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New Solutions to the Gauge Hierarchy Problem

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Abstract

Applying dimensional analysis to the Higgs mass leads one to predict new physics interactions that generate this mass at a scale of the order of 1 TeV. The question of what these interactions could be is known as the gauge hierarchy problem. Resolving this question has been a central aim of particle physics for the past few decades. Traditional solutions introduce new particles with masses below 1 TeV, but that prediction is now challenged by experiment. In this article, I review recent new approaches to the problem that do not require new particles at the TeV mass scale. I first discuss the relaxation approach, whereby the Higgs mass is made dynamical and is small at the absolute minimum of its potential. I then discuss the historical approach, whereby details about inflation and/or reheating after inflation cause the Higgs mass to be smaller than otherwise expected. Finally, I discuss solutions that use conditional probability, whereby conditioning on the fact that the cosmological constant is small automatically leads one to select vacua where the Higgs mass is also small.

Contents

1. INTRODUCTION

One of the most important features of physics is dimensional analysis. Dimensional analysis allows us to estimate the answer to many questions without detailed knowledge of the system at hand. Almost always, the dimensionless numbers in front of any dimensional analysis estimate happen to be $\mathcal{O}(1)$, validating its use. As long as one understands all of the relevant symmetries of the problem and the question being asked makes sense, dimensional analysis provides the answer. In this article, I review the application of dimensional analysis to the Higgs boson. The application of dimensional analysis to various problems in particle physics has a long and storied history (see, e.g., References 1–21).

The gauge hierarchy problem is the problem of estimating the distance or mass scale of the physics responsible for setting the Higgs boson mass. In particular, why is the Higgs mass much less than the other fundamental scale we observe in nature, the Planck length? At its heart, the gauge hierarchy problem is a question about estimating the distance scale where our standard description of the Higgs boson breaks down. As we will be using dimensional analysis to estimate this distance scale, it is worth first reviewing some of the spectacular successes that can be achieved with it. Almost universally, the first example of dimensional analysis that people learn is how to estimate the time it takes for a ball that has been dropped to hit the floor. Because this depends only on the distance to the floor, d, and the strength of gravity, g, the simple dimensional analysis estimate gives $t \sim \sqrt{d/g}$. However, if all of the relevant physics has not been identified, an incorrect result can be obtained. For example, if the coefficient of air resistance, b, is important then the time would instead be estimated as $t \sim bd/(mg)$.

A classical physics example that is analogous to what we will do below in the quantum theory is to use the multipole expansion to guess the size of an object. The multipole expansion involves taking an arbitrary distribution of charges and performing a Taylor series on the potential in the long-distance limit:

$$V = \frac{1}{r} \sum_{l,m} \left(\frac{a}{r}\right)^l C_{l,m} Y_{l,m}(\Omega). \qquad 1.$$

One can guess the size at which our roughly spherically charged cow is nonspherical. The first term in the potential Q/r gives the rough size of the typical charges involved in the system while the second-order term, the dipole term, scales as p/r^2. The variable p has dimensions of charge times distance, and the size of the system can be guessed to be

$$R \sim p/Q. \tag{2.}$$

When performing classical experiments, we use estimates of this sort all the time to guess the size of an object. Applying this estimate to molecules, protons, nuclei, and so forth always gives the rough size of the object. Particle physicists formalize the statement that such dimensional analysis estimates are accurate with the term naturalness.

Perhaps the most successful application of naturalness is anticipating the scale of quantum mechanics à la Weisskopf (22, 23). The first step in this line of reasoning is obtaining the classical electron radius. Consider the mass of the electron. Einstein's most famous equation, $E = mc^2$, shows that energy and mass are one and the same. Thus, any source of potential energy associated with the electron should contribute to its mass. One source of potential energy is the electromagnetic self-energy of the electron. At the classical level, this energy is infinite. Assuming that at some distance R, one of the assumptions breaks down (e.g., $1/r^2$ electric field, point-like electron), dimensional analysis tells us that the self-energy has a potential energy $V \sim \alpha/R$, which gives a contribution $\delta m_e \sim \alpha/R$ to the mass of the electron (using particle physics units where $\epsilon_0 = \mu_0 = c = \hbar = 1$). In the extreme limit, the entirety of the electron mass is due to the electromagnetic self-energy $m_e \sim \delta m_e$, giving

$$R_{\rm cl} = \frac{\alpha}{m_e}. \tag{3.}$$

$R_{\rm cl}$ is also known as the classical radius of the electron. If the electron were modeled as a charged ball, $R_{\rm min}$ would be the smallest the radius of the electron could possibly be.

An important point is that the above argument does not specify what actually happens at the scale $R_{\rm cl}$. Dimensional analysis simply informs the physicist when something will happen, not what will happen at said location. While the classical assumption was that electrons would cease to be point-like before the scale $R_{\rm cl}$, instead what we observed was that all of classical mechanics broke down, and quantum mechanics emerged instead. This prediction bears out experimentally: When $R \sim 100R_{\rm cl}$, quantum mechanics kicks in and completely changes the computation. While dimensional analysis cannot predict what will occur at R, it does correctly predict that one of the assumptions will break down.

