What Sociologists Should Know About Complexity

Scott E. Page

University of Michigan, Ann Arbor, Michigan 48109, and The Santa Fe Institute, Santa Fe, New Mexico 87501; email: spage@umich.edu

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Abstract

I discuss the concept of complexity and the burgeoning field of complex systems and their relevance to sociology. I begin by comparing and contrasting various definitions of complexity and then describe the attributes of systems capable of producing complexity: diversity, networked interactions, interdependent behavior, and adaptation. Next, I survey the contributions of complexity sciences with the most resonance with sociology. I organize those contributions into four categories: dynamics, aggregation, distributions, and functional properties of structure and diversity. On the basis of that survey, I conclude that incorporating complexity science into sociology requires the introduction of new models and methodologies as well as a more expansive approach to empirical research, and that the benefits of a deeper engagement with complexity will be substantial.

INTRODUCTION

In this article, I describe a collection of ideas, models, results, theories, and techniques all loosely subsumed under the label of complexity science and discuss their sociological relevance and their potential impact on the field. The protodiscipline of complexity science garners substantial attention in both academic journals and the popular press owing partly to the growing complexity of the world and partly to the abundance of high-dimensional data. Advocates argue that complex systems methods enlarge the set of questions we can consider, the types of data we can gather and employ, and the classes of phenomena that we can identify and analyze. They also argue that taking a complexity perspective obliges us to reconsider closely held assumptions and preconceptions about how the world works and how systems aggregate.

Some scholars see the complexity sciences as transformative, as a "new kind of science" that will disrupt current scientific practices (Wolfram 2002). I stake a more modest claim and suggest that the complexity sciences hold promise as a complement to existing methods of social scientific inquiry by shining light and focus on unasked questions. They represent another arrow in our methodological quiver (Castellani & Hafferty 2009) that will improve social science and help us guide and design policy choices (Colander & Kupers 2014).

To be clear about my aims, I present a personal (and somewhat idiosyncratic) coverage of the measures, findings, concepts, and models from complex systems that have sociological relevance. Along the way, I comment on a selective smattering of published sociological examples, but I make no attempt at a comprehensive review of the literature at the intersection of sociology and complexity. This implies that I leave out some of the best examples of complexity research in sociology. I ask then that this piece be read as a review of a set of ideas and not as a more traditional review of the literature.

The relevance of complexity to sociology begins with a resonance. The discipline of sociology, as I perceive it from the outside, attempts to make sense of interactions of individuals and aggregations of individuals—groups, classes, and societies—and to explore how those interactions are shaped by human psychology, culture, social structure, and formal institutions. Briefly put, sociology sees actors as situated—within networks, categories, and place. Unlike in, say, economics, these contexts or fields play a central role (Bourdieu 1984, 1993).

Sociology engages both change and progress as opposed to presuming equilibrium. From a sociological perspective, our social, economic, and political lives are dynamic—admitting new ideas, categories, and technologies. Those innovations can produce complex (nonstationary) dynamics. Finally, parts of sociology accept the possibility that a group can have emergent properties that exist independent of its members (Byrne & Callaghan 2014, Little 2012, Sawyer 2005).

Many of these same organizing ideas and beliefs of the field of sociology occupy central positions within complex systems (Xie 2007). Briefly put, complex systems consist of situated, adaptive, diverse individuals whose interactions produce higher-order structures (self-organization) and functionalities (emergence). Complexity scientists, like sociologists, are interested in properties of those systems. Are they robust? How do behaviors aggregate? What structures form? What patterns emerge? What distributions across types arise? How durable are entities? Do these systems become more complex over time? What is known to date is both impressive and partial. We know that complex systems tend to be nonpredictable, hard to describe or define, prone to large events, and often produce long-tailed distributions across types (Miller & Page 2007, Mitchell 2009).

I can with some confidence claim (using Venn terminology) that a meaningful part of sociology overlaps with a substantial subset of the complexity sciences. That is, although much of what sociologists study can be thought of as occurring within a complex system, not all of it does. Further, although complex systems include sociological concerns, much of what is studied within complex systems has little to do with sociology. For example, complex systems scientists study social interactions that are less central to sociology, such as traffic patterns (Nagel & Paczuski 1995), financial collapse (Goldin & Mariathasan 2014), and international politics (Jervis 1998). Moreover, the bulk of complex systems research does not include humans at all. It focuses on physical, biological, and ecological systems. Thus, one can find complex systems models of magnets, neurons, bees, forest fires, and avalanches.

