ANNUAL REVIEWS

Annual Review of Nuclear and Particle Science

Testing Lepton Flavor Universality with Pion, Kaon, Tau, and Beta Decays

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Annu. Rev. Nucl. Part. Sci. 2022. 72:69–91

First published as a Review in Advance on June 15, 2022

The *Annual Review of Nuclear and Particle Science* is online at nucl.annualreviews.org

https://doi.org/10.1146/annurev-nucl-110121-051223

Keywords

lepton flavor universality violation, π decays, K decays, τ decays, beta decays, CKM unitarity, Standard Model, new physics, Cabibbo angle anomaly, flavor anomalies

Abstract

We present an overview of searches for violation of lepton flavor universality with a focus on low energy precision probes using π, K, τ, and nuclear beta decays. We review the current experimental results, summarize the theoretical status within the context of the Standard Model, and discuss future prospects (both experimental and theoretical). We review the implications of these measurements for physics beyond the Standard Model by performing a global model-independent fit to modified W couplings to leptons and four-fermion operators. We also discuss new physics in the context of simplified models and review Standard Model extensions with a focus on those that can explain a possible deviation from unitarity of the Cabibbo–Kobayashi–Maskawa quark mixing matrix.

Contents

1. INTRODUCTION

The Standard Model (SM) of particle physics describes the known constituents of matter: the three generations (or flavors) of quarks and leptons as well as their interactions (excluding gravity). Its final missing ingredient, the Higgs boson, was discovered at the Large Hadron Collider (LHC) at CERN in 2012 (1, 2). However, it is clear that the SM cannot be the ultimate fundamental theory of nature. In addition to many theoretical arguments for the existence of new physics (NP), the SM, for instance, can account neither for the existence of dark matter (DM) or dark energy established at cosmological scales nor for neutrino masses or the existence of exactly three generations of fermions.

Therefore, the search for physics beyond the SM (BSM) is a prime subject of current research. There are, in general, two ways to search for new particles and interactions: direct searches at high energy colliders (such as the LHC) and indirect searches for quantum effects in precision observables (3). Concerning the latter, an especially promising avenue is to search for the violation of (approximate) symmetries of the SM. In this way, such searches are very sensitive to NP that does not necessarily respect these symmetries and thus leads to sizable effects even if the mass scale is quite high. Furthermore, the symmetries of the SM can be exploited to obtain more precise predictions, as in most cases theoretical and parametric uncertainties are reduced.

In the SM, the gauge interactions are the same for all flavors; in other words, they respect lepton flavor universality (LFU), which in fact is broken only by the Higgs Yukawa couplings. As these couplings are very small (at most, of the order of 1% for the τ lepton), LFU is an approximate accidental symmetry of the SM (at the Lagrangian level). However, the impact of the lepton masses, originating from the Higgs Yukawa couplings after electroweak (EW) symmetry breaking on the lifetimes of charged leptons, is enormous as a result of kinematic effects. Therefore, LFU violation (LFUV) implies interactions with different couplings to electrons, muons, and τ leptons (disregarding phase-space effects) that directly distinguish among the charged leptons at the Lagrangian level.

Recent experimental tests of LFU have accumulated intriguing hints of physics effects not included in the SM (for a short review, see 4).[1] In particular, measurements of the ratios of branching

[1]Note that interesting hints of new scalar particles have recently emerged at the LHC (5–7).

ratios (BRs) $R[D^{(*)}] = \mathrm{Br}[B \to D^{(*)}\tau\nu_\tau]/\mathrm{Br}[B \to D^{(*)}\ell\nu_\ell]$, where $\ell = \mu$ or e (8–10), and $R[K^{(*)}] = \mathrm{Br}[B \to K^{(*)}\mu^+\mu^-]/\mathrm{Br}[B \to K^{(*)}e^+e^-]$ (11–13), deviate from the SM expectation by more than 3σ (14–18) and 4σ (19–22), respectively.[2] In addition, anomalous magnetic moments $(g-2)_\ell$ ($\ell = e, \mu, \tau$) of charged leptons are intrinsically related to LFUV because they are chirality-flipping quantities. Here, there is a long-standing discrepancy in $(g-2)_\mu$ of 4.2σ (24–26), which can be considered a hint of LFUV because, when compared with $(g-2)_e$, the bound from the latter on flavor-blind NP is much more stringent. In addition, there is a hint of LFUV in the difference in the forward–backward asymmetries (ΔA_{FB}) in $B \to D^*\mu\nu$ versus $B \to D^*e\nu$ (27, 28). In another possible indication of LFUV, the CMS Collaboration (29) observed an excess in nonresonant dielectron pairs with respect to dimuons. Furthermore, the possible deficit in first-row unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, known as the Cabibbo angle anomaly (CAA), can also be viewed as a sign of LFUV (30, 31).

The connection between the CAA and LFUV can be seen as follows. The determination of V_{ud} from beta decays, which is most relevant for a possible explanation of the CAA, is affected by a modified $W\mu\nu$ coupling (31). Importantly, a modification of the $W\mu\nu$ coupling, if not compensated for by an effect in $We\nu$, would also affect, for example, the ratios of decay rates $R^\pi_{e/\mu} = \Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ and $R^\tau_{e/\mu} = \Gamma(\tau \to e\nu\overline{\nu})/\Gamma(\tau \to \mu\nu\overline{\nu})$, which provide the best tests of LFU. In fact, recent global fits to EW observables and tests of LFU show a preference for a value of $R^\pi_{e/\mu}$ that is smaller than its SM expectation (30, 32). Furthermore, $R(K^*)$ can be correlated to $R^\pi_{e/\mu}$ (33), and a combined explanation of the deficit in first-row CKM unitarity and the CMS excess in dielectrons even predicts that $R^\pi_{e/\mu}$ should be smaller than its SM value (34).

These considerations provide an additional motivation for us to review, summarize, and reexamine the different searches for LFUV in the charged current, with a focus on π, K, τ, and beta decays. In the next section, we discuss the experimental and theoretical status of these processes. We then consider the impact on NP searches, first in a model-independent way by a global analysis of modified $W\ell\nu$ coupling and four-fermion operators and then by considering different NP models, with a focus on those that can explain the CAA in Section 3.[3] We then give an outlook for future experimental and theoretical prospects in Section 4 before we conclude in Section 5.

2. STANDARD MODEL THEORY AND OBSERVABLES

2.1. Light Meson Decays

The ratios of the decay rates

$$R^P_{e/\mu} = \frac{\Gamma[P \to e\bar{\nu}_e(\gamma)]}{\Gamma[P \to \mu\bar{\nu}_\mu(\gamma)]}, \qquad 1.$$

where $P = \pi$ or K, provide some of the most stringent tests of LFU of the SM gauge interactions. In the SM, the decay rates $\Gamma[P \to e\bar{\nu}_e(\gamma)]$ are helicity suppressed because of the V–A structure of

[2] Even though the ratios $R(D)$ and $R(D^*)$ point toward NP as they show deviations from μ–τ LFU, we do not include them in the observables discussed in this review. The NP effects required are so large that these ratios cannot be explained by modified $W\ell\nu$ couplings, which are more stringently constrained by τ decays. Therefore, effects in two-quark–two-lepton operators are required in order to explain $R(D)$ and $R(D^*)$, which in general have no direct correlations with the tests of LFUV discussed in this review, unless a flavor symmetry is assumed. However, some of these scenarios give rise to large radiative corrections to τ decays, such that they are excluded by low energy probes of LFUV (23).

[3] Here we focus on models with heavy NP components such that the effective Lagrangian contains only SM fields. However, light but massive right-handed neutrinos (35–38), majorons (39), and light DM candidates (40, 41) can also have an impact on the tests of LFUV studied in this review; these effects are searched for in π, K, and τ decay experiments (42–44).

the charged current. Moreover, their ratios can be calculated with extraordinary precision at the 10^{-4} level (45–48) because, to a first approximation, the strong interaction dynamics cancel out in the ratio $R^P_{e/\mu}$ and the hadronic structure dependence appears only through EW corrections. Because of these features and the precise experimental measurements, the ratios $R^P_{e/\mu}$ are very sensitive probes of all SM extensions that induce nonuniversal corrections to $W\ell\nu$ couplings as well as $\bar{e}\nu\bar{u}d$ and $\bar{e}\nu\bar{u}s$ operators, in particular, if they generate a pseudoscalar current or induced scalar current (49).