The last example of dimensional analysis that I will review is estimating the energy scale where the description of the charged pion must be modified. The pion is inherently quantum mechanical, so the above computation must be modified. As above, if the pion were a point-like object, the electromagnetic energy would be infinite. We thus assume that something happens at an energy scale Λ and use dimensional analysis to estimate Λ. The correction to the mass of the pion coming from Einstein is $\delta m_\pi^2 \sim e^2\Lambda^2$. As above, this can be flipped on its head to give an estimate of the energy scale where something new happens to be $\Lambda^2 \lesssim m_\pi^2/e^2$, which turns out to be within $\mathcal{O}(1)$ of the mass of the ρ meson. As in the case of the electron, dimensional analysis has been used to estimate the scale at which something happens. Like the multipole example, we have used two terms in the multipole expansion of the pion, the charge and mass, to estimate something intrinsic to the nature of the pion.

We are finally at the point where we can describe the gauge hierarchy problem. The question is "At what length scale does our description of the Higgs field break down?" Experimentally, we know that the Higgs field is charged under the electroweak and hypercharge gauge groups. It also gives mass to the top quark and is thus coupled to it via a Yukawa coupling y_t. All of these interactions have been experimentally measured and lead to potential energies that via $E = mc^2$ contribute to the mass of the Higgs boson.

We can apply the previous arguments to the many different interactions of the Higgs boson to obtain the potential energies resulting from these interactions. We thereby obtain the following result:

$$\Lambda_{\text{Higgs}} \lesssim \frac{m_h}{\max(g, g', y_t)} \sim 10^{2-3}\ \text{GeV}. \qquad 4.$$

The mystery of the gauge hierarchy problem is "What happens at this scale?"[1]; if the answer is nothing, then what did we do wrong when estimating this energy scale?

The question of what happens at Λ_{Higgs} has two sides. The first is experimental, and the second is theoretical. Experimentally, the LHC has looked for new particles. If the new particles are colored, then their mass must be at the TeV scale (29–32). If the new particles are electroweak-charged, then they could still have a mass of hundreds of GeV (33–35). Precision measurements of Higgs boson higher multipole moments (i.e., higher-dimensional operators) indicate that the Higgs boson's size is at least smaller than 1/TeV (36, 37).

Combined, all of these experimental results suggest that it is likely, though not impossible, that if something new happened in our description of the Higgs boson at an energy scale $E \lesssim 10^3$ GeV, then we would have seen it. Meanwhile, any solution at an energy scale $E \gtrsim 10^3$ GeV remains untested. With reference to Equation 4, we see that we have explored $\mathcal{O}(50\%)$ of the energy range where our description of the Higgs boson is expected to break down. Further experimental input would be needed to firmly establish the failure of dimensional analysis (see, e.g., Reference 38).

From these experimental results, one can see that it is in fact premature to call the gauge hierarchy problem a "problem" as we have not fully explored the region of parameter space predicted by dimensional analysis. While the issue is often phrased as a problem, one should keep in mind that there is not a problem yet. Some resolutions might be excluded, but not all of them are.

The theoretical side of what happens at R_{Higgs} is an open and interesting question. There has been much work on this subject, which has led to many exotic and exciting ideas. One of the original ideas was technicolor (25, 26), which solved the problem by lacking a Higgs boson entirely. Another foundational approach was compositeness, which is discussed in more detail below.

After these initial approaches, there was a surge of newer ideas, though at this point they are considered traditional. Extra dimensions explored the possibility that the scale of quantum gravity was much lower than the Planck scale. Extradimensional solutions usually fall in the category of large extra dimension (39, 40) or warped 5D models (41, 42) (for a recent review of these models, see Reference 43).

One of the other traditional solutions is supersymmetry (2). It is connected to the abovementioned ideas as supersymmetry is merely fermionic extra dimensions. On the opposite end lies what is easily the most contentious traditional approach, anthropics (44–47). Anthropics postulates that the existence of intelligent life is a strong selection criterion that warps what we consider as a natural expectation.

The last of the traditional solutions centers around allowing for new particles at the predicted scale but making them hard to observe. Originally, what was thought to be present at the scale R_{Higgs} was new colored scalars, as motivated by supersymmetry and compositeness. With ideas such as extra dimensions and little Higgs (48–50), it was found that the new particles could be fermions as well. Finally, neutral naturalness models such as twin Higgs (51–53) and folded supersymmetry (54, 55) showed that these new particles did not need to have Standard Model gauge charges.

[1]While the focus here has been simply on dimensional analysis, many other approaches attempt to go even further and predict the $\mathcal{O}(1)$ numbers in Equation 4. This is usually done with a hard cutoff, though many different approaches and philosophies have been proposed (24–28).

It is thus a valid possibility that the new particles are exactly where they have been predicted and that only precision Higgs physics can reveal their existence (56, 57).

In this article, I focus more on the newer approaches to the electroweak hierarchy problem. The new approaches I discuss have all been inspired by the following line of thought. As described above, using dimensional analysis to successfully estimate the length scale of a problem is a common practice. There is absolutely nothing special about its application to the Higgs boson. It is an experimental fact that common problems usually feature common solutions. As such, it is natural to expect that whatever solves the electroweak hierarchy problem should be a common solution that we have observed before. As such, the new approaches discussed in this article take observed common solutions to previous examples of hierarchy problems and apply them to the Higgs boson.

If the solution to what happens at R_{Higgs} is a common solution that has also solved previous hierarchy problems, then we need to enumerate previous solutions to naturalness problems. At the classical level, there are several examples of previous solutions, but two stand out as the most commonly observed ones.

