question is whether distributivity always reaches down to individuals or whether distributivity to subgroups is also possible. Consider a scenario in which Mary and Sue each bought a lottery ticket, and John and Bill bought one together. Suppose that these three tickets turn out to be the winning ones. In response to questions such as *Which women won? Which men won? Who won?* one might give answers 8*a*, 8*b*, and 8*c*, respectively. Answer 8*a* is true on its distributive reading, answer 8*b* is true on its nondistributive reading, and answer 8*c* is true on what I call a nonatomic distributive reading (in contrast to the atomic distributive reading of answer 8*a* and most previous examples). This kind of reading has also been called a subgroup distributive reading or, following Heim (1994), an intermediate reading:

(8a) Mary and Sue won.

(8b) John and Bill won.

(8c) Mary and Sue and John and Bill won.

Despite such sentences as 6, 7, and 8*c*, many theories do not put atomic and nonatomic distributivity on an equal footing. Elucidating the relationship between the two kinds of distributivity is a central problem; an example adapted from Lasersohn (1989) helps us see what makes it hard. Suppose that there are four teaching assistants (TAs) in the local linguistics department, and that each of them taught a recitation and was paid \$7,000 for it last year. In this situation, both sentence 9*a* (on its distributive reading) and sentence 9*b* (on its nondistributive reading) are judged true, but sentence 9*c* is not:

(9a) The TAs were paid exactly \$7,000.

(9b) The TAs were paid exactly \$28,000.

(9c) The TAs were paid exactly \$14,000.

Clearly, sentence 9*c* does not exhibit nonatomic distributivity; if it did, it should be true, since the four TAs can be grouped into pairs each of which was paid \$14,000. The question is: Why not?

The following minimal pair brings the problem into sharp relief:

(10a) The men wrote operas.

(10b) The men wrote an opera.

The subject of these sentences is to be understood as referring to Mozart, Handel, and Gilbert and Sullivan, just like the subject of sentence 6. Given the known facts about these composers, sentence 10*a* is judged true but sentence 10*b* is not. The challenge for theories of distributivity, then, is to chart the right course between examples such as 6, 7, and 8*c* on the one hand and examples such as 9*c* and 10*b* on the other (Kratzer 2008; Winter 2001, p. 256f).

In this review, I focus on distributivity of verbs and VPs over individuals, with an emphasis on the challenges posed by nonatomic distributivity. I describe some of the dimensions along which formal theories of distributivity can be distinguished and how they account for the phenomena described so far (for a more extensive overview of relevant empirical facts, with an emphasis on distributivity across languages, see Champollion forthcoming and the references given in Section 5).

2. SOME CHOICE POINTS FOR THEORIES OF DISTRIBUTIVITY

When considering a large number of theories, it is useful to set aside certain distinctions and focus on others. Some distinctions I ignore here concern the formal modeling of plurality. I use the term *pluralities* as a theory-neutral way to refer to collections of entities denoted by Keys, no matter how these collections are represented formally. Most accounts of distributivity are couched in formal systems that represent pluralities explicitly as possible referents of variables. This article focuses on such systems and sets aside those that treat pluralities only implicitly via the resources of plural logic, such as those presented by Schein (2008) and Oliver & Smiley (2016) (see Moltmann 2016 for a comparison of these two kinds of systems). As described in Section 2.1, for ease of comparison I cast all theories I review in terms of mereological sums as opposed to sets. The distinctions I focus on concern different kinds of distributivity operators (Section 2.2), whether they apply to predicates of individuals or of events (Section 2.3), and whether they take scope at the verb level or at the VP level (Section 2.4).

2.1. Sets Versus Sums

Within approaches that represent pluralities explicitly, we can distinguish between approaches that represent pluralities as sets and those that represent them as mereological sums. The earliest formal semantic approaches to distributivity use sets (Bartsch 1973, Bennett 1974, Hausser 1974). In these approaches, singular individuals can be represented either as ordinary entities or as singleton sets. Some approaches use a nonstandard interpretation of set theory, following Quine (1937), that collapses the distinction between these two options (e.g., Schwarzschild 1996). Standard set theory is used by Winter (2001) and Heycock & Zamparelli (2005).

However, most modern formal semantic research on distributivity uses mereological sums, following Link (1983). Mereology is the philosophical study of parthood (Leśniewski 1916). The overview articles by Champollion & Krifka (2016) and Varzi (2016) contain detailed presentations. As applied to the semantics of natural language, mereological models countenance not only singular individuals, such as Alice, Bob, and Chris, but also plural individuals, such as the sum of Alice and Bob or the sum of Alice, Bob, and Chris. The most widely used and formally best-understood version of mereology in formal semantics is classical extensional mereology (CEM), but other versions of mereology are in use as well (Moltmann 1997, 1998).

In mereology, an operation $\oplus$ is assumed to combine entities into sums. In CEM, this operation is assumed to be associative, commutative, and idempotent. If we replace singular individuals by singleton sets, CEM makes the sum operation behave just like union in ordinary set theory.

A central notion in mereology is the parthood relation $\leq$. In CEM, this is assumed to be a partial order. Parthood and sum are interdefinable, in the sense that $a \leq b$ is equivalent to $a \oplus b = b$. In CEM, parthood is analogous to the subsethood relation in ordinary set theory. Models of CEM are lattice structures that are isomorphic to complete Boolean algebras with the bottom element removed. The powerset algebra of any set of singular individuals, minus the empty set, is isomorphic to the mereology that has these individuals as its atoms (where an atom is defined as an individual that has no proper parts, that is, no parts other than itself).

One of the differences between set theory and mereology is that the empty set is a subset of any set, but there is no such thing as an "empty part," which would be a part of every entity. Another difference concerns the lack of structure in sums. Set theory allows for nested sets and makes distinctions that do not correspond to anything within CEM. For example, $\{\{a, b\}, c\}$, $\{a, \{b, c\}\}$, and $\{a, b, c\}$ are distinct sets, but there is no such difference in CEM; since the sum operation is associative, the mereological object $a \oplus b \oplus c$ can be taken to correspond to any of these sets. This difference might seem substantive, but as discussed below, semanticists who use mereology sometimes avail themselves of "group formation" operators that reinsert into sums the structure that has been lost in the transition from set theory to CEM.