The most recent theoretical calculations of $R^P_{e/\mu}$ (47, 48) are based on chiral perturbation theory (ChPT), the low energy effective field theory (EFT) of QCD (50–52), generalized to include virtual photons and light charged leptons (53). This framework provides a controlled expansion of the decay rates in terms of a power counting scheme characterized by the dimensionless ratio $Q \sim m_{\pi,K,\mu}/\Lambda_\chi$, where $\Lambda_\chi \sim 4\pi F_\pi \sim 1.2$ GeV ($F_\pi \simeq 92.4$ MeV is the π decay constant), and the electromagnetic coupling e. In this setup, one can write

$$R^P_{e/\mu} = \bar{R}^P_{e/\mu}\left[1 + \Delta^P_{e^2Q^0} + \Delta^P_{e^2Q^2} + \Delta^P_{e^2Q^4} + \cdots + \Delta^P_{e^4Q^0} + \cdots\right], \tag{2.}$$

where

$$\bar{R}^P_{e/\mu} = \frac{m_e^2}{m_\mu^2}\left(\frac{m_P^2 - m_e^2}{m_P^2 - m_\mu^2}\right)^2. \tag{3.}$$

Here we have kept all the terms needed to reach an uncertainty of $\sim 10^{-4}$ for the ratio. The leading electromagnetic corrections $\Delta^P_{e^2Q^0}$ correspond to the pointlike approximation for πs and Ks, and their expressions are well known (54). The hadronic structure dependence first appears through the correction $\Delta^P_{e^2Q^2} \sim (\alpha/\pi)(m_P/\Lambda_\chi)^2$, which features both the calculable double-chiral logarithms and an a priori unknown low energy coupling constant, which was estimated in large-N_C QCD (where N_C is the number of colors) (47, 48) and found to contribute negligibly to the error budget.

2.1.1. Pion decays. In the π case ($P = \pi^\pm$), one usually defines the ratio to be fully photon inclusive, such that it is infrared safe. As a consequence, one has to include in $R^P_{e/\mu}$ terms arising from the structure-dependent contribution to $\pi \to \ell\bar{\nu}_\ell\gamma$ (55), which are formally of $O(e^2Q^4)$ but are not helicity suppressed and behave as $\Delta^P_{e^2Q^4} \sim (\alpha/\pi)(m_P/\Lambda_\chi)^4(m_P/m_e)^2$. Finally, at the level of uncertainty considered, one needs to include higher-order corrections in α, namely $\Delta^P_{e^4Q^0}$. The leading logarithmic correction $\Delta^P_{e^4Q^0,\mathrm{LL}} = (7/2)(\alpha/\pi \log m_\mu/m_e)^2$ was calculated in Reference 45, and the effect of subleading contributions was estimated in Reference 47 as $(\alpha/\pi)^2 \log m_\mu/m_e \sim 0.003\%$. Numerically, one finds $\Delta^\pi_{e^2Q^0} = -3.929\%$, $\Delta^\pi_{e^2Q^2} = 0.053(11)\%$, $\Delta^\pi_{e^2Q^4} = 0.073(3)\%$, and $\Delta^{(\pi)}_{e^4Q^0} = 0.055(3)\%$, which lead to the SM expectation[4]

$$R(\mathrm{SM})^\pi_{e/\mu} = (1.23524 \pm 0.00015) \times 10^{-4}. \tag{4.}$$

We reiterate that (*a*) this prediction includes structure-dependent hard bremsstrahlung corrections to $\Gamma[\pi^+ \to e^+\nu(\gamma)]$, which are not helicity suppressed, and (*b*) the dominant uncertainty

[4]Due to a larger uncertainty estimate in $\Delta^\pi_{e^4Q^0}$, namely $\Delta^\pi_{e^4Q^0} = 0.055(10)\%$, Reference 56 quotes a final result of $R(\mathrm{SM})^\pi_{e/\mu} = (1.2352 \pm 0.0002) \times 10^{-4}$.

of the SM prediction arises from a low energy constant in ChPT, followed by the nonleading logarithmic corrections of $O(\alpha^2)$.

The most accurate measurement of $R^{\pi}_{e/\mu}$, reported by the TRIUMF PIENU Collaboration (57), is

$$R(\exp)^{\pi}_{e/\mu} = [1.2344 \pm 0.0023(\text{stat.}) \pm 0.0019(\text{syst.})] \times 10^{-4} \qquad 5.$$

at the 0.24% precision level. The Particle Data Group (PDG) (3) average, including previous experiments done at TRIUMF (57–59) and the Paul Scherrer Institute (60), is

$$R(\overline{\exp})^{\pi}_{e/\mu} = (1.2327 \pm 0.0023) \times 10^{-4}. \qquad 6.$$

The comparison between theory and experiment given in Equations 4 and 6 provides a stringent test of the e–μ universality of the weak interaction. We choose to express the results in terms of the effective couplings A_ℓ, which enter by multiplying the low energy charged-current contact interaction

$$L_{\mathrm{CC}} = A_\ell \bar{u}\gamma^\mu P_L d \bar{\nu}_\ell \gamma_\mu P_L \ell, \qquad 7.$$

where $P_L \equiv (1-\gamma_5)/2$. In the SM, at tree level the couplings are given by $A_\ell = -2\sqrt{2}G_{\mathrm{F}}V_{ud}$ and thus satisfy LFU; in other words, $A_\ell/A_{\ell'} = 1$. The measurement of $R^{\pi}_{e/\mu}$ results in

$$\left(\frac{A_\mu}{A_e}\right)_{R^{\pi}_{e/\mu}} = 1.0010 \pm 0.0009, \qquad 8.$$

which is in excellent agreement with the SM expectation. A deviation from $A_\ell/A_{\ell'} = 1$ can originate from various mechanisms. In the literature it is common to interpret deviations from $A_\ell/A_{\ell'} = 1$ in terms of flavor-dependent couplings g_ℓ of the W boson to the leptonic current, in which case $A_\ell \propto g_\ell$. We discuss this scenario in detail in Section 3.1.1. We note that, in the context of modified W couplings, LFU tested with $R^{\pi}_{e/\mu}$ probes the couplings of a longitudinally polarized W boson, whereas tests using purely leptonic reactions like $\tau \to \ell\nu_\tau\nu_\ell$ ($\ell = e, \mu$) probe the couplings of a transversely polarized W boson and are thus complementary.

2.1.2. Kaon decays. LFU can also be tested using the ratios

$$R^{K}_{e/\mu} = \frac{\Gamma\left[K^+ \to e^+\nu(\gamma)\right]}{\Gamma\left[K^+ \to \mu^+\nu(\gamma)\right]}, \text{ and} \qquad 9.$$

$$R^{K\to\pi}_{e/\mu} = \frac{\Gamma\left[K \to \pi e\nu(\gamma)\right]}{\Gamma\left[K \to \pi\mu\nu(\gamma)\right]}. \qquad 10.$$

Here, for $R^{K\to\pi}_{e/\mu}$ both neutral and charged K decays (e.g., $K_L \to \pi^\pm\ell^\mp\nu$ and $K^\pm \to \pi^0\ell^\pm\nu$) are used.

The calculation of $R^{K}_{e/\mu}$ is similar to that of $R^{\pi}_{e/\mu}$ described in the previous section. An important difference concerns the definition of the infrared-safe decay rate, which requires including part of the radiative decay mode. The radiative amplitude is the sum of the inner bremsstrahlung (T_{IB}) component of $O(eQ)$ and a structure-dependent (T_{SD}) component of $O(eQ^3)$ (55). While the experimental definition of $R^{(\pi)}_{e/\mu}$ is fully inclusive, the one for $R^{K}_{e/\mu}$ includes the effect of T_{IB} in $\Delta^{(K)}_{e^2Q^0}$ (dominated by soft photons) and excludes the effect of T_{SD}. With this definition, one finds $\Delta^{K}_{e^2Q^0} = -3.786\%$, $\Delta^{K}_{e^2Q^2} = 0.135(11)\%$, $\Delta^{K}_{e^2Q^4} = 0$, and $\Delta^{K}_{e^4Q^0} = 0.055(3)\%$, and the SM expectation is (47, 48)

$$R(\mathrm{SM})^{K}_{e/\mu} = (2.477 \pm 0.001) \times 10^{-5}, \qquad 11.$$

where the final uncertainty accounts for higher-order chiral corrections of expected size $\Delta_{e^2Q^2} \times m_K^2/(4\pi F_\pi)^2$.