Classically, perhaps the most common resolution to hierarchy problems is compositeness—namely, despite the oft-used spherical cow approximation, in detail a cow is not a simple object but is made of nontrivial components. This is the resolution whenever the multipole expansion is used, and this long-distance approximation is constantly used even in other systems, such as effective field theories. Finite size is the constant resolution to these erstwhile "problems." The problem and solution are so common that they often are not even phrased as a problem and resolution but simply as facts of life.

Compositeness is such a clear and obvious resolution to the gauge hierarchy problem that it has been proposed before, and solutions along these lines are called composite Higgs models (58, 59). These solutions are well explored in other reviews (60, 61) and discussions and, as such, are not discussed further here despite their clear attractiveness.

In classical examples, the only other solution that is anywhere near as common as compositeness is relaxation. Like compositeness, it is often so obvious that the problem and solution are not considered as such. As an example of the interplay between relaxation and naturalness, consider carbon dioxide (13). Dimensional analysis predicts that the dipole moment of CO_2 should be $p \sim qd \sim 10^{-8}\ e{\cdot}\mathrm{cm}$, where q is the typical charge of the oxygen/carbon and d is the size of the molecule. Unfortunately, we have measured the dipole moment of CO_2 to be exactly zero, indicating quite a large failure of dimensional analysis. This occurs because the angle between the two C=O bonds can change, and the molecule relaxes to a linear equilibrium configuration O=C=O. In this equilibrium configuration, the electric dipole moment is exactly zero. This elegant solution is a relaxation solution and is so natural that the CO_2 dipole problem is never even called a problem.

Relaxation approaches to the Higgs mass are so new that there is currently only a single relaxation model that solves the Higgs mass problem (62). This model is still new enough that it is not clear conceptually how to generalize this example into a class of models. Solutions of this type are ubiquitous classically, but their application to the electroweak hierarchy problem has been so recent that the solution is not even fully understood yet.

The next class of solutions being explored is historical solutions. These are solutions where context and history matter. If a theorist wanders into an experimental laboratory, the theorist will likely be extremely confused as there will be a large abundance of fine-tuned 3D theories being probed. For example, the magnetic field might be at just the critical point such that the Landau–Ginzburg scalar is nearly massless. However, considering the graduate student present controlling the magnetic field and the fact that the lab studies phase transitions, the theorist will no longer be confused.

Historical solutions to the electroweak hierarchy problem use our history to solve the problem. There are two general classes of models of this sort. Both use our known history (63), which involves inflation followed by the Universe reheating and reaching thermal equilibrium sometime before big bang nucleosynthesis. One class is based on the reheating process after inflation; this article discusses the example of NNaturalness (64). In these models, the reheating process results in the Universe being populated only by Higgs bosons that are abnormally light.

The second class of historical solutions is known colloquially as the relaxion (65).[2] In these models, the early Universe dynamics of the moduli controlling the mass of the Higgs boson leads to the moduli spending a large amount of time at the minimum where the Higgs boson mass is small. If we exist during this long period of time before the moduli tunnels to another vacuum, then it explains why the Higgs mass is so small.

The last class of solutions is based on conditional probabilities. Our observable Universe contains many features. It is entirely possible that questions about the Higgs boson could be correlated with other unknowns. There exist many approaches along these lines. The oldest and most famous is anthropics, where the answer to a question depends on our existence. For example, the Universe is mostly empty. The answer to the question of why we exist in an atypical location in the Universe where matter exists likely has to do with the fact that the empty vacuum is not intelligent. However, the requirements for intelligent life are subtle and uncertain.

More recently, people have attempted to tie the electroweak hierarchy problem to the cosmological constant problem (47, 66, 67): the question of why the cosmological constant is 10^{120} times smaller than dimensional analysis predicts. These new approaches work by correlating the cosmological constant with the Higgs mass. Only when the Higgs mass is near its observed value is the cosmological constant small, and thus while a small Higgs mass may seem surprising, it is not surprising once one realizes that we are in a universe with a small cosmological constant.

An important part of any new theory is the experimental signatures associated with it. The new solutions discussed here all have new experimental signatures. Traditionally, solutions to the electroweak hierarchy problem involve new particles at high energies that require high-energy colliders to observe. The experimental signatures of these new approaches involve precision Higgs physics, fifth-force experiments, astrophysical probes, and cosmological observables such as N_{eff} and modified structure formation. This wide variety of probes indicates that tests of naturalness are far more varied than previously appreciated.

Aside from the solutions mentioned above, there are many other more speculative proposals. Some of these involve ultraviolet (UV)/infrared (IR) mixing where UV symmetries can reach into the IR (68–71). Others use quantum gravity-based conjectures, such as the weak gravity conjecture (72–74). Finally, some other approaches embrace the measure problem of quantum gravity and postulate specific measures (75–78). Unfortunately, space is limited, and these other models therefore are not discussed in this review.

Section 2 discusses the relaxation solution. Section 3 discusses historically based solutions. Conditional probability–based solutions are discussed in Section 4. Section 5 concludes.

2. RELAXATION SOLUTIONS

Relaxation solutions are the solution to many would-be dimensional analysis problems. The approach of any relaxation solution is simple. Let the small quantity of interest become dynamically

[2] Contrary to what the name suggests, these are historical solutions as opposed to relaxation solutions.

variable, and either hope or arrange that at the absolute minimum of the system, the small number is explained.