Another difference between set theory and mereology is that the standard axiomatization of set theory forbids infinitely descending chains of sets of the form $\ldots \in S_3 \in S_2 \in S_1$. By contrast, nothing in mereology forbids analogous infinitely descending chains of sums $\ldots s_3 \leq s_2 \leq s_1$. Indeed, it is possible for every object in a mereology to have proper parts; this gives rise to an "atomless" mereology, which is often used to model the semantics of mass terms and other domains such as events and temporal and spatial intervals. Link (1983) regards mereology as a better fit than set theory for the mass domain, and extends it to the semantics of plurals in order to capture certain parallels between the count and mass domains. However, treatments of mass terms that use standard set theory have been proposed since then (Chierchia 1998, 2010), leading some (e.g., Winter 2001) to question whether mereology is a well-motivated choice, despite its widespread adoption. The use of mereology for events and intervals has not been questioned in the same way, however. Because events and intervals play an important role in the semantics of verbs, mereology is arguably a natural choice for theories that aim to account for parallels between the nominal and verbal domains (e.g., Bach 1986, Krifka 1998, Champollion 2017). In this article, I adopt sums as a lingua franca, even though doing so occasionally requires me to reformulate proposals that were originally based on sets rather than sums.

2.2. The Star Operator Versus the D Operator

The vast majority of formal semantic approaches to distributivity model it using one or more silent operators, with major questions revolving around the choice and the division of labor between different operators. In order to understand how the need for operators arises, it is useful to consider an alternative view that makes no use of them. In the earliest formal semantic treatments, distributivity is as a property of predicates, with no operators involved. Thus, for Bartsch (1973), Scha (1981), and Hoeksema (1983), predicates in general admit both individuals and pluralities of individuals in their extension; a distributive predicate is a predicate which, whenever it applies to a plurality, also applies to each of the individuals in that plurality. Hoeksema (1983) suggests capturing the difference between distributive and nondistributive predicates by using meaning postulates to put restrictions on admissible models. On this type of approach, the sentence *Alice and Bob smiled* is represented (assuming mereology) as smile($a \oplus b$). A meaning postulate to the effect that *smile* is distributive rules out models in which this predicate applies to the sum $a \oplus b$ without also applying to a and to b separately.

In contrast to these approaches, Link (1983) suggests that distributive predicates such as *smile* or *sleep* admit only atoms in their extension. This property sets them apart from collective predicates such as *meet*, which contain only proper sums of individuals, and from "mixed" (neither distributive nor collective) predicates such as *win* and *carry the piano*, which contain atoms as well as sums (for an overview of the distributive–collective opposition, see Champollion forthcoming). When a distributive predicate combines with a plural argument, an operation called algebraic closure, represented by a star (*), is applied to the predicate. For example, the sentence *Alice and Bob smiled* is represented as *smile($a \oplus b$). This operation can be defined as follows (the formulation is taken from Sternefeld 1998):

Star operator

For any set P, $^*P$ is the smallest set such that

(11a) $P \subseteq {}^*P$, and

(11b) if $a \in {}^*P$ and $b \in {}^*P$, then $a \oplus b \in {}^*P$.

Because this definition is inductive, not only binary sums but also sums of arbitrary large finite subsets of P end up in $^*P$. Alternative definitions of the star operator make sure that even the sums of infinitely large subsets of P make it into ${}^*P$ (e.g., Link 1983, Champollion 2017). When considering only finitely large P, this difference does not matter.

Link (1987, 1991) takes the same approach as Link (1983) for distributive predicates like *smile* and *sleep*, but treats mixed predicates such as *build a sand castle* through a different mechanism, namely a VP-level operator that shifts nondistributive predicates into distributive predicates:

D operator

(12) $[\![D]\!] = \lambda P \lambda x \forall y[[y \leq x \wedge \mathrm{atom}(y)] \rightarrow P(y)]$

This operator takes a predicate P over individuals and returns a predicate that applies to any individual whose atomic parts each satisfy P. Its intuitive meaning and function correspond to those of the English adverb *each*: When the operator is inserted into sentence 2, the resulting meaning can be paraphrased by sentences 4*a* and 4*b*. Roberts (1987) and others suggest an analysis of adverbial *each* in terms of Link's D operator.

2.3. Events Versus No Events

The D operator has been adapted to various frameworks (e.g., Lasersohn 1995). Champollion (2016a) adapts it to the Neo-Davidsonian setting, in which verbs and VPs are assumed to denote sets of events rather than individuals (Davidson 1967, Parsons 1990):

Event-based D operator

(13) $[\![\mathrm{D}_\theta]\!] = \lambda V \lambda e.\, e \in {}^*\{e' \mid V(e') \wedge \mathrm{atom}(\theta(e'))\}$

The operator applies to a set of events V, typically denoted by a verb or VP, and returns another event predicate, which contains events e that either are or consist of events e' in V. Each of these events e' is mapped to some atomic entity by what I call the dimension parameter θ, which can be resolved to a thematic role such as *agent*, *theme*, or *goal* (Parsons 1990). Typically, but not always, this is the thematic role of the subject. Similar operators are proposed by Lasersohn (1998), Kratzer (2008), and LaTerza (2014a,b); event-based distributivity is also discussed by Schein (1993) and Lasersohn (1995), among others.

Because thematic roles relate events to individuals, they are available only in the event semantic setting. By letting the dimension parameter vary, one can account for nonsubject Keys, such as that in the prepositional phrase in sentence 5*b*; in eventless frameworks, one would instead resort to type shifting or quantifier raising (Roberts 1987, Lasersohn 1995). An advantage of the event semantic framework is that an operator such as the one in definition 13 can make the sum event e available for modification by further arguments, or by adjuncts such as the one in the following sentence (Schein 1993, Eckardt 1998):

(14) From 2pm to 4pm, the children [D built a sand castle].