In contemplating the intersection between sociology and complexity, I see the greatest potential for sociological gain in the acquisition and application of models. I say this because the sandbox of complexity scientists contains an abundance of them, many of which have direct or indirect applications to the social world. Some models explore theoretical questions such as whether a system will produce an equilibrium, complexity, or randomness (Page 2008, Hidalgo et al. 2014, Wolfram 2002) or how a system is impacted by network structure (Newman 2010, Watts 2004), interdependence (Durlauf & Ioannides 2010), diversity (Page 2007, 2010), or learning (Epstein 2014, Vriend 2000). Many of these models can be connected to spatial and temporal data and then applied to topics ranging from social organization (Helbing 2010) to the spread of culture, ideas, and disease (Tassier 2013).

Some refer to complex systems models as transdisciplinary because they satisfy the one-to-many property; by that I mean that models developed for one purpose can be applied to many others. Models developed to explore the spread of diseases have been applied to the spread of crime (Akers & Lanier 2009), obesity (Akers & Lanier 2009), and Justin Bieber (Tweedle & Smith 2011). This application is accomplished by thinking of a behavior, say, downloading a Bieber song, as akin to a disease. Just as a friend may give you the measles, she may also give you Bieber fever. The difference is that Bieber fever spreads over the Internet, whereas the measles spread over a physical social network. But to first approximation, both phenomena can be captured as spread over a network.

In the sections that follow, I describe concepts, ideas, and insights produced by these models. These fall under the broad categories of dynamics, aggregation, distributions, and the functional properties of diversity and structure. I discuss each in turn and then comment on the changes required within sociology for complexity sciences to take root and on the potential for taking new, big data to these models. In order to make the remainder of the essay understandable, I first briefly describe what is meant by complexity and complex systems and introduce necessary terminology.

COMPLEXITY AND COMPLEX ADAPTIVE SYSTEMS

The concept of complexity can be applied to a wide-ranging set of systems and phenomena. The brain, the global financial system, the tax code, international politics, middle school, protein folding, ecosystems, and ant colonies can and have been described as complex. Given this diversity of applications of complexity, one cannot expect a single measure or statistic to capture the essence of complexity (Mitchell 2009, Page 2010). This creates a problem and an opportunity. The problem is that for the field to transition from what organizational theorist Michael Cohen once called "a festival of metaphors" to an actual science requires well-defined terms and concepts, and that means measures. But no single measure can capture the myriad meanings and nuances of the concept of complexity. The opportunity lies in the potential for a variety of measures to collectively play that role, in that each measure captures part of what is meant by "complex."

The existing set of measures can be seen either as a motley collection of failed attempts or as an ensemble of perspectives, in which each sheds light on some aspects of complexity. I believe the latter characterization to be the more accurate. We gain a deep understanding of the concept by engaging multiple definitions and exploring their overlaps and distinctiveness. For that reason, I begin with a brief characterization of the various types of complexity measures.

The measures can be best explained by having a set of data in mind. Consider a set of concurrent time series of data, perhaps a collection of daily prices for the stocks in the S&P 500 or the daily-changing playlists on the smartphones of the students at Tappan Middle School in Ann Arbor, Michigan. Each set could be characterized as complex according to a variety of measures. First, each would be difficult to describe, and by that I literally mean that each would take many words to communicate. In the formal language of computer science, each has a substantial description length or Kolmogorov complexity.

Second, the stock prices and playlists would be neither a simple pattern nor random (Wolfram 2002). Each set would both have order and lack order. Third, if one could identify and remove the randomness from each set of time series, elaborate structures would remain. Physicists call the remaining structure predictive information or excess entropy (see Prokopenko et al. 2009). Excess entropy is the nonrandom uncertainty that you could eventually figure out if you had enough time and the right model. A large amount of excess entropy implies that predicting future values would be a challenge.

Further, each set of series would be difficult to reproduce, whether by a bottom-up evolutionary system or in a top-down engineered fashion. Minimal models that would create statistically indistinguishable collections of time series would have to contain many relevant states.[1] Such processes are said to have high statistical complexity (Shalizi et al. 2004). Finally, these data can also be interpreted as containing substantial information (Adami 2002). This is particularly true of playlists. They embed information about the environment within which the choices were made, an insight that should be familiar to sociologists but was more novel to other disciplines.