The PDG (3) average of previous measurements done by the NA62 (61) and KLOE (62) experiments is

$$R(\overline{\text{exp}})^K_{e/\mu} = (2.488 \pm 0.009) \times 10^{-5}. \qquad 12.$$

The comparison of theory and experiment given in Equations 11 and 12 corresponds to a test of e–μ universality:

$$\left(\frac{A_\mu}{A_e}\right)_{R^K_{e/\mu}} = 0.9978 \pm 0.0018. \qquad 13.$$

The analogous LFU test based on the ratios $R^{K\to\pi}_{e/\mu}$ has been discussed by the FLAVIAnet Collaboration (63). For a given neutral or charged initial-state K, the Fermi constant, V_{us}, short-distance radiative corrections, and the hadronic form factor at zero momentum transfer cancel out when taking the ratio $R^{K\to\pi}_{e/\mu}$. Therefore, in the SM this ratio is determined entirely by phase-space factors and long-distance radiative corrections (64–67). The ratios for K_L and $K^\pm$ are consistent, leading to the following values for A_μ/A_e (63, 68, 69):

$$\begin{aligned}\left(\frac{A_\mu}{A_e}\right)_{R^{K_L\to\pi}_{e/\mu}} &= 1.0022 \pm 0.0024, \quad \text{and} \\ \left(\frac{A_\mu}{A_e}\right)_{R^{K^\pm\to\pi^\pm}_{e/\mu}} &= 0.9995 \pm 0.0026,\end{aligned} \qquad 14.$$

and the following average for $K_{\ell 3}$ decays:

$$\left(\frac{A_\mu}{A_e}\right)_{R^{K\to\pi}_{e/\mu}} = 1.0009 \pm 0.0018. \qquad 15.$$

The numbers given above correspond to the recent analysis in Reference 69, which uses experimental input from Reference 63 (updated in Reference 68 with reduced errors in the charged modes) and theoretical input on $K_{\ell 3}$ radiative corrections from References 66 and 67, which incorporates a new analysis of K_{e3} modes with reduced uncertainties (67).

Note that μ–e universality can also be determined from B decays such as $\text{Br}(B \to D^*\mu\nu)/\text{Br}(B \to D^*e\nu)$. Even though the relative precision at the percent level (70–72) is not competitive with that obtained from K and π decays, these measures of LFUV are interesting in light of the anomalies in $R[D^{(*)}]$ and ΔA_{FB} (27, 28, 71) because they test different four-fermion operators.

2.2. Beta Decays and CKM Unitarity

The observables testing LFUV discussed so far involve ratios of purely leptonic or semileptonic meson decays with an electron or muon in the final state. While consideration of the ratios of (semi)leptonic decay rates offers theoretical advantages [e.g., the elements V_{ud} and V_{us} of the CKM (73, 74) matrix, part of the radiative corrections, and hadronic matrix elements cancel], the high-precision study of absolute semileptonic decay rates can also uncover LFUV effects. For example, in the context of corrections to the $W \to \ell\nu_\ell$ vertex, the semileptonic transition $d(s) \to ue\bar{\nu}_e$ [$d(s) \to u\mu\bar{\nu}_\mu$] is sensitive to corrections to the muon (electron) coupling (31, 75, 76; see 77 for a discussion within supersymmetric models). In absolute decay rates, these BSM LFUV corrections contaminate the extraction of the CKM elements V_{ud} and V_{us} from measured decay rates. This means that beta decays and the study of CKM unitarity are intertwined with the study of LFUV

(31). In light of this connection, we briefly summarize the status of first-row CKM unitarity tests. We discuss the implications for LFUV BSM interactions in Section 3.

Unitarity of the CKM matrix (73, 74) implies $\Delta_{\mathrm{CKM}} \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$, where V_{ud}, V_{us}, and V_{ub} represent the mixing of u with d, s, and b quarks, respectively. In practice, $|V_{ub}|^2 < 10^{-5}$ can be neglected, and CKM unitarity reduces to the original Cabibbo universality, with the identifications $V_{ud} = \cos\theta_{\mathrm{C}}$ and $V_{us} = \sin\theta_{\mathrm{C}}$, where θ_{C} is the Cabibbo angle (73). The determination of V_{uD} ($D = d, s$) from various hadronic weak decays $h_i \to h_f \ell \nu_\ell$ ($\ell = e, \mu$) relies on the following schematic formula for the decay rate Γ:

$$\Gamma = G_{\mathrm{F}}^2 \times |V_{uD}|^2 \times |M_{\mathrm{had}}|^2 \times (1 + \delta_{\mathrm{IsoB}} + \delta_{\mathrm{RC}}) \times F_{\mathrm{kin}}, \qquad 16.$$

where G_{F} is the Fermi constant extracted from muon decay and F_{kin} is a phase-space factor. Theoretical input comes in the form of (*a*) the hadronic matrix elements of the weak current, M_{had}, usually calculated in the isospin limit of QCD (in which u and d quark masses are equal and electromagnetic interactions are turned off), and (*b*) small percent-level corrections, $\delta_{\mathrm{IsoB,\,RC}}$, due to strong isospin breaking (here, IsoB) and electromagnetic radiative corrections (RC) induced by the exchange of virtual photons and the emission of real photons, characterized by the small expansion parameters $\epsilon_{\mathrm{IsoB}} \sim (m_u - m_d)/\Lambda_{\mathrm{QCD}}$ and $\epsilon_{\mathrm{EM}} \sim \alpha/\pi$, respectively ($\alpha \sim 1/137$ is the electromagnetic fine-structure constant).

Currently, as shown in **Figure 1**, our knowledge of V_{ud} is dominated by $0^+ \to 0^+$ nuclear beta decays. The most recent survey (78) of experimental and theoretical input leads to $V_{ud} = 0.97373(31)$. This value incorporates a reduction in the uncertainty on the so-called inner radiative

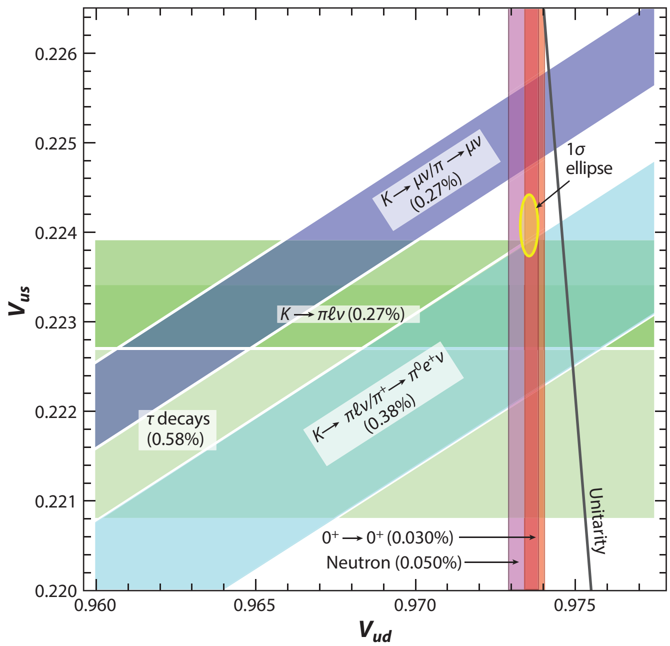

Figure 1

Summary of constraints on V_{ud} and V_{us} (assuming the Standard Model hypothesis) from nuclear, nucleon, meson, and τ lepton decays. For each constraint, the 1σ uncertainty on V_{us} or V_{ud} is given in parentheses. The 1σ ellipse from a global fit (with $\chi^2/\mathrm{d.o.f.} = 2.8$) (*yellow*) corresponds to $V_{ud} = 0.97357(27)$ and $V_{us} = 0.22406(34)$, implying $\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 - 1 = (-19.5 \pm 5.3) \times 10^{-4}$. Abbreviation: d.o.f., degree of freedom.

corrections (79, 80) and an increase in uncertainty due to nuclear structure–dependent effects with input from References 81–83.[5]

Thanks to higher-precision measurements of the lifetime (84) and beta asymmetry (85) (for a recent review, see 86), neutron decay is becoming competitive with superallowed beta decays concerning the precision with which V_{ud} can be extracted. Use of the PDG average for the neutron lifetime (including a scale factor $S = 1.6$ to account for tensions among experimental measurements)[6] and the post-2002 PDG average[7] determinations of the axial coupling $g_A = 1.2762(5)$ (3) leads to $V_{ud} = 0.97338(33)_\tau(32)_{g_A}(10)_{\mathrm{RC}} = 0.97338(47)$, with errors originating from the lifetime τ_n, g_A, and radiative corrections (3), respectively. Ongoing and planned neutron experiments aim to reduce the uncertainty in τ_n and g_A by a factor of a few (see 89 and references therein), which will put the extraction of V_{ud} from neutron decay at the same precision level as superallowed nuclear beta decays. Future prospects for improving the extraction of V_{ud} from π–beta decay are discussed in Section 4.