Champollion (2016b) builds on the event-based D operator in definition 13 and on the flexibility of event semantics to analyze the word *each* in its various uses as a determiner (sentence 3), as an adverbial (sentence 4*a*), and as a distance-distributive adnominal (sentence 4*b*). For previous analyses of *each*, see Choe (1987), Moltmann (1997), Zimmermann (2002), and Dotlačil (2012), among many others. The dimension parameter of the event-based D operator can then be used to model the ambiguity in sentences such as example 5*a*. Depending on whether it is set to the thematic role of the subject *the boys* or to that of the object *the girls*, the sentence entails either that there were two stories per boy or that there were two stories per girl. The parameter thus captures the difference between covert and overt distributivity over subjects and over nonsubjects in a uniform way.

2.4. Verb-Level Versus VP-Level Distributivity Operators

In Link 1983, the star operator applies only to individual words, such as verbs and nouns. In Link 1987, by contrast, the D operator can apply to entire VPs as well. This explains why in sentence 1*b*, repeated below in modified form, the indefinite can range over multiple dresses even though it is morphologically singular:

(15) Annie, Bonnie, and Connie are wearing a dress.

The distributive reading of sentence 15 cannot be modeled by assuming that only its main verb is interpreted distributively, as this would entail that there is a dress that the three girls are wearing; rather, the D operator must apply to the VP, as in sentence 16*a*. The universal quantifier that this operator introduces takes scope over the existential quantifier over dresses:

(16a) Annie, Bonnie, and Connie [D [are wearing a dress]].

(16b) $\forall y\ [[\mathrm{atom}(y) \wedge y \leq (a \oplus b \oplus c)] \rightarrow \exists x\ [\mathrm{dress}(x) \wedge \mathrm{wear}(y, x)]]$

For Link (1987), the D operator models distributivity at the VP level, while the star operator models distributivity at the verb level. Winter (2001), Champollion (2016a), and de Vries (2017) advocate similar approaches; Moltmann (1997) draws a related distinction. To be sure, the star operator is defined on one-place predicates, so it would not be able to apply to the transitive verb *lift* if that verb is modeled as a two-place predicate; but this is an accidental property of the specific framework. In Neo-Davidsonian event semantics, even transitive verbs are modeled as one-place predicates, and the event-based D operator can apply to them as well. A generalization of the star operator to two-place predicates is discussed in Section 4.3, below.

3. ATTEMPTS TO UNIFY THE D AND STAR OPERATORS

Although the D and star operators differ, they also overlap to some extent. Could one of them replace the other? This question is taken up by Landman (1989a,b, 1996, 2000) and by Winter (2001), who approach it from two different perspectives. Landman uses only the star operator, whereas Winter uses only the D operator. As discussed in this section, both authors also use resources other than those provided by CEM alone. Let us first take a closer look at the difference between the two operators and then consider the attempts to unify them.

To sidestep scope-related differences between the D and star operator, let us consider a VP that consists of only one nondistributive verb, *win*. In the scenario considered in Section 1, Mary and Sue each bought a winning lottery ticket, and John and Bill bought one together. Suppose there are no other winning tickets. It seems natural to represent the denotation of *win* by set 17*a* (this assumption is revised below). Compare the results of applying the star operator to that predicate, in set 17*b*, with applying the D operator to it, in set 17*c*:

(17a) $[\![\mathrm{win}]\!] = \{m, s, j \oplus b\}$

(17b) $[\![{}^{*}\mathrm{win}]\!] = \{m, s, m \oplus s, j \oplus b, m \oplus j \oplus b, s \oplus j \oplus b, m \oplus s \oplus j \oplus b\}$

(17c) $[\![\mathrm{D(win)}]\!] = \{m, s, m \oplus s\}$

The star operator expands the original set 17*a* by including all the sums built from its members, while the D operator in effect restricts set 17*a* to its atomic elements and then adds only those sums that are built from them. The two operators have the same result when they are applied to predicates that have only atoms in their extensions, but this is not what we have assumed in set 17*a*.

In order to appreciate what is at stake, it is useful to look at the truth conditions of various sentences in our scenario according to the different analyses, assuming for the time being that conjoined proper names refer to sums. Consider again sentences 8*a*–*c*, repeated below. Sentence 8*a* is predicted to be true when either of the operators applies, and false when no operator does:

	Mary and Sue won.	= 8*a*
(18a)	$\text{win}(m \oplus s)$	False
(18b)	$\text{D}(\text{win})(m \oplus s)$	True
(18c)	$^*\text{win}(m \oplus s)$	True

Sentence 8*b* is predicted to be true when no operator or the star operator applies, and false when the D operator applies:

	John and Bill won.	= 8*b*
(19a)	$\text{win}(j \oplus b)$	True
(19b)	$\text{D}(\text{win})(j \oplus b)$	False
(19c)	$^*\text{win}(j \oplus b)$	True

Sentence 8*c* is predicted to be false when no operator or the D operator applies, and true when the star operator applies:

	Mary and Sue and John and Bill won.	= 8*c*
(20a)	$\text{win}(m \oplus s \oplus j \oplus b)$	False
(20b)	$\text{D}(\text{win})(m \oplus s \oplus j \oplus b)$	False
(20c)	$^*\text{win}(m \oplus s \oplus j \oplus b)$	True

Formula 20*a* is false because there was no instance of winning in which the four individuals were involved together. Formula 20*b* is false because the four individuals did not *each* win. As for formula 20*c*, this is a nonatomic distributive reading. It essentially says that the group of children can be divided into atomic or nonatomic parts, such that each of these parts won.

Clearly, the D operator does not give rise to nonatomic distributive readings, whereas the star operator does. This might be taken as an argument for giving up the D operator, but, as shown below, things are not so straightforward. In Section 4, I present reasons to model nonatomic distributivity with operators that differ from both D and star. The remainder of this section, though, stays with the two operators introduced so far, and asks whether we can give up one or the other.