These many formal characterizations reveal the many facets of complexity: having defined structure that may embed meaning and being not easily defined, predicted, explained, built, or evolved. As I have stated more pithily elsewhere (Page 2010), complexity can be thought of as BOAR (between ordered and random) as well as DEEEP (difficult to explain, evolve, engineer, or predict). All these ideas of what complexity is have been formalized as mathematical measures and agree with intuitive notions of complexity, i.e., that something that is complex is not simple. It's hard to understand.

Complexity, as just defined, is a phenomenological property. The existence of complexity begs the question of how it arises. Most often, it is produced by complex systems. These systems tend to have four attributes. First, they include diverse entities. Diversity may be innate, as is the case of the various species in an ecosystem, or it may emerge from the process of interactions, as occurs in cell differentiation from (nearly) identical strands of DNA.

Second, the diverse entities in a complex system interact within an interaction structure. In some cases this structure is best represented as a fixed network—a person's place in an organizational hierarchy may be fixed in the short term. In other instances, the structure may be more ephemeral. The people whom we physically encounter on a daily basis are a dynamic, perpetually novel network.

Third, individual behaviors are interdependent, i.e., one person's action or behavior influences the behavior of others. That influence could be substantial (e.g., the invasion of one country by another induces multiple immediate reactions by others) or more subtle (e.g., seeing a stranger with a new style of haircut or dress may change what a person perceives as socially acceptable).

[1] By a relevant state I mean one that the process visits with a nontrivial probability.

Last, and perhaps most important, the entities adapt or learn. The entities change not only their behaviors, but also their connections and interdependencies. Adaptation can be at the level of the entities themselves, as is the case in a social system, or it can be at the population level, as is the case in ecologies. In social systems, both types of change occur. Individuals learn. Organizations that do not perform may not survive for long. How the entities learn, whether individual or social, whether by experimentation, mimicry, or selection, matters (Vriend 2000). The learning rule determines the dynamics of the systems, which in turn influences what happens next (Golman & Page 2009).

Systems with these properties, so-called complex systems, have the potential to produce complexity. But they need not. An economy is a complex system. But parts of the economy are stable and predictable. Thus, it would be more accurate (though pedantic) to use the terminology "systems capable of complexity" rather than "complex systems."

To state this insight more formally, a complex system, whether it be composed of ants, firms, countries, or multinational corporations, has the potential to produce multiple classes of outcomes: equilibria, patterns, complexity, and randomness (Wolfram 2002). Here patterns include both periodic patterns, such as business cycles, and trends, such as reductions in crime, a growing acceptance of inequality, or declines in social capital (Putnam 2000).

A given complex system will often produce multiple classes of outcomes. Consider the world market for oil. A graph of oil production over the past 60 years shows an approximately linear trend (the Gulf Wars create the deviations). Oil production tracks global economic growth so it exhibits a simple pattern. A graph of oil prices over that same period reveals complexity. The time series is between ordered and random. It is also difficult to describe, explain, or predict (see **Figure 1**). In a word, it is complex.

The past few decades have produced sustained and modest progress on general theories of complex systems (Tao 2012). I use the plural because no single general theory probably exists. This position rests on what some call the nonelephant problem, in reference to a quip attributed to both John von Neumann and Stanislaw Ulam that the set of nonlinear functions is akin to the set of nonelephant animals (Fisher 1987). The set of possible complex systems may just be too large for there to exist properties that hold across all realizations.

A problem with any general theory of complex systems is that it would seem to propose that identical phenomena occur in the brain, economies, cities, weather systems, ecologies, financial markets, intellectual communities, and ant colonies. Could this really be so? On the one hand, obviously not. Ant colonies do not produce hurricanes or suffer epileptic seizures. On the other hand, yes. The set of nonelephants shares many properties. Almost all exhibit clustering in volatility, and most are paradoxically both robust (Jen 2005) and prone to large events (Ramo 2009). Many complex systems also produce long-tailed, i.e., nonnormal, distributions. Distributions of city sizes, lengths of traffic jams, firm sizes, war deaths, species abundance, and numbers of citations all have long tails. Although no single causal model can explain the similar distributions, there do appear to be a handful of fundamental causes, e.g., positive reinforcement, self-organized criticality, that explain most cases, a point I return to at length later below.