The most precise value of V_{us} is extracted from $\Gamma(K \to \pi\ell\nu)$, while $R_A \equiv \Gamma(K \to \mu\nu)/\Gamma(\pi \to \mu\nu)$ currently provides the most precise determination of V_{us}/V_{ud} (90). A comprehensive discussion of the experimental and theoretical input up to 2010 can be found in Reference 91 (and references therein). Since then, experimental input on the $K^\pm$ BRs and form-factor parameters has been updated, as reviewed in Reference 68, while the most recent theoretical input on the hadronic matrix elements can be found in Reference 92 (and references therein). Radiative corrections are included according to References 66 and 67. With this input, one obtains $V_{us} = 0.2231(6)$ from $K_{\ell 3}$ decays and $V_{us}/V_{ud} = 0.2313(5)$ from R_A (69, 91). A recent study (93) pointed out that V_{us}/V_{ud} can also be obtained through the ratio of vector channel decays $R_V \equiv \Gamma[K \to \pi\ell\nu(\gamma)]/\Gamma[\pi^+ \to \pi^0 e^+\nu(\gamma)]$, leading to $V_{us}/V_{ud} = 0.22908(87)$, with uncertainty dominated by the π–beta decay width but nonetheless within a factor of two compared with the R_A determination. Future prospects on this front are discussed in Section 4. Finally, V_{us} can also be extracted from inclusive and exclusive semileptonic decays of the τ lepton, with final-state hadrons carrying strangeness quantum numbers. This process leads to a somewhat lower and less precise V_{us} value of 0.2221(13) (3, 94).

Figure 1 summarizes graphically the results on V_{ud} and V_{us} discussed so far and reveals that, while nuclear and neutron decay lead to a consistent picture for V_{ud}, tensions exist among current determinations of V_{us} ($K_{\ell 3}$ versus $K_{\ell 2}$ and K versus τ lepton). Moreover, an overall tension with CKM unitarity is apparent. A global fit leads to $V_{ud} = 0.97357(27)$ and $V_{us} = 0.22406(34)$, implying

$$\Delta_{\mathrm{CKM}} = (-19.5 \pm 5.3) \times 10^{-4}, \tag{17}$$

a 3.7σ effect. Due to the tension in the input data, the χ^2 per degree of freedom is 2.8, corresponding to a scale factor of $S = 1.67$ under the assumption that there is no NP effect. In Section 3, we discuss the implications of this tension for LFUV interactions.

2.3. Tau Decays

Tests of LFU can also be obtained by comparing different τ decay rates with those of muons or πs (Ks). For τ, we have semileptonic as well as purely leptonic decays at our disposal. While the

[5] Note that the V_{ud} value quoted by the PDG (3) does not yet reflect the increased error in the nuclear structure–dependent radiative corrections and therefore has an uncertainty $\delta V_{ud} = 0.00014$.

[6] The PDG excludes the beam lifetime measurements from the current "PDG average," quoting $\tau_n = 879.4(6)$ s. This value would change to $\tau_n = 879.6(8)$ s if the beam lifetime measurement result were included.

[7] Here we follow the analysis described in Reference 3. A recent comprehensive analysis of beta decays and τ decays can be found in References 87 and 88. Adopting input from these references would lead to very small changes in the global fit presented below.

former can test only τ–μ universality efficiently, the latter allow us to assess μ–e, τ–e, and τ–μ universality. If we define

$$R^{\tau}_{\mu/e} = \frac{\mathrm{Br}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{\mathrm{Br}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}, \qquad 18.$$

$$R^{\tau\pi(K)}_{\tau/\mu} = \frac{\mathrm{Br}[\tau \to \pi(K)\nu_\tau]}{\mathrm{Br}[\pi(K) \to \mu\nu_\mu]}, \qquad 19.$$

$$R^{\tau}_{\tau/\mu} = \frac{\mathrm{Br}(\tau^- \to e^- \bar{\nu}_e \nu_\tau)}{\mathrm{Br}(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}, \text{ and} \qquad 20.$$

$$R^{\tau}_{\tau/e} = \frac{\mathrm{Br}(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{\mathrm{Br}(\mu^- \to e^- \bar{\nu}_e \nu_\mu)}, \qquad 21.$$

then the LFU ratios can be expressed in terms of experimentally measured rates and theoretical input. For μ–e universality, we have

$$\left(\frac{A_\mu}{A_e}\right)_\tau = \sqrt{R^{\tau}_{\mu/e} \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)}}, \qquad 22.$$

where $f(x) = -8x + 8x^3 - x^4 - 12x^2 \log x$. The above expression receives radiative corrections of $O(\alpha/\pi) \times (m_\mu/m_\tau)^2$ (95), which are therefore suppressed. For τ–μ universality, we have[8]

$$\left(\frac{A_\tau}{A_\mu}\right)_h = \frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2} \sqrt{R^{\tau h}_{\tau/\mu} \frac{2 m_h m_\mu^2 \tau_h}{(1+\delta_h) m_\tau^3 \tau_\tau}}, \qquad 23.$$

where $h = \pi$ or K. An alternative method to test τ–μ universality, similar to the μ–e case, compares the electronic and muonic decay rates and can be expressed as

$$\left(\frac{A_\tau}{A_\mu}\right)_\tau = \sqrt{R^{\tau}_{\tau/\mu} \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^2}{m_\tau^3} (1+\delta_W)(1+\delta_\gamma)}. \qquad 24.$$

In the above equations, $m_{e,\mu,\tau}$ are the masses of e, μ, and τ; $\tau_{\tau,h}$ are the lifetimes of the particles τ and h; and $\delta_{h,W,\gamma}$ are the weak and electromagnetic radiative corrections (for details, see 95 and references therein). Experimentally, these tests have been carried out at B factories, where, at the nominal center-of-mass energy of 10.58 GeV/c^2, thanks to a cross section of 0.919 nb, these machines are de facto τ factories that produce large numbers of τ pairs.

Both the BaBar and CLEO Collaborations performed LFU tests according to Equations 22 (96) and 23 (97), while only CLEO performed the measurement according to Equation 24. In the reaction $e^+e^- \to \tau^+\tau^-$ at a B factory, one can use the decay of the τ^+ to tag and study the τ^- (and vice versa). Typically one uses either the so-called 3×1 τ topology, with the decay $\tau^+ \to \pi^+\pi^+\pi^-\bar{\nu}_\tau$ as a tag and then a study on the other τ^- is performed, or the so-called 1×1 topology, in which both τs decay with one prong (lepton or hadron) and a neutrino. While the latest BaBar measurement focused only on the 3×1 topology, the latest study from the CLEO Collaboration also used the 1×1.

[8] In the case of purely leptonic decays, we write the LFU test in terms of the ratios $A_\ell/A_{\ell'} \equiv A_{\ell\ell''}/A_{\ell'\ell''}$, with $L = A_{\ell'\ell}\bar{\ell}'\gamma^\mu P_L \nu_{\ell'} \bar{\nu}_\ell \gamma_\mu P_L \ell$.

Tests of LFU are precise measurements for which, in addition to sizable quantities of data, one needs to control systematic effects when determining the BRs. In the most recent results from BaBar, for example, $R^{\tau}_{\mu/e}$ and $(A_\mu/A_e)_\tau$ were determined with a precision of $0.4\%(0.16\%_{\text{stat}} \oplus 0.36\%_{\text{syst}})$ and 0.2% (96), respectively, where the leading systematic uncertainty (0.32%) originated from particle identification. Similarly, $R^{\pi}_{\tau/\mu}$ and $(A_\tau/A_\mu)_\pi$ were determined with a precision of $0.63\%(0.14\%_{\text{stat}} \oplus 0.61\%_{\text{syst}})$ and 0.57%, where again the dominant systematic source originated from particle identification.

The results from BaBar and CLEO have also been used to obtain the latest Heavy Flavor Averaging Group combination, which includes 176 measurements and 89 constraints in τ processes (98). For purely leptonic τ decays, these are

$$\left(\frac{A_\tau}{A_\mu}\right)_\tau = 1.0010 \pm 0.0014, \tag{25}$$

$$\left(\frac{A_\tau}{A_e}\right)_\tau = 1.0029 \pm 0.0014, \text{ and} \tag{26}$$

$$\left(\frac{A_\mu}{A_e}\right)_\tau = 1.0018 \pm 0.0014. \tag{27}$$

During the preparation of this review, new values of $(A_\tau/A_\mu)_h$ ($h = \pi, K$) that were obtained by computing radiative corrections, including the lightest multiplets of spin-1 heavy states in ChPT, were reported (99). These new values are

$$\left(\frac{A_\tau}{A_\mu}\right)_\pi = 0.9964 \pm 0.0038, \text{ and} \tag{28}$$

$$\left(\frac{A_\tau}{A_\mu}\right)_K = 0.9857 \pm 0.0078. \tag{29}$$

These values have the correlation coefficients (98)

$$\begin{array}{cccccc}
(A_\tau/A_\mu)_\tau & 1 & & & & \\
(A_\tau/A_e)_\tau & 0.51 & 1 & & & \\
(A_\mu/A_e)_\tau & -0.50 & 0.49 & 1 & & \\
(A_\tau/A_\mu)_\pi & 0.23 & 0.25 & 0.02 & 1 & \\
(A_\tau/A_\mu)_K & 0.11 & 0.10 & -0.01 & 0.06 & 1 \\
 & (A_\tau/A_\mu)_\tau & (A_\tau/A_e)_\tau & (A_\mu/A_e)_\tau & (A_\tau/A_\mu)_\pi & (A_\tau/A_\mu)_K,
\end{array} \tag{30}$$

and there is 100% correlation among $(A_\tau/A_\mu)_\tau$, $(A_\tau/A_e)_\tau$, and $(A_\mu/A_e)_\tau$ (98).