3.1. Landman: No D Operator

Landman (1989a) argues that set 17*a* is not the right way to represent the meaning of *win* in our model. Instead, Landman proposes that, following Link (1984), certain pluralities should be modeled not as sums but rather as groups. These entities are technically atomic entities, in the sense that they do not have any mereological proper parts, but they are related to sum individuals that are taken to correspond to their members. On this view, for example, a committee that consists of John and Bill is modeled as a group whose members are John and Bill. The sum of the two members of the committee is represented as $j \oplus b$, but the committee itself is modeled as the group $\uparrow(j \oplus b)$, where $\uparrow$ is taken to be a function that maps sums to atoms; thus, the atom $\uparrow(j \oplus b)$ is distinct from the sum $j \oplus b$ (see also Moltmann 1997 for a related theory based on a nontransitive parthood relation; for a criticism of that theory, see Pianesi 2002).

Landman uses the sum–group distinction to model distinctions between collective and cumulative readings and other phenomena. Groups essentially introduce into mereology the structure that is available in set theory. In a scenario where John and Bill jointly bought a winning ticket and Mary bought another one, one might think of the winners as a plurality in which John and Bill are more tightly connected to each other than any other pair. Because the sum operation is associative, in the absence of group formation it is not possible to express this type of plurality. For example, $m \oplus (j \oplus b)$ is the same entity as $(m \oplus j) \oplus b$, which is why we write it as $m \oplus j \oplus b$. The group formation operator breaks associativity: $m \oplus \uparrow(j \oplus b)$ is not the same entity as $m \oplus j \oplus b$.

The use of mereology and groups has been influential in formal semantics, which is reflected in the following exposition. But as noted in Section 2.1, one can express many of the relevant ideas in set theory as well, where the difference between the "flat" set $\{m, j, b\}$ and the "nested" set $\{m, \{j, b\}\}$ makes it possible to capture the "tight connection" between John and Bill. While I keep to sums and groups throughout, not much changes in the following exposition if we mentally replace terms such as $j \oplus b$ and $m \oplus \uparrow(j \oplus b)$ by $\{j, b\}$ and $\{m, \{j, b\}\}$, and the symbol $\leq$ by $\in$ or $\subseteq$ (for details on this last point, see Schwarzschild 1996, section 1.1). Landman (1989a, p. 568f.) contrasts this set-theoretic view, which he labels "ontological," with a view he calls "structural," on which the lattice structures that are the models of mereology are isomorphic to collections of sets but do not actually consist of sets (see also Link 1987 and Winter 2001, chapter 2).

With the sum–group distinction in hand, Landman proposes that if John and Bill won the lottery together, the winner should be thought of as the group of John and Bill, not as their sum. More generally, Landman proposes that basic predicates—that is, those to which no operators have applied—never take sums in their extension; only the output of the star operator can produce such predicates. Verbal predicates may apply to singular individuals and to groups, but not to sums. For example, the predicate *win*, before any operators apply to it, has the extension in set 21*a*, rather than the one in set 17*a*. Since $\uparrow(j \oplus b)$ is an atom, it is treated by the D operator exactly like other atoms; as a result, when applied to set 21*a*, the D and star operators return the same result, as shown in set 21*b*. This assumes that the members of a group, unlike the parts of a sum, are not accessible for distribution (but see de Vries 2015):

(21a) $[\![\text{win}]\!] = \{m, s, \uparrow(j \oplus b)\}$

(21b) $[\![{}^*\text{win}]\!] = [\![\text{D(win)}]\!]$

$$= \{m, s, \uparrow(j \oplus b), m \oplus s, m \oplus \uparrow(j \oplus b), s \oplus \uparrow(j \oplus b), m \oplus s \oplus \uparrow(j \oplus b)\}$$

On the basis of this observation, Landman (1989a) argues that the D operator is superfluous and that the star operator can take over its function. As Landman (2000, p. 152) puts it, we can reduce distributivity to semantic plurality. This refers to the fact that Landman, following Link (1983), uses the star operator not only in the verbal domain to create distributive interpretations but also in the nominal domain to capture the semantic contribution of the plural morpheme.

Although Landman does not discuss it explicitly, in some of his examples the star operator is applied to predicates denoted by entire VPs that contain an indefinite, such as *carry a piano upstairs*. This means that in his system the star operator cannot be regarded as a property of verbs. As discussed in Section 4.2, applying the star operator to VPs overgenerates, which arguably means that Landman's (1989a) project of reducing distributivity to plurality is only partly successful. Furthermore, the reliance on groups to model the meaning of plural definites has been called into question (Barker 1992, Schwarzschild 1996). It is therefore doubtful whether the D operator can be replaced by the star operator.

3.2. Winter: No Star Operator

Winter (2001) takes the opposite approach. In this system, it is the star operator that is removed; the D operator (which Winter calls *pdist*, for predicate distributivity) takes its place, and is used both as a means to analyze distributive readings and as a way to capture the semantic contribution of the plural morpheme. Here, for ease of comparison I recast the relevant parts of Winter's theory in terms of mereology and group formation; Winter's actual system is largely set based.

If one assumes that all singular nouns have only atoms (of which some may be groups) in their extension, the two operators return the same result whenever they are applied to sets of atoms, as shown above. On this view, sentence 8*c* (*Mary and Sue and John and Bill won*) can be analyzed as in formula 22*a*, which is equivalent to formula 22*b*. Since $\uparrow(j \oplus b)$ is an atomic part of $m \oplus s \oplus \uparrow(j \oplus b)$, but neither j nor b is, the result entails that that Mary won, that Sue won, and that John and Bill as a group won:

(22a) $\mathrm{D}(\mathrm{win})(m \oplus s \oplus \uparrow(j \oplus b))$

(22b) $\forall x\, [\mathrm{atom}(x) \wedge x \leq m \oplus s \oplus \uparrow(j \oplus b)] \rightarrow \mathrm{win}(x)]$

Whether sentences like 8*c* have readings such as the one captured by formula 22*b* is a critical factor in the question of whether the D operator can be removed from the grammar.