Thus, any relaxation solution to the Higgs mass problem adds an additional field, ϕ, whose main purpose is to change the measured value of the Higgs mass, $M_H^2(\phi)$. The second part of any relaxation solution is to check whether when ϕ is at its absolute minimum, the value of M_H is approximately zero. We may now check whether the Standard Model has this property. At tree level, the potential for the Higgs boson is

$$V = M_H^2 HH^\dagger + \frac{\lambda}{4}(HH^\dagger)^2 + cM_H^2\Lambda^2, \quad 5.$$

where the first two terms are responsible for spontaneous symmetry and the last term comes from the zero-point energy of the Higgs boson, with c being a constant whose exact value depends on the UV completion. After integrating out the Higgs boson, we find that the vacuum energy's dependence on M_H^2 is

$$V(M_H) = -\frac{M_H^4}{\lambda}\Theta(-M_H^2) + cM_H^2\Lambda^2. \quad 6.$$

Only if the total Higgs mass is negative will the Higgs boson induce a nonzero tree-level potential for M_H^2. We see that there are two main issues with promoting $M_H^2 \to M_H^2(\phi)$. The first issue is that the $cM_H^2\Lambda^2$ term is UV sensitive and tends toward $|M_H^2| \to \Lambda^2$, which is not the small value that one hopes for. The second issue is that the sign of $\delta V = -\frac{M_H^4}{\lambda}\Theta(-M_H^2)$ is opposite what we would have hoped. Even if the first issue is resolved, a Higgs mass of zero is a maximum rather than a minimum. Finally, one would like to go beyond tree-level results and hopefully still solve the problem once loop-level corrections are taken into account.

2.1. A Model

Solving the two problems ubiquitous to relaxation solutions is difficult but not impossible. I now present the one model that evades these issues (62). As mentioned above, any relaxation solution starts by postulating a new periodic field ϕ with period f_ϕ whose role will be to change the mass of the Higgs boson. For reasons that will be clear later, our example will contain four separate Higgs bosons: H_1, H_2, H_3, and H_4.

As with most solutions to the electroweak hierarchy problem, symmetries play an important role. Here, we will introduce a $\mathbb{Z}_4$ symmetry. The choice of a $\mathbb{Z}_4$ symmetry is not unique, and the following discussion can easily be extended to any $\mathbb{Z}_N$ symmetry for any $N > 2$. Under this discrete symmetry, the relevant fields transform as

$$H_i \to H_{i+1} \quad \text{and} \quad \frac{\phi}{f_\phi} \to \frac{\phi}{f_\phi} + \frac{2\pi}{4}. \quad 7.$$

The four Higgs bosons are cyclically permuted while the angular field ϕ moves a quarter of the way through its period.

The last relevant symmetry is a shift symmetry on ϕ, $\phi/f_\phi \to \phi/f_\phi + \alpha$. This shift symmetry is explicitly broken by a dimension two spurion κ^2. Under this shift symmetry, ϕ and the spurion transform as

$$\frac{\phi}{f_\phi} \to \frac{\phi}{f_\phi} + \alpha \quad \text{and} \quad \kappa^2 \to \kappa^2 e^{-i\alpha}. \quad 8.$$

Because of this symmetry, any nonderivative interaction of the field ϕ involves the combination $\kappa^2 e^{i\phi/f_\phi}$ or $\kappa^2\cos(\frac{\phi}{f_\phi} + \theta)$ with some phase θ.

We now write the most general Lagrangian consistent with the symmetries. The potential of this Lagrangian is

$$\begin{aligned} V &= M_{H_1}^2(\phi)H_1H_1^\dagger + M_{H_2}^2(\phi)H_2H_2^\dagger + M_{H_3}^2(\phi)H_3H_3^\dagger + M_{H_4}^2(\phi)H_4H_4^\dagger \\ &\quad + \lambda\left(\left(H_1H_1^\dagger\right)^2 + \left(H_2H_2^\dagger\right)^2 + \left(H_3H_3^\dagger\right)^2 + \left(H_4H_4^\dagger\right)^2\right), \\ M_{H_k}^2(\phi) &= M_H^2 + \kappa^2 \cos\left(\frac{\phi}{f_\phi} + \frac{k\pi}{2}\right). \end{aligned} \tag{9}$$

For simplicity, we have neglected cross quartics between the various Higgs bosons, but they do not change anything qualitatively and merely make the algebra more difficult.

Already we notice that one of the two problems with relaxation solutions has been solved. The UV-sensitive part of the Lagrangian is

$$\delta V = cM_{H_1}^2(\phi)\Lambda^2 + cM_{H_2}^2(\phi)\Lambda^2 + cM_{H_3}^2(\phi)\Lambda^2 + cM_{H_4}^2(\phi)\Lambda^2 = \text{const.} \tag{10}$$

Importantly, this piece of the Lagrangian is independent of ϕ. As a result, this part of the Lagrangian does not play a role in determining the minimum of ϕ.

We are interested in minimizing the potential with respect to all five fields. To simplify things, we first integrate out the four Higgs bosons at tree level. After integrating out the Higgs bosons, we obtain the potential

$$V(\phi) = -\sum_i \frac{M_{H_i}^4(\phi)}{\lambda}\Theta(-M_{H_i}^2(\phi)). \tag{11}$$

This tree-level potential has several surprising features and is represented by the solid red lines in **Figure 1*a*,*b***. If all four Higgs masses squared are negative, $M_{H_i}^2 < 0$, then the potential is ϕ independent. It is only when one of the Higgs masses squared becomes positive that the potential becomes ϕ dependent. Unlike the first term of Equation 6, a Higgs mass of zero is now a minimum rather than a maximum.