3. BEYOND THE STANDARD MODEL ANALYSIS

Let us now interpret the experimental bounds for LFUV in the charged current in terms of constraints on NP. To do so, we first study effective operators (i.e., modified $W\ell\nu$ couplings and four-fermion operators) and then consider simplified models that give rise to the corresponding Wilson coefficients. In this context, we highlight a possible correlation between the CAA and nonresonant dilepton searches at the LHC, and finally study NP models with a focus on those motivated by the CAA.

3.1. Effective Field Theory

We now consider NP effects parameterized by effective interactions.

3.1.1. Modified $W\ell\nu$ couplings. All observables discussed in this review are sensitive to modified W couplings to leptons. To investigate their effects, we therefore use the parameterization[9]

$$\mathcal{L} \supset -i\frac{g_2}{\sqrt{2}}\bar{\ell}_i\gamma^\mu P_L\nu_j W_\mu^-\left(\delta_{ij}+\epsilon_{ij}\right)+\text{h.c.}, \qquad 31.$$

where $i, j = e, \mu$, or τ; δ_{ij} is the Kronecker delta; and the SM $SU(2)_L$ gauge coupling g_2 is recovered in the limit $\epsilon_{ij} \to 0$. Here we have neglected possible effects of the PMNS (Pontecorvo–Maki–Nakagawa–Sakata) matrix that drop out in the limit of vanishing neutrino masses. Furthermore, below we disregard flavor-violating couplings (ε_{ij}, with $i \neq j$) because they are tightly bounded by radiative lepton decays $\ell \to \ell'\gamma$ and lead to effects in LFUV observables that do not interfere with the SM and are thus suppressed. Note that in Equation 31 we simply parameterize the BSM effect by ε_{ij} but do not consider the $SU(2)_L$ gauge invariance in SM EFT, which we discuss in Section 3.1.4.

For the phenomenological analysis, note that all LFUV observables (encoded in direct ratios) depend, at leading order, on differences $\epsilon_{aa} - \epsilon_{bb}$ ($a \neq b$), while the deficit in first-row CKM unitarity, related to the determination of V_{ud}, is to a good approximation sensitive only to $\epsilon_{\mu\mu}$ (31). In order to extract V_{ud} from beta decays, the Fermi constant determined from the muon lifetime (100) is needed:

$$\frac{1}{\tau_\mu} = \frac{(G_F^{\mathcal{L}})^2 m_\mu^5}{192\pi^3}(1+\Delta q)(1+\varepsilon_{ee}+\varepsilon_{\mu\mu})^2. \qquad 32.$$

Here $G_F^{\mathcal{L}}$ is the Fermi constant appearing in the Lagrangian (excluding BSM contamination), and Δq subsumes the phase space, QED, and EW radiative corrections. Therefore, the Fermi constant measured in muon decay and extracted under the SM assumption (G_F) is related to the one at the Lagrangian level as

$$G_F = G_F^{\mathcal{L}}(1+\varepsilon_{ee}+\varepsilon_{\mu\mu}). \qquad 33.$$

Thus,

$$V_{ud}^\beta = V_{ud}^{\mathcal{L}}\left(1-\varepsilon_{\mu\mu}\right), \qquad 34.$$

where $V_{ij}^{\mathcal{L}}$ denotes CKM matrix elements without any BSM contamination, which by definition fulfills CKM unitarity, and V_{ud}^β is the CKM element extracted from beta decays within the SM. Taking into account that first-row and -column CKM unitarity relations are very much dominated by V_{ud}, being by far the biggest element of the CKM matrix, we find that to a good approximation

$$\varepsilon_{\mu\mu} \approx 0.00098 \pm 0.00027, \qquad 35.$$

which reflects the corresponding 3.7σ tension.

We can now reparameterize the NP effects by writing

$$\epsilon_{ee}-\epsilon_{\mu\mu}, \qquad \epsilon_{\tau\tau}-\epsilon_{\mu\mu}, \qquad \text{and} \qquad \epsilon_{\mu\mu}, \qquad 36.$$

such that differences are direct measures of LFU and are constrained by the corresponding ratios. As a result, we can perform a global fit in the $\epsilon_{\tau\tau} - \epsilon_{\mu\mu}$ versus $\epsilon_{ee} - \epsilon_{\mu\mu}$ plane, which is uncorrelated

[9] In the conventions of Reference 95, we have $1 + \epsilon_{ii} - \epsilon_{jj} = g_i/g_j$ or, equivalently, $g_i = g_j(1 + \epsilon_{ii} - \epsilon_{jj})$, where $i, j = e, \mu$, or τ.

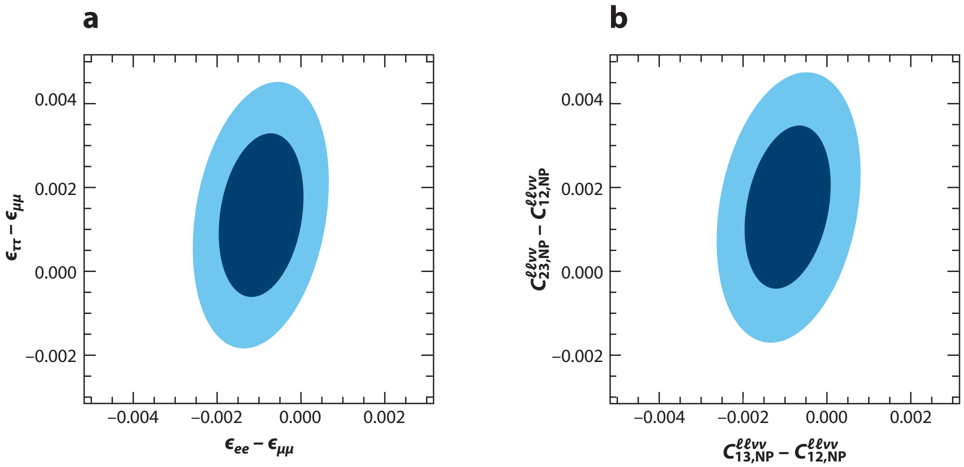

Figure 2

(*a*) Global fit in the $\epsilon_{\tau\tau} - \epsilon_{\mu\mu}$ versus $\epsilon_{ee} - \epsilon_{\mu\mu}$ plane, including K, π, and τ decays, quantifying LFU in the charged current. (*b*) Global fit in the $C^{\ell\ell\nu\nu}_{23,\mathrm{NP}} - C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}$ versus $C^{\ell\ell\nu\nu}_{13,\mathrm{NP}} - C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}$ plane from leptonic τ and muon decays. Uncertainties are shown for 1σ (*dark blue*) and 2σ (*light blue*). Abbreviations: LFU, lepton flavor universality; NP, new physics.

with $\epsilon_{\mu\mu}$, taking into account all LFU ratios discussed above (including correlations among them). **Figure 2*a*** shows the result. In this depiction, while the hypothesis of LFU in the charged current is compatible with data at the 2σ level, we observe a slight preference for negative values of $\epsilon_{ee} - \epsilon_{\mu\mu}$.

3.1.2. Four-lepton operators. It is clear that four-lepton operators enter only purely leptonic decays. Furthermore, because (in the limit of vanishing masses of the final-state leptons) only left-handed vector operators with the same flavor structure as the SM lead to interference with the SM in these decays, we can focus on them and write

$$\mathcal{L}_{4\ell} = \frac{-g_2^2}{2m_W^2} C^{\ell\ell\nu\nu}_{fi} \bar{\ell}_f \gamma_\mu P_L \ell_i \bar{\nu}_i \gamma^\mu P_L \nu_f, \qquad 37.$$

where $C^{\ell\ell\nu\nu}_{fi} = 1 + C^{\ell\ell\nu\nu}_{fi,\mathrm{NP}}$. The effects of $C^{\ell\ell\nu\nu}_{fi,\mathrm{NP}}$ are similar to those of modified $W\ell\nu$ couplings, and we can consider the three parameters $C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}$, $C^{\ell\ell\nu\nu}_{13,\mathrm{NP}} - C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}$, and $C^{\ell\ell\nu\nu}_{23,\mathrm{NP}} - C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}$. However, in this case C_{12} not only is determined from the CAA but also has an impact on the global EW fit because it modifies the determination of the Fermi constant from muon decay (101, 102). In fact, they turn out to prefer opposite signs:

$$\begin{aligned} C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}\Big|_{\mathrm{CAA}} &\approx 0.00098 \pm 0.00027, \text{ and} \\ C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}\Big|_{\mathrm{EW}} &\approx -0.00067 \pm 0.00033. \end{aligned} \qquad 38.$$

Both $C^{\ell\ell\nu\nu}_{13,\mathrm{NP}} - C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}$ and $C^{\ell\ell\nu\nu}_{23,\mathrm{NP}} - C^{\ell\ell\nu\nu}_{12,\mathrm{NP}}$ are determined from the ratios of rates $\tau \to \mu\nu\nu/\tau \to e\nu\nu$, $\tau \to \mu\nu\nu/\mu \to e\nu\nu$, and $\tau \to e\nu\nu/\mu \to e\nu\nu$, while all ratios involving mesons remain unaffected. Therefore, we find the global fit shown in **Figure 2*b***.