In the remainder of this article, I use the term *structured plurality* for pluralities that involve the $\uparrow$ operator as well as for sets that are nested rather than flat, and I use the term *unstructured plurality* for $\uparrow$-less pluralities and their corresponding flat sets. The analyses by Landman (1989a) and Winter (2001) require the subject of sentence 8*c* to denote a structured plurality akin to $m \oplus s \oplus \uparrow(j \oplus b)$, as opposed to the unstructured plurality $m \oplus s \oplus j \oplus b$.

To be clear, analyzing coordinated noun phrases as structured pluralities does not by itself entail a commitment to the use of sets versus sums; nor does it entail a specific analysis of the word *and*. For example, Winter (2001) shows that through the use of sets and various silent operators, structured pluralities arise naturally within the framework of generalized quantifier theory (Partee & Rooth 1983, Keenan & Faltz 1985). He argues that the word *and* denotes neither mereological sum or set union nor set formation, but rather intersection of generalized quantifiers. Structured pluralities are possible referents of coordinated noun phrases in Winter's system, but only in their incarnation as nested sets; the structure of these nested sets, in effect, mirrors some or all of the syntactic structure of the coordinated noun phrases that denote them.

4. THE STATUS OF NONATOMIC DISTRIBUTIVITY

As noted in Section 1, there are open questions concerning the status of nonatomic distributivity, its relation to atomic distributivity, and the way it arises in the formal system. Part of what makes these questions hard to answer is that the use of devices such as the group-forming $\uparrow$ operator makes it possible to represent pluralities as atoms. This raises the question of whether all Keys can denote structured pluralities.

4.1. Can Definite Plurals Denote Structured Pluralities?

Positing structured pluralities as possible referents of Keys can help account for certain instances of nonatomic distributivity, particularly in the case of coordinated noun phrases, as in sentence 8*c*. However, the standard examples of Keys are definite plurals, which do not provide as much internal syntactic structure as coordinated noun phrases. Consider the following sentence in a situation similar to that described above, and assume that John, Bill, Mary, and Sue are the children:

(23) The children won.

In this situation, the standard analysis of the definite plural *the children* is that it refers to the unstructured plurality $m \oplus s \oplus j \oplus b$, the sum of the four children (e.g., Link 1983). Could the same noun phrase *the children* also refer to the structured plurality $m \oplus s \oplus \uparrow(j \oplus b)$ instead? If we want to remove the star operator from the grammar and replace it by the D operator, as Winter (2001) proposes, it is crucial for the answer to be yes; otherwise, the D operator would force distribution down to John and Bill individually. At the same time, the same definite plural would also have to be able to refer to the unstructured plurality, in order to model sentences such as *The children took a deep breath*, where the predicate distributes down to each of the children.

That definite plurals are ambiguous between structured and unstructured pluralities had previously been proposed by Gillon (1987, 1990). The sentence which motivated this proposal, sentence 10*a* (*The men wrote operas*), is structurally similar to sentence 23, except that it contains a transitive verb. Its subject is to be understood as referring to Mozart, Handel, and Gilbert and Sullivan, like the subject of sentence 6.

Lasersohn (1989, 1995) argues, contra Gillon, that definite plurals cannot refer to structured pluralities, as this would overgenerate nonatomic distributive readings. Consider again the scenario described in Section 1, where Mary, Sue, John, and Bill are the TAs and each of them was paid $7,000. Sentences 9*a–c* are repeated below, along with the judgments associated with them:

(24a)	The TAs were paid exactly $7,000.	True
(24b)	The TAs were paid exactly $28,000.	True
(24c)	The TAs were paid exactly $14,000.	False

If the definite plural *The TAs* could refer to structured pluralities, the contrast between these three sentences would be unexpected; all three would be predicted to be true. The reason is that there are ways to structure the plurality in question—for example, $\uparrow(m \oplus s) \oplus \uparrow(j \oplus b)$—which, when used as referents of the subject of sentence 24*c*, would make it true no matter whether it is the D operator or the star operator that applies to the VP. Lasersohn (1989) concludes from this type of example that, in this situation, *The TAs* can refer only to the unstructured plurality $m \oplus s \oplus j \oplus b$, and not to the structured plurality $\uparrow(m \oplus s) \oplus \uparrow(j \oplus b)$, contrary to Gillon (1987). This also means that the VP *were paid exactly $28,000* in sentence 24*b* applies to a sum without applying to its parts, contrary to Landman (1989a). Schwarzschild (1996) develops a more extensive argument that definite plurals cannot in general denote structured pluralities (see also Lasersohn 1995, chapter 9; Moltmann 1997, 2005; and Winter 2001 for discussion).

4.2. Applying the Star Operator to VPs Overgenerates

With structured pluralities taken off the table again as possible referents of definite plurals, a question arises as to how the nonatomic distributive readings of sentences 6 and 23 should be modeled. The D operator is clearly of no use: In the absence of structured pluralities, it will distribute the VP down to individual children and composers. In the case of sentence 23, the star operator fares better: Since the subject denotes $m \oplus s \oplus j \oplus b$, the sentence is analyzed in the same way as formula 20*c* and is correctly predicted to be true. In the case of sentence 6, one might be tempted to follow a similar route. First, assume that the predicate *write operas* applies to Mozart, to Handel, and to the sum of Gilbert and Sullivan:

(25) $[\![\text{write operas}]\!] = \{m, h, g \oplus s\}$

Next, assume that the star operator applies to the predicate *write operas*:

(26) $[\![{}^*(\text{write operas})]\!] = \{m, h, g \oplus s, m \oplus h, m \oplus g \oplus s, h \oplus g \oplus s, m \oplus h \oplus g \oplus s\}$

Finally, apply this starred predicate to *the men*, which we now take to denote the unstructured plurality $m \oplus h \oplus g \oplus s$. Since the predicate applies to this plurality, we correctly predict that sentence 6 is true.

But there is a problem. In the model considered above, under the assumption that Mary and Sue are female and John and Bill are male, the following sentences are true:

(27a) The female TAs were paid \$14,000.

(27b) The male TAs were paid \$14,000.