Other common properties of complex systems include self-organization, emergence, boundaries and levels (Holland 2014), path dependence, threshold phenomena (a.k.a. tipping points), production of novelty (barely 10% of the firms in the 1955 Fortune 500 remain on that list today), and tension in balancing exploration with exploitation. These phenomena can all be found in social, biological, ecological, economic, and virtual worlds. This is why, as noted above, complexity science is promoted as transdisciplinary, as providing concepts that apply to multiple domains. So even though complex systems may lack a single unifying theory, the field contains a sufficient number of coherent and related understandings to constitute a body of knowledge.

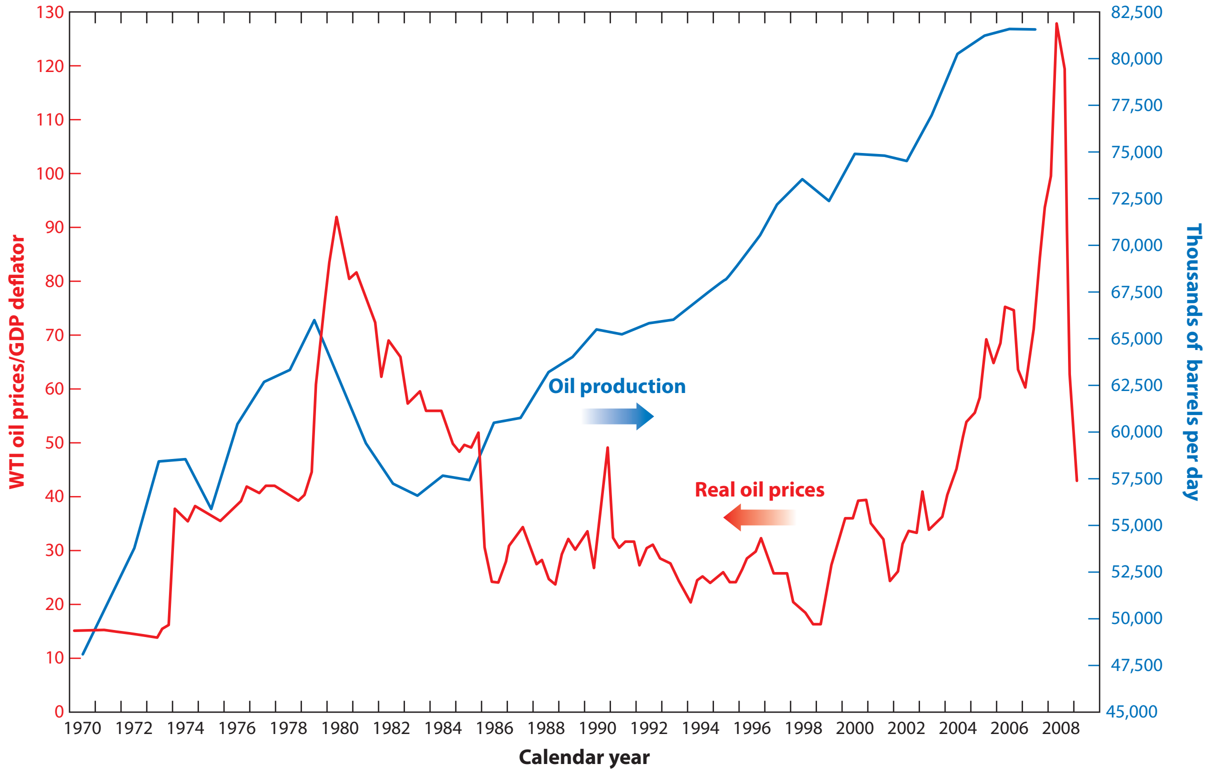

Figure 1

Complex and patterned phenomena produced by the same system. Modified with permission from the *Wall Street Journal*, Bureau of Economic Analysis, BP Statistical Review of World Energy, June 2008. Abbreviations: GDP, gross domestic product; WTI, West Texas Intermediate.

THE AGGREGATION DIAGRAM

To frame the contributions of complexity theory, I rely on the aggregation diagram (**Figure 2**). This diagram captures the fact that a complex systems approach to science begins from the bottom-up (Iwasa et al. 1987). The fundamental units of analysis are agents, not a set of variables (Macy & Willer 2002).[2] The agents are endowed with behaviors, situated in social space, and set loose (Epstein 2014, Miller & Page 2007). The aggregate behavior of the system might be an equilibrium, a pattern, near randomness, or complexity. Think of dropping off a child at camp, arranging said child's gear in a cabin, and then driving off and letting whatever happens happen.