3.1.3. Two-quark–two-lepton operators. Concerning two-quark–two-lepton operators, both left-handed vector operators and scalar ones are relevant because they interfere with the SM contribution. In fact, the latter have enhanced effects in $R^P_{e/\mu}$:

$$\left(\frac{A_\mu}{A_e}\right)_{R^P_{e/\mu}} = \frac{C^{V_L\mu}_{fi} - C^{V_R\mu}_{fi} + \dfrac{m_P^2}{(m_{u_f}+m_{d_i})m_\mu}\left(C^{S_R\mu}_{fi} - C^{L\mu}_{fi}\right)}{C^{V_Le}_{fi} - C^{V_Re}_{fi} + \dfrac{m_P^2}{(m_{u_f}+m_{d_i})m_e}\left(C^{S_Re}_{fi} - C^{S_Le}_{fi}\right)}, \tag{39.}$$

with $u_f = u$ and $d_i = d(s)$ for $P = \pi(K)$, defined via the Lagrangian

$$L_{ud\ell\nu} = \frac{-g_2^2}{2m_W^2} V_{fi} \sum_{A=L,R} \left(C^{V_A\ell}_{fi}\bar{u}_f\gamma^\mu P_A d_i \bar{\ell}\gamma_\mu P_L \nu_\ell + C^{S_A\ell}_{fi}\bar{u}_f P_A d_i \bar{\ell} P_L \nu_\ell\right), \tag{40.}$$

where $C^{V_L\ell}_{fi} = 1 + C^{V_L\ell}_{fi,\mathrm{NP}}$. All other Wilson coefficients are zero within the SM.

From the CAA, we find

$$C^{V_Le}_{11,\mathrm{NP}} = -0.00098 \pm 0.00027, \tag{41.}$$

and from LFUV ratios, the bounds can be directly read off by using Equations 8 and 13. For the three-body decays involved in $R^{K\to\pi}_{e/\mu}$, the last term of Equation 39 should be omitted, and the sign in front of the right-handed vector Wilson coefficient changes. Note that, in principle, constraints from ratios like $\mathrm{Br}(K\to\mu\nu)/\mathrm{Br}(\pi\to\mu\nu)$ (for an overview, see, e.g., 103) have to be taken into account, especially in the case of scalar operators, even though these are not measures of LFUV.

3.1.4. $SU(2)_L$ gauge invariance. Interestingly, assuming that there is NP above the EW scale which respects $SU(2)_L$ gauge symmetry, any modification of the left-handed charged current also leads to a modification of a neutral current (76). The relevant effective operators in explicit $SU(2)_L$ gauge-invariant language (for definitions of the conventions, see 75, 104) are contained in the Lagrangian

$$\begin{aligned} L =& \frac{1}{\Lambda^2}\Big([C^{(3)}_{\ell q}]_{ijkl}(\bar{L}_i\gamma^\mu\tau^I L_j)(\bar{Q}_k\gamma_\mu\tau^I Q_l) + [C^{(3)}_{\ell\ell}]_{ijkl}(\bar{L}_i\gamma^\mu\tau^I L_j)(\bar{L}_k\gamma_\mu\tau^I L_l) \\ &+ C^{(1)ij}_{\phi\ell}\phi^\dagger i\overleftrightarrow{D}_\mu\phi\,\bar{L}^i\gamma^\mu L^j + C^{(3)ij}_{\phi\ell}\phi^\dagger i\overleftrightarrow{D}^I_\mu\phi\,\bar{L}^i\tau^I\gamma^\mu L^j\Big). \end{aligned} \tag{42.}$$

Making the identification with the operators discussed in the previous subsections, we find that, for modified $W\ell\nu$ couplings,

$$\epsilon_{fi} = \frac{v^2}{\Lambda^2}C^{fi}_{\phi e}, \tag{43.}$$

where $v = \sqrt{2}\times 174$ GeV and Λ is the mass scale of NP. This means that, at the same time, $Z\to\nu\nu$ and/or $Z\to\ell\ell$ effects appear. However, in order to avoid effects in stringently constrained Z couplings to charged leptons (105), we can assume $C^{(1)}_{\phi\ell} = -C^{(3)}_{\phi\ell}$. In this case, the constructive effect in $Z\to\nu_\mu\nu_\mu$ can be compensated for by a destructive effect in $Z\to\nu_e\nu_e$, as only the sum of the three flavors is measured, resulting in a very good fit to data (30).

For four-lepton operators,

$$C^{\ell\ell\nu\nu}_{fi,\mathrm{NP}} = -\frac{4m_W^2}{g_2^2\Lambda^2}[C^{(3)}_{\ell q}]_{ffii}. \tag{44.}$$

Here the implication is that a purely leptonic charged-current operator with first-generation leptons generally gives rise to a neutral current with electrons, which is subject to LEP bounds from nonresonant dilepton searches (106).

Finally, for four-fermion operators, working in the down-basis, we find

$$C_{fi,\mathrm{NP}}^{V_L \ell_j} = -\frac{4m_W^2}{g_2^2 \Lambda^2}[C_{\ell q}^{(3)}]_{jjfi}. \tag{45}$$

This means that for left-handed $\bar{u}d\bar{\nu}e$ operators affecting beta decays, $\bar{u}u\bar{e}e$ and/or $\bar{d}d\bar{e}e$ operators are also generated. This has interesting implications for an explanation of the CAA, as it leads to an additional neutral $\bar{u}u\ell\ell$ and $\bar{d}d\ell\ell$ current after EW symmetry breaking. $[Q_{\ell q}^{(3)}]_{1111}$ therefore also contributes to nonresonant dielectron production at the LHC, which is tailored to search for heavy NP that is above the direct production reach (107, 108). The latest dilepton results from ATLAS and CMS are presented in References 109 and 29, respectively. CMS observed a slight excess in the dielectron cross section at high invariant lepton mass and computed the double ratio

$$\frac{R^{\mathrm{data}}_{\mu^+\mu^-/e^+e^-}}{R^{\mathrm{MC}}_{\mu^+\mu^-/e^+e^-}} \tag{46}$$

in order to reduce the uncertainties (110). This means that CMS provides the relative signal strength for muons versus electrons, $R^{\mathrm{data}}_{\mu^+\mu^-/e^+e^-}$, divided by the SM expectation obtained from Monte Carlo simulations, $R^{\mathrm{MC}}_{\mu^+\mu^-/e^+e^-}$. Using these results, we find that the best-fit value for the Wilson coefficient is

$$\frac{[C_{\ell q}^{(3)}]_{1111}}{\Lambda^2} \approx \frac{1.0}{(10\ \mathrm{TeV})^2}, \tag{47}$$

with $\Delta\chi^2 \equiv \chi^2 - \chi^2_{\mathrm{SM}} \approx -10$ and $0.3/(10\ \mathrm{TeV})^2 \lessapprox [C_{\ell q}^{(3)}]_{1111} \lessapprox 1.8/(10\ \mathrm{TeV})^2$. Note that this value is compatible with the corresponding ATLAS bounds and $(A_\mu/A_e)_{R^\pi_{e/\mu}}$ in Equation 8. Treating the ATLAS exclusion as a hard cut, we therefore find that at 95% CL

$$\frac{0.6}{(10\ \mathrm{TeV})^2} \lessapprox \frac{[C_{\ell q}^{(3)}]_{1111}}{\Lambda^2} \lessapprox \frac{1.4}{(10\ \mathrm{TeV})^2}, \tag{48}$$

which predicts

$$1.0004 \lessapprox \left(\frac{A_\mu}{A_e}\right)_{R^\pi_{e/\mu}} \lessapprox 1.0009 \tag{49}$$

at 95% CL.

3.2. Simplified New Physics Models

The following NP models (for a complete categorization of tree-level extensions of the SM, see 111) give contributions to the effective operators discussed in the last subsection. These have a relevant impact on our observables limiting LFUV.