Assuming that the subjects of these two sentences refer to $m \oplus s$ and to $j \oplus b$, their VP *were paid \$14,000* must denote a predicate that has at least these two pluralities in its extension. Now suppose that the star operator can apply to this predicate. Then the plurality $m \oplus s \oplus j \oplus b$ should be in the extension of the result as well. But then, given that the subject of sentence 24*c* refers to $m \oplus s \oplus j \oplus b$, we would predict that sentence to be true after all, contrary to fact.

The problem becomes even more severe when we compare sentence 10*a* with its counterpart 10*b*, both repeated here:

(28a) The men wrote operas.

(28b) The men wrote an opera.

When *The men* is taken to refer to Mozart, Handel, and Gilbert and Sullivan, sentence 28*b* is not judged to be true in the scenario described in Section 1, even though sentence 28*a* is. In particular, sentence 28*b* lacks the nonatomic distributive interpretation that sentence 28*a* has. Heim (1994, p. 12) describes this situation as follows:

> So we are in a bit of a dilemma: We do seem to need representations in terms of the star operator to treat the cases of apparent intermediate readings that are attested, but if we freely generate them, this has all sorts of undesirable consequences.

4.3. Generalizing the Star Operator to Transitive Verbs

On the basis of examples such as sentences 24*b* and 28*b*, Lasersohn (1989) and others conclude that, unlike the D operator, the star operator cannot apply to entire VPs. This accounts for the lack of a nonatomic distributive interpretation of sentence 28*b*, but it does not explain why sentence 28*a* is true. For this purpose, Lasersohn (1989) proposes the use of lexical meaning postulates to the effect that whenever a writes x and b writes y, it is also the case that $a \oplus b$ writes $x \oplus y$ (Krifka 1989, Kratzer 2008). Let o_1 be the sum of all the operas that Mozart wrote, and similarly for o_2 (Handel) and o_3 (Gilbert and Sullivan). It follows that $m \oplus h \oplus g \oplus s$ wrote $o_1 \oplus o_2 \oplus o_3$. The effect of this type of meaning postulate can be captured by a generalization of the star operator to binary relations, which is often expressed through the double-star operator ** (e.g., Beck & Sauerland 2000, Beck 2012). Here is one way to define this operator (Sternefeld 1998):

Double-star operator

For any two-place relation R, let $^{**}R$ be the smallest relation such that

(29a) $R \subseteq {}^{**}R$, and

(29b) if $\langle a, x\rangle \in {}^{**}R$ and $\langle b, y\rangle \in {}^{**}R$, then $\langle a \oplus b,\ x \oplus y\rangle \in {}^{**}R$.

As in the case of the star operator, alternative definitions of the double-star operator exist (e.g., Vaillette 2001, Champollion 2017).

For example, let the denotation of *write* be the two-place relation 30*a*. To avoid clutter, I omit the individual operas that make up o_1, o_2, and o_3. This two-place relation could equivalently be represented as a function that takes two arguments one at a time, corresponding to the object and subject of the verb *write*, and returns a truth value. I ignore this point since it does not affect the discussion (for details, see Heim & Kratzer 1998, section 2.4). The double-star operator maps relation 30*a* to relation 30*b*:

(30a) $[\![\text{write}]\!] = \{\langle m, o_1\rangle, \langle h, o_2\rangle, \langle g \oplus s, o_3\rangle\}$

(30b) $[\![{}^{**}\text{write}]\!] = \{\langle m, o_1\rangle, \langle h, o_2\rangle, \langle g \oplus s, o_3\rangle, \langle m \oplus h, o_1 \oplus o_2\rangle, \langle m \oplus g \oplus s, o_1 \oplus o_3\rangle, \langle h \oplus g \oplus s, o_2 \oplus o_3\rangle, \langle m \oplus h \oplus g \oplus s, o_1 \oplus o_2 \oplus o_3\rangle\}$

The double-star operator can be used to model the difference between sentences 28*a* and 28*b* as follows. Sentence 28*a* is represented by existentially quantifying over an entity in the denotation of *operas*. The meaning of *operas*, in turn, is represented by applying the star operator to the denotation of *opera*, and it holds of the sum entity $o_1 \oplus o_2 \oplus o_3$:

(31a) The men **wrote operas.

(31b) $\exists x\, [{}^{*}\text{opera}(x) \wedge {}^{**}\text{write}(m \oplus h \oplus g \oplus s, x)]$

As for sentence 28*b*, it is represented in the same way except that the existential quantifier ranges only over entities in the denotation of *opera*:

(32a) The men **wrote an opera.

(32b) $\exists x\, [\text{opera}(x) \wedge {}^{**}\text{write}(m \oplus h \oplus g \oplus s, x)]$

These entities are individual operas but not sums of operas; therefore, $o_1 \oplus o_2 \oplus o_3$ is not in the denotation of *opera*. As for individual operas, such as the atomic parts of o_1, while they are in the denotation of *opera*, neither relation 30*a* nor relation 30*b* relates any of them to the sum $m \oplus h \oplus g \oplus s$ denoted by *the men*. Thus, sentence 32*a* is correctly predicted to be false on its nondistributive reading. As for its distributive reading, resulting from applying the D operator to *wrote an opera*, it is also correctly predicted to be false: This operator distributes down to atoms, but neither the atom g nor the atom s stands in either relation 30*a* or relation 30*b* to anything.

The double-star operator is motivated by sentences like example 33*a*, which can be analyzed as in formula 33*b*, on the assumption that the operator applies directly to the verb. This is an instance of a summative or cumulative reading (e.g., Scha 1981, Krifka 1989, Sternefeld 1998):

(33a) [[Tom and Dick] **like [Sue and Jane]].

(33b) ${}^{**}\text{like}(t \oplus d, s \oplus j)$

With the double-star operator in place, let us consider example 34, a variation of example 20*a–c* that replaces intransitive by transitive *win*, in the same scenario as that sentence. When no operator applies, as in formula 34*a*, the sentence is predicted to be false. The same is the case even if the D operator is applied to the VP, as in formula 34*b*. Only if the double-star operator applies to the verb, as in formula 34*c*, is the sentence predicted to be true:

Mary and Sue and John and Bill won the lottery.