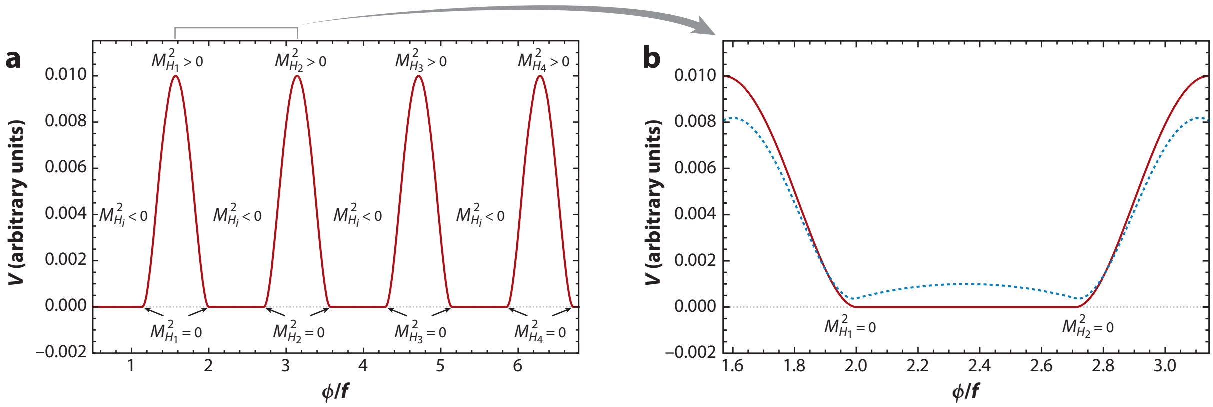

Figure 1

The potential for the field controlling the Higgs mass. (*a*) The solid red line is the tree-level potential. The potential is flat as long as all of the Higgs bosons have negative masses squared, and it goes up whenever one of the masses squared becomes positive. The edges of the flat region indicate when one of the four Higgs masses passes through zero. (*b*) A magnified view of the potential. The solid red line is the tree-level potential, and the dashed blue line includes additional corrections, such as the one-loop potential of the Standard Model. The degenerate minimum is broken, and the minimum is near the point where one of the Higgs masses is small.

At tree level, there is an infinite number of minima with a Higgs mass of zero still being special. As shown in **Figure 1**, a Higgs mass of zero is at the edge of the degenerate minimum. All that is needed is another contribution to the potential of ϕ that very slightly breaks the degeneracy and creates a minimum right near the edge where the potential ceases to be flat. As shown in **Figure 1**, the smaller this additional contribution is, the smaller the Higgs mass will be. If we can arbitrarily suppress these additional contributions, the Higgs mass will be arbitrarily small compared with the UV cutoff.

There are two additional contributions to the potential of ϕ. The first is a higher-dimensional term of the form

$$\delta V = \frac{\kappa^8}{\Lambda^4} \cos(4\phi). \quad 12.$$

This term is highly suppressed as $\kappa < \Lambda$. As we take $N > 4$, this term becomes exponentially suppressed and quickly becomes completely irrelevant.

The other way that the degeneracy breaks is that our assumption that the Higgs potential is given by Equation 5 is correct only at tree level. While not clear from the analysis so far, any polynomial corrections to Equation 5 will not remove the flatness of $V(\phi)$ when all of the Higgs masses are negative.[3] Polynomial corrections, such as the quadratic or logarithmic divergence induced by a loop of top quarks, pose no challenge to the mechanism.

The leading nonpolynomial correction to the Higgs potential comes from the one-loop Coleman–Weinberg potential resulting from a loop of top quarks, which also removes the degeneracy in a manner shown in **Figure 1**. Because this model does not exponentially suppress the nonanalytic pieces of the one-loop Coleman–Weinberg potential of the Higgs boson, this model can at best make the Higgs mass M_H about 10 times lighter than it otherwise would be. While this is a significant gain experimentally, it is not the parametric realization of the approach one would hope for.

As there is only a single relaxation solution to the electroweak hierarchy problem, much still needs to be done to explore this approach. The model described in this section is only partially satisfactory. Although it explains why the Higgs mass is small, the light Higgs mass is small and positive rather than the observed small and negative. While there are more complicated approaches that solve this particular problem, it remains an open question whether there is a simple and elegant solution. Aside from this example, it would be intriguing if there were other solutions of this type.

2.2. Experimental Implications of Relaxation Models

The experimental implications of relaxation models are tied to the new particle that every solution of this type requires. The scalar ϕ necessarily changes the Higgs mass and thus necessarily mixes with the Higgs boson. A scalar that mixes with the Higgs boson is present in many different theories, and thus its experimental consequences are well known.

At extremely low masses, the new scalar is macroscopic in distance and would be probed by fifth-force experiments. Mixing with the Higgs boson results in a force that violates the equivalence principle. The MICROSCOPE experiment is the current best experiment probing *EP*-violating fifth forces at long distance (79). At low masses, fifth-force experiments fall behind stellar constraints. Excess cooling of red giants and supernovae places tight constraints until the

[3] More precisely, corrections to the flatness of $V(\phi)$ are exponentially suppressed in the large N limit.

mass of ϕ is below $\sim$10 MeV (80). Finally, at masses above $\sim$10 MeV, collider constraints dominate. These can take the form of rare meson decays or exotic decays of the Higgs boson (81, 82).