3.2.1. *W′* boson. In order to be relevant for our observables, a W' boson must be the component of a $SU(2)_L$ triplet X_a^μ with hypercharge 0 such that it can couple to left-handed fermions:

$$L_{X_a^\mu} = -g_{ji}^{\ell} X_a^\mu \bar{L}_j \gamma_\mu \frac{\tau^a}{2} L_i - g_{ji}^{q} X_a^\mu \bar{Q}_j \gamma_\mu \frac{\tau^a}{2} Q_i. \tag{50}$$

These couplings can contribute to LFUV observables in several ways (32):

- Modified $W\ell\nu$ coupling via mixing with the SM W boson. In this case, it generates $C_{\phi\ell}^{(3)}$ such that $W\ell\nu$, $Z\nu\nu$, and $Z\nu\nu$ couplings are affected. As discussed in the preceding subsection, this leads to limited effects in $W\ell\nu$ due to the stringent constraints from Z decays. Global fits to $C_{\phi\ell}^{(3)}$ can be found in References 32 and 112.

- Tree-level effects in $\ell \to \ell' \nu\nu$. In this case, the Wilson coefficient $C_{\ell\ell}^{(3)}$ is generated, resulting (after EW symmetry breaking) in

$$C_{fi}^{\ell\ell\nu\nu,\mathrm{NP}} = \frac{g_{ff}^{\ell}}{g_{ii}^{\ell}} \frac{m_W^2}{m_{W'}^2}, \qquad 51.$$

and the bounds of Section 3.1.2 can be used.
- Tree-level effects in $d \to u e\nu$. Similarly, if W' possesses couplings to quarks and leptons, it leads to tree-level effects $d \to u e\nu$ via the Wilson coefficient $C_{qq}^{(3)}$, and the bounds from K and π decays apply.

For example, left-handed W' bosons appear as excitations of the SM W boson in composite (113, 114) or extradimensional models (115) as well as in theories with several $SU(2)_L$ gauge groups (116, 117).

3.2.2. Vector-like leptons. Vector-like leptons (VLLs), such as right-handed neutrinos (118), affect $W\ell\nu$ coupling via their mixing with SM leptons and are EW symmetry breaking. There are five representations of VLLs that can couple to SM leptons and the Higgs boson and mix with the former after EW symmetry breaking. They are represented as

	$SU(3)$	$SU(2)_L$	$U(1)_Y$
ℓ	1	2	$-1/2$
e	1	1	-1
ϕ	1	2	$1/2$
N	1	1	0
E	1	1	-1
$\Delta_1 = (\Delta_1^0, \Delta_1^-)$	1	2	$-1/2$
$\Delta_3 = (\Delta_3^-, \Delta_3^{--})$	1	2	$-3/2$
$\Sigma_0 = (\Sigma_0^+, \Sigma_0^0, \Sigma_0^-)$	1	3	0
$\Sigma_1 = (\Sigma_1^0, \Sigma_1^-, \Sigma_1^{--})$	1	3	-1

. 52.

These result in the following Wilson coefficients:

$$\begin{aligned}
\frac{C_{\phi\ell}^{(1)ij}}{\Lambda^2} &= \frac{\lambda_N^i \lambda_N^{j\dagger}}{4M_N^2} - \frac{\lambda_E^i \lambda_E^{j\dagger}}{4M_E^2} + \frac{3}{16}\frac{\lambda_{\Sigma_0}^{i\dagger}\lambda_{\Sigma_0}^{j}}{M_{\Sigma_0}^2} - \frac{3}{16}\frac{\lambda_{\Sigma_1}^{i\dagger}\lambda_{\Sigma_1}^{j}}{M_{\Sigma_1}^2},\\
\frac{C_{\phi\ell}^{(3)ij}}{\Lambda^2} &= -\frac{\lambda_N^i \lambda_N^{j\dagger}}{4M_N^2} - \frac{\lambda_E^i \lambda_E^{j\dagger}}{4M_E^2} + \frac{1}{16}\frac{\lambda_{\Sigma_0}^{i\dagger}\lambda_{\Sigma_0}^{j}}{M_{\Sigma_0}^2} + \frac{1}{16}\frac{\lambda_{\Sigma_1}^{j\dagger}\lambda_{\Sigma_1}^{i}}{M_{\Sigma_1}^2},\\
\frac{C_{\phi e}^{ij}}{\Lambda^2} &= \frac{\lambda_{\Delta_1}^{i\dagger}\lambda_{\Delta_1}^{j}}{2M_{\Delta_1}^2} - \frac{\lambda_{\Delta_3}^{i\dagger}\lambda_{\Delta_3}^{j}}{2M_{\Delta_3}^2},
\end{aligned} \qquad 53.$$

where λ are the couplings of the VLLs to the SM leptons and the Higgs doublet (for details, see 32). Note that only $C_{\phi\ell}^{(3)}$ generates a charged current and affects our LFUV observables, whereas the other coefficients enter $Z\ell\ell$ and $Z\nu\nu$ couplings affecting the global EW fit (119). While for $C_{\phi\ell}^{(3)}$ we can apply the bounds from the preceding subsection, in order to take into account all effects, a global fit is necessary (32).

VLLs are predicted in many SM extensions, such as Grand Unified Theories (120–122), composite models, or models with extra dimensions (123–130). They are also involved in the type I (118, 131) and type III (132) seesaw mechanisms.

3.2.3. Singly charged $SU(2)_L$ singlet scalar.

Because it is a $SU(2)_L \times SU(3)_C$ singlet, ϕ^+ has hypercharge $+1$ and allows only for Yukawa-type interactions with leptons. The model is particularly interesting in the context of LFUV. Because of the hermicity of the Lagrangian, it has antisymmetric (i.e., off-diagonal) couplings,

$$\mathcal{L} = -\frac{\lambda_{ij}}{2}\,\bar{L}^c_{a,i}\,\varepsilon_{ab}\,L_{b,j}\,\Phi^+ + \text{h.c.}, \tag{54}$$

but not with quarks. Here L is the left-handed $SU(2)_L$ lepton doublet, c stands for charge conjugation, a and b are $SU(2)_L$ indices, i and j are flavor indices, and ε_{ab} is the two-dimensional antisymmetric tensor. Note that, without loss of generality, λ_{ij} can be chosen to be antisymmetric in flavor space, $\lambda_{ji} = -\lambda_{ij}$, such that $\lambda_{ii} = 0$ and our free parameters are λ_{12}, λ_{13}, and λ_{23}. This choice yields

$$C^{\ell\ell\nu\nu}_{fi,\text{NP}} = \frac{\left|\lambda^2_{fi}\right|}{g_2^2}\,\frac{m_W^2}{m_\phi^2}. \tag{55}$$

This means that the effect of $\ell \to \ell'\nu\bar{\nu}$ is necessarily constructive, such that the CAA can be solved (133–135) and $C^{\ell\ell\nu\nu}_{23,\text{NP}}$ has the right sign preferred by the fit. However, in order to not violate the bounds from $\mu \to e\gamma$, λ_{13} must be very close to zero (133).

Singly charged scalars have been proposed within the Babu–Zee model (136, 137). They were studied in References 138–148 as part of a larger NP spectrum, mostly with the aim of generating neutrino masses at loop level.

3.2.4. Scalar $SU(2)_L$ triplet.

The scalar $SU(2)_L$ triplet couples to a lepton doublet and a charge-conjugated one as

$$\mathcal{L}_{\mathcal{T}} = -\frac{\kappa_{ij}}{2}\,\bar{L}^c_{a,i}\,\varepsilon_{ab}\tau^I_{cd}\,L_{d,j}\,\Phi^T + \text{h.c.} \tag{56}$$

The result is

$$C^{\ell\ell\nu\nu}_{fi,\text{NP}} = -\frac{\left|\kappa^2_{fi}\right|}{g_2^2}\,\frac{2m_W^2}{m_\phi^2}, \tag{57}$$

such that the effect is always destructive in $\ell \to \ell'\nu\bar{\nu}$.

This scalar can generate neutrino masses within the type II seesaw model (149–153). While in general the tiny neutrino masses require very small couplings κ (for TeV-scale masses), the contribution to the Weinberg operator (154) can be suppressed, for instance, by an approximate baryon number symmetry, such that phenomenologically relevant effects in LFUV are possible.

3.2.5. $SU(2)_L$ neutral vector boson (Z').

A Z' boson, which is an $SU(2)_L$ singlet, interferes with the SM amplitudes for $\ell \to \ell'\nu\nu$ only if it has couplings to lepton doublets:

$$L_{Z'} = -ig^{\ell}_{fi}\bar{L}_f\gamma^\mu L_i Z'_\mu. \tag{58}$$

Furthermore, these couplings must be flavor violating, such that

$$C^{\ell\ell\nu\nu}_{fi,\text{NP}} = \frac{2m_W^2}{g_2^2 M_{Z'}^2}|g^{\ell}_{fi}|^2. \tag{59}$$

Note that the effect in $\ell \to \ell'\nu\bar{\nu}$ is necessarily constructive (155).

There is a huge literature on Z' bosons (for an overview, see 156). Again, these could be excitations of the SM Z boson and γ, but in this case they also originate from a gauged $U(1)$ flavor symmetry like L_μ–L_τ or B–L.