(34a)	$\text{win}(m \oplus s \oplus j \oplus b, l)$	False
(34b)	$\text{D}(\lambda x.\text{win}(x, l))(m \oplus s \oplus j \oplus b)$	False
(34c)	${}^{**}\text{win}(m \oplus s \oplus j \oplus b, l)$	True

As this example shows, there are cases in which even the application of a verb-level operator (such as the double-star operator) can cause an entire VP to appear to exhibit nonatomic distributivity. This is the case for example 34 because its object, a definite singular, is scopally inert; therefore, distributing over only the verb or over the entire VP makes no difference to truth conditions. If the object were replaced by an indefinite such as *a lottery*, the double-star operator would be unable to cause that indefinite to covary, and the modified sentence would continue to require that everyone won the same lottery. This is analogous to sentences like example 28*b*, which on its nondistributive reading requires everyone to have written the same opera.

Winter (2001, p. 255) concludes from similar examples that, as a matter of methodology, distribution over subparts of a plural individual "is easier to attest when the plural NP in question is semantically interpreted as a quantifier taking scope over another element in the sentence (an indefinite, a pronoun, a disjunction etc.)." Winter refers to such cases as "Q-distributivity" (where Q refers to quantificational), as opposed to "P-distributivity" for those cases of distributivity which can, in principle, be derived from a property of the lexical item involved. Even unmodified indefinites should perhaps be avoided; for further details on this test, see de Vries (2015).

Because only VPs but not verbs can contain indefinites, pronouns, or disjunctions, any operator that is meant to capture Q-distributivity must be able to apply at the VP level. As for cases that can be handled by application of an operator to the verb, such as examples 20, 28*a*, and 34, we can regard these operators as part of the denotation of the verb itself, in the spirit of the meaning-postulate approach put forward by Scha (1981), Hoeksema (1983), and Lasersohn (1989). This is also the view taken by Winter (2001), who therefore regards all such cases as P-distributivity (see also Kratzer 2008).

4.4. Beyond D and Star: The Part Operator

As we have seen, the nonatomic distributive reading of sentence 28*a* can be analyzed in terms of P-distributivity. This raises the question of whether cases of genuine nonatomic Q-distributivity can be found. Such cases could not be modeled by applying any operator at the verb level, because verb-level operators cannot capture Q-distributivity, nor could they be modeled by applying the D operator at any level, because the D operator can capture only atomic distributivity. Lasersohn (1998) offers example 7, repeated below as example 35*a*, which is understood as stating that each pair of shoes costs \$50. Similarly, example 35*b* (J. Bledin, personal communication) can be used as a clue to solving a magic square, a type of puzzle that involves filling in a grid with natural numbers. In that context, it states that each row, column, or diagonal in the square sums up to 25:

(35a) The shoes cost \$50.

(35b) The numbers sum to 25.

On the assumption that the subject of sentence 35*a* refers to the sum $s_1 \oplus s_2 \oplus s_3 \oplus s_4$, where each s_i is a shoe, this reading cannot be modeled via the double-star operator, as this would merely capture the fact that the price of the sum of two entities is the sum of their prices. For example, if the pair of shoes $s_1 \oplus s_2$ costs \$50 and the pair of shoes $s_3 \oplus s_4$ also costs \$50, then the sum of these two pairs costs \$100. For the sake of the example, I am ignoring the difference between mereological sum and arithmetic sum.

The relevant reading of sentence 35*a* cannot be modeled using the D operator either, at least not in the absence of further assumptions such as structured pluralities. This operator, when applied to the VP *cost \$50*, would take the predicate it denotes and distribute it down to each atomic part of $s_1 \oplus s_2 \oplus s_3 \oplus s_4$—that is, down to each shoe.

From similar examples, Schwarzschild (1991, 1996) concludes that distributivity over nonatomic entities is possible when these entities are made salient by context (see also Moltmann 1997 for a similar proposal). To avoid problems that would result from using groups, Schwarzschild (1996) proposes modeling this behavior by modifying the D operator to include a variable over covers, which are generalizations of partitions. In mereological terms, a cover of a plurality x is a set of entities whose sum is x. Unlike partitions, this definition allows for overlap, as in the magic square case in example 35*b*. Schwarzschild (1996) renames the modified operator Part, to set it apart from Link's D operator. In definition 36, the cover variable is represented as a subscripted C:

Schwarzschild's nonatomic distributivity operator Part

(36) $[\![\text{Part}_C]\!] = \lambda P \lambda x \forall y \, [[C(y) \wedge y \leq x] \rightarrow P(y)]$

To illustrate, the structure of sentence 35*a* would be taken to be as in example 37*a*. Here, C is a free variable that is resolved to a salient cover of the subject, for example $\{s_1 \oplus s_2, s_3 \oplus s_4\}$, which results in formula 37*b*:

(37a) The shoes Part_C [cost \$50].

(37b) $\forall y \, [y \in \{s_1 \oplus s_2, s_3 \oplus s_4\} \rightarrow y \in [\![\text{cost } \$50]\!]]$

Various authors disagree on whether the fact that C must be a cover of x should be written into the operator or can be derived from more general considerations. The event semantic version of the Part operator shown in definition 38, from Champollion (2016a), makes this a moot point, since the free predicate C can be shown to be a cover of the event e by virtue of appearing in the scope of a star operator (Vaillette 2001):

Event-based Part operator

(38) $[\![\text{Part}_{\theta,C}]\!] = \lambda V \lambda e \,.\, e \in {}^*\{e' \mid V(e') \wedge C(\theta(e'))\}$

This is a generalization of the event-based D operator shown in definition 13. Champollion (2016a) recasts the cover variable C as a granularity parameter. The D operator can be thought of as a special case of the Part operator in which this granularity parameter is hardwired to the predicate *atom*, as opposed to being anaphoric on a pragmatically salient predicate. This view forms part of a more comprehensive theory of distributivity, aspect, and measurement (Champollion 2017; for an overview of the main ideas of this theory, see also Champollion 2015). When applied to distributivity, this broader view posits that distributivity always takes place along a certain dimension θ and down to a certain granularity C. Dimension and granularity are understood as parameters whose values can vary across constructions and sometimes across instances of one and the same construction.