3. HISTORICAL SOLUTIONS

Another class of new solutions being explored is historical solutions. Historical solutions are those in which the history of how we got to where we are is important. An atypical situation can easily be explained if one knows the history. Two main classes of historical solutions have been explored so far.

The first historical approach uses a long period of metastability of the vacuum with a small Higgs mass. Plausible initial conditions lead to a long period of time where the Universe is in the vacuum with a very small Higgs mass. These approaches are typically called relaxion models.

The second historical approach uses reheating. After inflation, the Universe was left cold and empty. To resemble the observed Universe today, it needs to have undergone a period called reheating in which it was heated up to a hot thermal bath. If this process was somehow weighted toward producing light particles, then it would not be surprising for the Higgs boson we see to be lighter than otherwise expected. The first in this class of models was one called NNaturalness.

3.1. Relaxion

The relaxion historical approach to the electroweak hierarchy problem proceeds by using metastable vacua (65). Plausible initial conditions combined with frictional effects lead the Universe to spend a long amount of time in the minimum with a small Higgs mass. As long as we happen to live in this period of time, then the small Higgs mass is explained.

The relaxion Lagrangian includes a scalar ϕ that controls the Higgs mass. This scalar ϕ has a shift symmetry that is broken by the spurion κ. As such, any nonderivative interaction involving ϕ must be proportional to κ. A second shift-symmetry-breaking spurion is the QCD θ angle, which implies that ϕ couples to the $G\tilde{G}$ operator. The Lagrangian of interest is

$$\mathcal{L} \supset (M_H^2 + \kappa\phi)HH^\dagger + \frac{\lambda}{4}(HH^\dagger)^2 + \kappa\phi\Lambda^2 + \left(\frac{\phi}{f} - \theta\right) G\tilde{G}. \qquad 13.$$

Here, $M_H^2 \sim \Lambda^2$ is the large natural value of the Higgs mass as opposed to m_h, its small observed value. As promised, as the vacuum expectation value (VEV) of ϕ changes, the Higgs mass changes. The last term is of particular importance. When QCD confines, this generates a potential for ϕ that scales parametrically as

$$\delta V(\phi) \sim y_u |H| \Lambda_{\mathrm{QCD}}^3 \cos\left(\frac{\phi}{f} - \theta\right). \qquad 14.$$

This potential is highly suppressed when the VEV of the Higgs boson is zero, $|H| = 0$, and increases as the VEV of the Higgs boson increases past the confinement scale of QCD, Λ_{QCD}. As is well known, the axion potential coming from QCD is suppressed by the small Yukawa coupling of the up quark, y_u.

The potential of ϕ is shown in **Figure 2**. All of the dynamics of ϕ occur during inflation. Initially, $M_H^2(\phi)$ is positive and ϕ rolls down the potential. Because of Hubble friction, ϕ is slow rolling down the potential with a speed $\dot{\phi} \sim \kappa\Lambda^2/H$. Eventually, ϕ stops in the first minimum that it encounters. This first minimum occurs when the linear term $\kappa\phi\Lambda^2$ balances against the growing

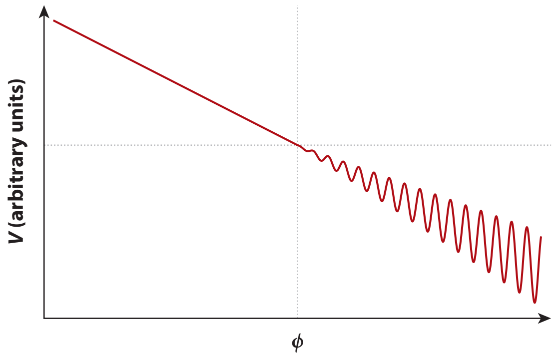

Figure 2

The potential for the relaxion particle. Parameters are chosen such that the first potential on the left has the observed value of the Higgs boson mass.

oscillations given in Equation 14:

$$\kappa\Lambda^2 \sim y_u \langle H \rangle \frac{\Lambda^3_{\text{QCD}}}{f}. \qquad 15.$$

As long as κ, f, and Λ are chosen appropriately, then the minimum at which ϕ stops has the Higgs mass at the observed small value. Once ϕ has stopped at the correct minimum, it can escape only by tunneling. This process takes an exponentially large amount of time, and so it is plausible that inflation ended and reheating took place before tunneling had occurred, which would explain why the Higgs mass is so small.

Since the original model, there has been a vast plethora of refinements on the idea of the relaxion. The original model described above predicts a large θ angle that is excluded by data. Models have been proposed to solve some of the issues with the previous model. Some use a new friction mechanism, some remove inflation, some introduce multiple relaxion scalars, and others use supersymmetry. Many other options have been explored (83–92).

The experimental implications of the relaxion model are very similar to those discussed in Section 2.2. The reason is not particularly surprising. In both cases, there is a scalar ϕ that controls the mass of the Higgs boson. As a result, in both cases, it mixes with the Higgs boson. All of the experimental implications come from this mixing effect. As such, it is clear that the experimental implications of the historical and the relaxation approaches have significant overlap. In the context of the relaxion, much of the work has focused on cases in which the relaxion is dark matter or a subset of it (93–95).