3.2.6. Leptoquark. Ten representations of leptoquarks (five scalar and five vector) exist, with gauge-invariant couplings to quarks and leptons (157). Among them, six representations generate a charged current, two a vector current, two a scalar current, and two simultaneously a vector and a scalar current. While those with a scalar current are stringently constrained by $R^{\pi}_{e/\mu}$, for vector currents, other observables such as dilepton pairs, low energy parity violation, or K decays are, in general, more constraining (for a recent comprehensive analysis, see 158, 159).

Leptoquarks arise in the Pati–Salam model (160), in $SU(5)$ Grand Unified Theories (161, 162), and in the R-parity-violating minimal supersymmetric model (MSSM) (for a review, see, e.g., 163). They have been studied in the context of LFUV in K, τ, and π decays in References 49 and 164–167.

3.2.7. Charged Higgs. Charged Higgs bosons have been considered in the context of leptonic meson decays (168), especially $R^{K}_{\mu/e}$ in the context of the MSSM (169, 170). Furthermore, the type X two-Higgs-doublet model is constrained by loop effects in $\tau \to \mu\nu_\tau\nu_\mu$ (171–173), which are relevant at large values of tan β.

4. FUTURE PROSPECTS

4.1. Pion and Kaon Experiments

The PIENU (174, 175) and PEN (176, 177; see **http://pen.phys.virginia.edu/**) experiments aim to further improve the precision of the $\pi \to e\nu$ BR $R^{\pi}_{e/\mu}$. However, even when these goals are realized, an experimental improvement by more than an order of magnitude in uncertainty is warranted to confront the SM prediction and to search for non-SM effects. A developing proposal, PIONEER (178), aims to improve precision for $R^{\pi}_{e/\mu}$ by an order of magnitude, making the experimental uncertainty comparable to the theoretical uncertainty in Equation 4. Reaching very high precision will require high statistics as well as extensive evaluation of systematic uncertainties, backgrounds, biases, and distortions in the data selection criteria. Like PIENU (57) and PEN, PIONEER will use stopped πs that decay at rest. Principal features of the experiment include a fully active silicon tracking stopping target (179) and a high-resolution calorimeter, both of which contribute to suppression of systematic effects.

PIONEER also aims to improve the precision of the BR of π–beta decay $\pi^+ \to \pi^0 e^+\nu(\gamma)$ (180). π–beta decay, while providing theoretically the cleanest determination of the CKM matrix element V_{ud}, is currently not competitive: $V_{ud} = 0.9739(28)_{\mathrm{exp}}(1)_{\mathrm{theor}}$, where the experimental uncertainty comes almost entirely from the $\pi^+ \to \pi^0 e^+\nu(\gamma)$ BR (180) (the π lifetime contributes $\delta V_{ud} = 0.0001$) and the theory uncertainty has been reduced from $(\delta V_{ud})_{\mathrm{theor}} = 0.0005$ (181–183) to $(\delta V_{ud})_{\mathrm{theor}} = 0.0001$ via a lattice QCD calculation of the radiative corrections (184). As pointed out in Reference 93, even a threefold improvement in the precision of the π–beta decay BR compared with Reference 180 would enable a 0.2% accuracy in the determination of the ratio V_{us}/V_{ud} from $R_V \equiv \Gamma[K \to \pi\ell\nu(\gamma)]/\Gamma[\pi^+ \to \pi^0 e^+\nu(\gamma)]$, competitive with the existing determination based on $R_A \equiv \Gamma(K \to \mu\nu)/\Gamma(\pi \to \mu\nu)$. Regarding other K decays, the TREK experiment may provide additional information on $R^{K}_{e/\mu}$ (185), and NA62 may make further measurements relevant to LFUV (37, 186).

4.2. Tau Experiments

The Belle II experiment is currently collecting data at and near the center-of-mass energy of 10.58 GeV/c^2 and is expected to obtain a total integrated luminosity of 50 ab^{-1}, equivalent to 45 billion τ pairs (187, 188). Although precision measurements are ultimately dominated by the capabilities of experiments to limit systematic factors, an improvement in the determination of all LFU parameters in τ decays is to be expected.

The proposed Future Circular Collider (FCC) plans to collide electrons and positrons (FCC-ee) at different center-of-mass energies, including a four-year high-statistics run at the Z pole (189). Assuming an instantaneous luminosity of 2.3×10^{36} cm^{-2} s^{-1} and four interaction regions, this would translate into 1.7×10^{11} τ pairs produced in $Z \rightarrow \tau^+\tau^-$ reactions available for precision studies of τ properties and polarization at the FCC-ee (190). A rich τ physics program would also be expected by the proposed Circular Electron Positron Collider (CEPC), where 30 billion τ pairs could be produced at the Z pole (191).

For both the FCC-ee and the CEPC, the same considerations regarding the control of systematic effects hold as for Belle II. Although collecting high-statistics samples of τ is a *conditio sine qua non* to perform precision tests of the SM via suppressed or forbidden processes in τ decays, systematic effects must be understood and kept under control. All the abovementioned experiments could reach a statistical precision at the level of 10^{-4}–10^{-5} on a number of LFU parameters, given the large number of τ decays that will be produced and analyzed, but systematic effects will dominate the final precision. Planned future experiments will have the advantage of designing the detectors to minimize potential systematic effects.

4.3. Theory

While the ratios measuring LFU are, in general, theoretically very clean, the uncertainty in the extraction of V_{ud} from superallowed beta decays is still limited by the theoretical error. Moreover, expected experimental improvements in the neutron lifetime and decay asymmetry will enhance the relative impact of theoretical uncertainties in the extraction of V_{ud} from neutron decay. The theoretical uncertainty in both cases arises from electromagnetic radiative corrections. Progress in the next decade can be expected on a number of fronts. First, at the single-nucleon level, one expects results on radiative corrections to neutron decay from lattice QCD. The technology to perform these calculations in meson systems ($K^+ \rightarrow \mu^+\nu_\mu$ and $\pi^+ \rightarrow \pi^0 e^+\nu_e$) has been demonstrated (184, 192) but not yet applied to nucleons. Second, one can expect progress in the analysis of few- and many-body effects in nuclear transitions, both with dispersive techniques (83) and through the development of chiral EFT few-body transition operators to $O(G_F\alpha)$ coupled to first-principles many-body methods.

5. CONCLUSIONS

In this article, we have reviewed the status of the searches for LFUV in the charged current involving πs, Ks, τs, and beta decays. Averaging the values presented in Section 2.1.1, 2.1.2, and 2.3, we find that the ratios of the $W\ell$ couplings are

$$\begin{aligned}
\frac{g_\mu}{g_e} &= 1 + \epsilon_{\mu\mu} - \epsilon_{ee} = 1.0009 \pm 0.0006,\\
\frac{g_\tau}{g_\mu} &= 1 + \epsilon_{\tau\tau} - \epsilon_{\mu\mu} = 1.0013 \pm 0.0013, \text{ and}\\
\frac{g_\tau}{g_e} &= 1 + \epsilon_{\tau\tau} - \epsilon_{ee} = 1.0022 \pm 0.0013.
\end{aligned} \qquad 60.$$

Note that these individual results for g_i/g_j use values of the others that minimize the χ^2 of 1d fits and should thus be understood separately; in other words, no correlations among the three ratios are taken into account. These high-precision tests of LVU, which agree well with the SM, are particularly interesting in light of the experimental hints of LFUV in semileptonic B decays, the anomalous magnetic moment of the muon, and the CAA.

Taking into account the current experimental and theoretical results, we performed a combined fit to modified $W\ell\nu$ couplings, as shown in **Figure 2*a***. However, the results can also be interpreted in terms of four-lepton and two-quark–two-lepton operators. We have also reviewed the experimental and theoretical prospects for the many observables. The proposed PIONEER experiment aims to improve the test of e–μ universality in π decay and the π–beta decay determination of V_{ud}. Furthermore, BELLE II measurements will lead to significant improvements in LFU tests with τ leptons. Finally, the proposed FCC-ee and CEPC accelerators offer intriguing possibilities for flavor physics and, in particular, for τ decays, given the expected large samples of B mesons and τ leptons that could be produced by these facilities.

DISCLOSURE STATEMENT

The authors are not aware of any affiliations, memberships, funding, or financial holdings that might be perceived as affecting the objectivity of this review.

ACKNOWLEDGMENTS

D.B. is supported by Natural Sciences and Engineering Research Council of Canada grant SAPPJ-2018-0017. A.C. gratefully acknowledges support from the Swiss National Science Foundation under project PP00P21_76884. G.I. was supported by the European Research Council under grant agreement 947006 ("InterLeptons"). V.C. is supported by the US Department of Energy through the Office of Nuclear Physics and the Laboratory Directed Research and Development program at Los Alamos National Laboratory. Los Alamos National Laboratory is operated by Triad National Security, LLC, for the National Nuclear Security Administration of the US Department of Energy (contract 89233218CNA000001).

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