4.5. Distributivity at the Intersection of Semantics and Pragmatics

The introduction of a pragmatic component into the analysis of distributivity raises questions that go beyond formal semantics as such. Moltmann (1997, 1998, 2005) locates it in the part–whole relation, which she sees as determined by the information content of situations. Malamud (2012) proposes that the Part operator is anaphoric on a decision problem in the sense of van Rooij (2003). In Malamud's approach as well as Schwarzschild's, the Part operator imposes a stronger restriction on the identity of the cover than would be achieved by merely existentially quantifying over the cover variable. This restriction prevents that operator from generating unattested nonatomic readings for examples 9*c* and 28*b* and similar sentences.

In the discussion in connection with the *shoes* example (sentence 35*a*), I have assumed that pairs of shoes are represented as nonatomic entities, and that these are the only kinds of entities in the extension of the plural noun *shoes* (perhaps in addition to individual shoes; see Krifka 1989, Spector 2007, Zweig 2009). But what if pairs of shoes are atoms after all, perhaps derived via $\uparrow$? In that case, what looks like nonatomic distributivity could turn out to be atomic, and the D operator would suffice. A challenge to this view is that when a question like *How many shoes are on display?* is answered with an integer (e.g., *four*), that number is understood as counting shoes, not pairs.

How certain should we be that the intended referents of singular and plural definite descriptions in specific contexts are the singular and plural entities that semantic theories assign to them? Winter & Scha (2015, p. 97) note that in saying *This shoe costs \$50* one may speak loosely of the price of a pair of shoes, and relate this to fact that short general definite descriptions sometimes stand for related entities (Nunberg 1979). The definite description in sentence 35*a* is also short and general. As Winter & Scha (2015) observe (see also Winter 2000), this raises the question of whether the right context can still induce a nonatomic distributive interpretation when we replace the short definite description by a more specific one, as in sentence 39*a*, or by an explicit enumeration, as in sentence 39*b*:

(39a) These four shoes cost $50.

(39b) Shoes A, B, C, and D cost $50.

This empirical question remains open.

5. CONCLUSION

The study of distributivity in formal semantics has yielded a variety of approaches. I have focused primarily on the tension that arises from the need to constrain nonatomic distributivity without excluding it entirely. Here, I list a number of related phenomena that this review article does not discuss in detail.

Within formal semantics, research topics on distributivity include the best way to model distributivity in dynamic frameworks (e.g., van den Berg 1996, Brasoveanu 2011, Bumford & Barker 2013, Kuhn 2015); the nature of the relationship between distributivity and reciprocals (e.g., Roberts 1991; Moltmann 1992; Sternefeld 1998; Filip & Carlson 2001; Dotlačil 2010, 2013; Winter 2017); the status of "antidistributivity markers" such as *together* (e.g., Lasersohn 1990, 1995, 1998; Schwarzschild 1994, 1996; Moltmann 2004); the relationship of distributivity to event plurality and dependent indefinites (e.g., Kratzer 2008, Cabredo Hofherr & Laca 2012, Balusu & Jayaseelan 2013, Henderson 2014); the connection to homogeneity and nonmaximality in plural definites (Lasersohn 1999; Brisson 2003; Križ 2015, 2016); and the relationship between *each* and *all*, which raises open questions relating to the status of collective predicates and their relation to distributivity (e.g., Champollion 2017, chapter 10).

Distributivity can be approached from subfields of linguistics other than formal semantics. In Section 4.5, I have drawn some connections between formal semantics and pragmatics. These connections are also helpful in other areas. For example, Rothstein (2010) and Schwarzschild (2011) discuss distributivity in adjectives from a semantic point of view, while Scontras & Goodman (2017) and Glass (2018a) exploit the potential of pragmatic reasoning in determining whether a given adjectival predicate is interpreted as distributive. Other subfields in which distributivity has been studied include acquisition (e.g., Syrett & Musolino 2013, de Koster et al. 2017), lexical semantics (e.g., Glass 2018b), and processing (e.g., Frazier et al. 1999). For additional overviews of distributivity in formal semantics, see Link (1991); Lønning (1997); Lasersohn (2011); Szabolcsi (2010, chapters 7 and 8); Nouwen (2016); Winter & Scha (2015); and Champollion (forthcoming, section 2), who reviews empirical and crosslinguistic phenomena and largely steers clear of the formal details that are the focus of this review.

I close with a personal view on the elusive status of nonatomic Q-distributivity. In this article, I have followed the literature by focusing largely on distributivity in the domain of individuals. But in domains that arguably lack atoms, such as time, clearer examples of nonatomic Q-distributivity can be observed. If we set the value of the dimension parameter θ in the event-based version of the Part operator given above (operator 38) to a function from events to the time intervals at which they occur, the operator induces covariation of indefinites with salient time intervals such as days. Starting from the observation that indefinites normally fail to covary with *for*-adverbials, as shown by the oddness of example 40*a*, taken from Zucchi & White (2001), Champollion (2016a) uses this Part operator to explain why the indefinite *two pills* in example 40*b* can covary (think of a hospital context where a patient's daily intake is salient). This suggests that example 40*b* involves nonatomic distributivity:

(40a) ??John found a flea for ten minutes.

(40b) The patient took two pills for a month and then went back to one pill.

Champollion (2016b) argues that the relationship between the D and Part operators is mirrored in the relationship between the English word *each* and its crosslinguistic counterparts. For example, German *jeweils* can be interpreted as either *each* or *each time/on each occasion* (Zimmermann 2002). If these analyses are on the right track, then the difference between atomic and nonatomic distributivity accounts for crosslinguistic variation in the meaning of overt distributivity markers, and is therefore worth maintaining in the grammar.

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