3.2. NNaturalness

The idea behind NNaturalness is that decays can result in the Universe being populated by very light particles (64). There is a simple example of this mechanism at work. Consider N scalars with random interactions and random masses. If the Universe were populated by some number abundance of these scalars, then as time moved on, these scalars would start to decay. They would continue decaying until all that remained was the lightest scalar. This scalar could be lighter than all of the rest by accident. For example, if the scalar masses were chosen randomly, then it would be $\sim 1/\sqrt{N}$ lighter than its brothers and sisters. The Universe being full of this abnormally light scalar would not be a surprise as all of the heavier scalars have decayed.

NNaturalness takes this idea of decaying scalars and applies it to the Higgs boson. The one fact that makes applying these ideas to the Higgs boson more complicated is that the Standard Model has particles whose masses are not set by the Higgs boson, such as the massless photon. If the existence of massless particles were common, then many sectors would contain massless particles. As a result, the generalization of the above argument to the observed Universe would result in a very large $N_{\text{eff}} \sim N$ and would be excluded. N_{eff} is the effective number of fermions in the thermal bath at late time ($N_{\text{eff}} \sim 3$ is the Standard Model prediction). Extra light species affect the Hubble expansion and are heavily constrained.

To obtain a model that is not already excluded, NNaturalness uses a new particle called the reheaton, Φ, whose decays reheat the Universe from an empty bath of nothing to the bath of Standard Model particles we see today. The reheaton will kinematically prefer to decay into the sector with the lightest Higgs boson and thereby result in a universe that is predominantly populated by a sector whose Higgs boson is abnormally light.

The matter content of NNaturalness is as follows. Consider N copies of the Standard Model. In principle, these do not need to be copies but simply N objects of which the Standard Model is typical, but copies were considered to render the theory calculable. These N copies will couple to some other sector so that the Higgs mass is different in each sector. The final ingredient to the model is the reheaton Φ.

The only new interaction needed in NNaturalness is the coupling of the reheaton Φ to the N different sectors. We accomplish this by including the most relevant coupling of a scalar Φ to the N sectors:

$$\mathcal{L} \supset \kappa \sum_i \Phi H_i H_i^\dagger. \qquad 16.$$

With both the matter content and interactions of the theory, we can now flesh out the thermal history of the Universe. Initially, the Universe is populated by the reheaton Φ. After a time on the order of the lifetime of Φ, Φ will have decayed and populated the various N sectors. As long as the sector with most of the energy has abnormally light Higgs bosons, the electroweak hierarchy problem will have been resolved. As such, we may now study the branching ratios of Φ into the various sectors.

To highlight the mechanism, consider the case where $m_\Phi < m_b$, so that Φ is lighter than the Higgs boson. The decay of Φ proceeds via off-shell Higgs bosons into photons, A, or fermions, ψ:

$$\Gamma_\Phi(\Phi \to 4\psi) \sim \frac{\kappa^2 m_\Phi^7}{m_H^8} \qquad m_H^2 > 0 \qquad 17.$$

$$\Gamma_\Phi(\Phi \to 2A) \sim \frac{\kappa^2 m_\Phi^3}{(16\pi^2)^2 m_H^4} \qquad m_H^2 > 0 \qquad 18.$$

$$\Gamma_\Phi(\Phi \to 2\psi) \sim \frac{\kappa^2 m_\Phi}{m_H^2} \qquad m_H^2 < 0 \qquad 19.$$

As can be seen, Φ preferentially decays into the sector with the lightest Higgs boson and into sectors with a negative Higgs mass squared. The Φ particle simply prefers to decay via the least off-shell Higgs boson possible. As such, the largest decay width is into the sector with the lightest Higgs boson. Thus, this model predicts a universe whose energy density is dominated by the sector with the lightest Higgs boson.

The experimental implications for NNaturalness are that there should be some energy density in the other sectors. If this energy density is in heavy particles, they would manifest themselves as dark-matter-like particles. If this energy density is in light particles, they would manifest themselves as $N_{\rm eff}$. As discussed above, the simplest idea of decaying to a light Higgs boson is excluded by the $N_{\rm eff}$ measurement of Planck. Even in the successful implementation of the model, $N_{\rm eff}$ is nonzero. The model itself predicts that

$$1 \gtrsim N_{\rm eff} \gtrsim 10^{-1}. \qquad 20.$$

What is most striking about this prediction is that it is only logarithmically dependent on the number of copies N for $N < 10^4$ with smaller N giving logarithmically smaller $N_{\rm eff}$. For $N > 10^4$, the predictions for $N_{\rm eff}$ are essentially N independent. As a result, $N_{\rm eff}$ measurements constitute an extremely robust test of NNaturalness models. As such, this particular implementation of NNaturalness will be testable and excludable in upcoming cosmic microwave background experiments (96–98). As all historical solutions of this type rely on preferential production of light particles, $N_{\rm eff}$ constraints are likely to be the strongest test for all models of this type.

4. CONDITIONAL PROBABILITY–BASED SOLUTIONS

The final set of solutions discussed here is conditional probability–based solutions (66, 67, 99–101). While it may be puzzling to observe a small Higgs mass, in certain models it is not surprising to find one after imposing other criteria such as the smallness of the cosmological constant problem. The most famous of the conditional probability–based approaches is anthropics. The idea behind anthropics is that while the probability of some observed fact may be very small, $P(\text{obs}) \ll 1$, the probability given the existence of intelligent life to ask the question may be quite large, $P(\text{life}|\text{obs}) \approx 1$. This is a famous (and old) approach. As the focus of this review is on newer solutions to the problem, anthropics will not be discussed further.

More recently, people have built models in which the smallness of the cosmological constant is connected to the smallness of the Higgs mass. In these models, as long as one requires that we are in a vacuum with a small cosmological constant, the Higgs mass is small as well. In this manner, the problem of a small Higgs mass has been tied to the smallness of the cosmological constant. Whichever solution resolves the cosmological constant problem[4] would also solve the issue of the small Higgs mass.

4.1. More Vacua When $m_h \sim 100$ GeV

In this section, I discuss the simplest of the conditional probability–based solutions proposed in Reference 67. This approach uses a model in which there are more vacua with different cosmological constants when the Higgs VEV is around 100 GeV. Because there are more vacua when $m_h \sim 100$ GeV, the vacua where the cosmological constant is small are much more likely to have $m_h \sim 100$ GeV.

A simple toy model that exhibits this property is as follows. Consider a scalar ϕ with a $\mathbb{Z}_2$ symmetry acting on it coupled to a two-Higgs doublet model with doublets H_u and H_d. Under this symmetry, $\phi \to -\phi$ and $H_u H_d \to -H_u H_d$. The most general Lagrangian coupling these two

[4] Anthropics is the most commonly proposed explanation for why the cosmological constant is small. However, in this approach one does not need to commit to any particular solution to the cosmological constant problem, though certain solutions, such as relaxation solutions, cannot be used.

sectors together has

$$\mathcal{L} \supset \frac{1}{2} m_\phi^2 \phi^2 - \lambda \phi^4 + \kappa \phi H_u H_d. \qquad 21.$$

If $\langle H_u H_d \rangle = 0$ then there are two degenerate vacua. These two vacua have the same value for the cosmological constant. As a result, if the theory is at a minimum, there is only a single value of the cosmological constant. If $\langle H_u H_d \rangle \gtrsim m^3/(\sqrt{\lambda}\kappa)$, then there exists only a single minimum, and again there is only a single value of the cosmological constant. Only if $m^3/(\sqrt{\lambda}\kappa) \gtrsim \langle H_u H_d \rangle \gtrsim 0$ are there multiple vacua with different cosmological constants.

We can now use the building block of the above paragraph to construct a full model. If we take N scalars ϕ, all with similar masses and quartic couplings and all coupled to $H_u H_d$ as indicated in Equation 21, then we arrive at the following conclusion. If $\langle H_u H_d \rangle = 0$ or $\langle H_u H_d \rangle \gtrsim m^3/(\sqrt{\lambda}\kappa)$, then all minima have the same cosmological constant. The value of this constant would be a random natural value, 10^{120} times larger than the observed value. If $m^3/(\sqrt{\lambda}\kappa) \gtrsim \langle H_u H_d \rangle \gtrsim 0$, then there are 2^N vacua, all with different cosmological constants. As long as $2^N > 10^{120}$, one of these vacua will accidentally have the correct observed value and a small Higgs mass at the same time.

In the context of this landscape of vacua, once one conditions on having the correct value of the cosmological constant, the Higgs mass is expected to be small.[5] The reason is simply that most of the vacua that have different cosmological constants all have a small Higgs mass as indicated above. In this manner, the small Higgs mass is no longer a surprise once one accepts a small cosmological constant.

4.2. Experimental Implications of Conditional Probability Models

The experimental implications of conditional probability models are quite varied, and there is no single unique feature. In the model described in the above subsection, there are many light scalars, all of which are coupled to the Higgs boson, giving phenomenology similar to that described by relaxation and historical solutions. Additionally, it requires a two-Higgs doublet model that may be discovered at a collider. Other models, such as the one described in Reference 66, have other fine-tuned light scalars that may be dark matter and may be observed as oscillations of fundamental constants (93).

5. CONCLUSION

In this review, I have covered some of the newer approaches to the electroweak hierarchy problem. As discussed in Section 1, the electroweak hierarchy problem refers to the apparent failure of dimensional analysis as applied to the Higgs boson. Roughly half of the expected parameter space where something new should occur has already been explored, but the other half has yet to be explored and requires further experimental input, likely of the more traditional collider variety. If future collider experiments at higher energy continued to exclude the new particles required in TeV-scale models, this would firmly establish the failure of dimensional analysis as applied to the Higgs boson.

As dimensional analysis is one of the more experimentally tested and important aspects of physics, its failure would be a complete disaster. The most common reason behind the failure of dimensional analysis is an incomplete understanding of the problem at hand. As such, physicists

[5]More precisely, $\langle H_u H_d \rangle$ will be small and nonzero. A further model is needed to ensure that both VEVs are individually small and nonzero.

have been exploring alternative approaches to the problem to test what might have been overlooked. The new approaches covered in this review have been characterized by taking previous solutions to hierarchy problems and finding versions of these mechanisms that apply to the Higgs boson. These new approaches, while conceptually simple, have had a resounding impact on the experimental side.

These new approaches almost all have been tested in a manner very different from the older and more famous solutions to the problem. In many cases, there is a new light scalar coupled to the Higgs boson. This scalar could be tested through a wide range of precision measurements, from rare decays of the Higgs boson and mesons to small-scale experiments measuring fifth forces. Alternatively, some of these theories can be tested by cosmological measurements such as $N_{\rm eff}$.

It is therefore clear that the electroweak hierarchy problem remains an area of active research. New experiments are needed to firmly establish the presence of a problem as well as to test the variety of alternative explanations. New theories are constantly emerging, and it is entirely possible that the most plausible and compelling theory of the Higgs mass is yet to come.

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