This is shown in the diagram as follows: At a given instance in time (time t in the diagram), a set of micro-level agents (these could be people or organizations) takes actions. These actions are represented in the lower-left-hand corner of the figure. These actions aggregate to produce phenomena at higher levels (denoted by Y^t). I have drawn the diagram with only two levels; a more accurate version would include meso levels (Little 2012). People create groups, groups produce society, and so on.

[2]This primacy of agents (Wellman 2014) differentiates complex systems models from system dynamics models that rely on flows between aggregate state variables (Sterman 2000).

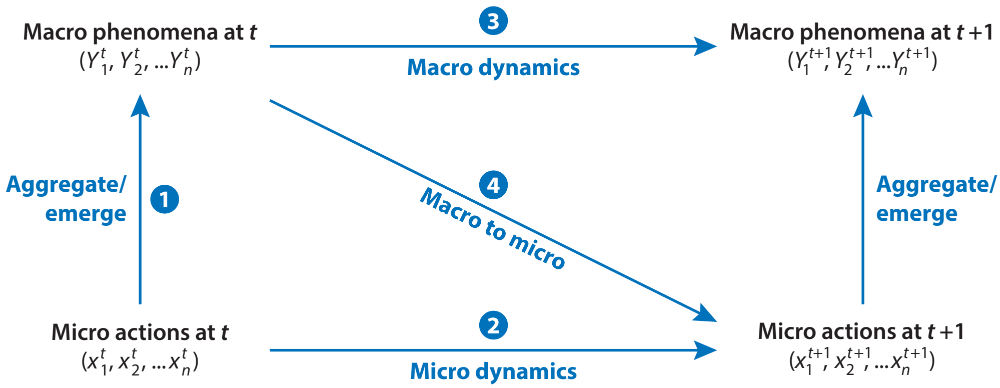

Figure 2

The aggregation diagram.

Erratum

As shown in the diagram, in the next period ($t + 1$), agents again take actions, hence the system is inherently dynamic. The diagram intentionally provides no hints about the assumed micro-level behavior. Within the broad tent of complex systems, one can assume a variety of behaviors, from rational to rule based, to psychologically biased, to culturally driven. The only requirement is that the behavior can be described using either a mathematical or an algorithmic representation. The behaviors will often depend on both the previous actions of others and the macro-level properties of the system. For example, an individual's choice to return to school or to take a new job may depend on the actions of other individuals as well as conditions in the macro economy.

The diagram also makes clear that the macro-level variables can produce their own dynamics. The inclusion of the causal arrow at the top of the diagram distinguishes the aggregation diagram from Coleman's boat (Little 2012). The possibility of distinct macro-to-macro (or meso-to-meso) dynamics rests partly on the existence of self-organized and emergent properties: meso- and macro-level properties that cannot exist at the micro level (Sawyer 2005). These may be system-level properties such as robustness or efficiency, or they may be more specific actions such as guiding a ship to port (Hutchins 1996). Rather than say the whole is more than the sum of its parts, complex systems scholars tend to quote the physicist Philip Anderson (1972), who wrote that "more is different." His phrase captures the fact that the macro and the micro can differ in essence.

The diagram obliges us to try to explain the macro-level properties and dynamics in terms of the micro-level interactions and to think in terms of a population of interacting entities that produce macro-level patterns. It is easier to make the causal linkage in physical systems because the fundamental units of analysis, such as carbon atoms or simple magnets, are often identical. The fundamental units of sociology—people—are unique, which makes understanding aggregation more difficult.

For this reason, the emergence of distinct macro-level patterns, properties, and functionalities is best expressed in physical systems. Water exhibits properties such as wetness and viscosity that the molecules do not possess. These macro-level properties can exhibit their own dynamics, such as the flow of water over rocks. Ideally, these emergence properties and their dynamics are understandable, such as when physicists understand wave formations or when social scientists can explain how flows of pedestrians emerge from micro-level behavior (Helbing 2010), but they need not be. Some emergent phenomena such as consciousness remain mysterious.

As should be clear from the aggregation diagram, a complex systems perspective makes no assumption about the types of allowable micro- and macro-level phenomena. A system could settle into an equilibrium, form patterns, exhibit complexity, or even produce one thing after another with no structure at all. If we can write down a potential function, a function with a maximum (minimum) such that whenever the system is not in equilibrium, the potential increases (decreases) by at least some fixed amount, then we know the system will equilibrate.

However, if a system includes interdependencies, such as when a trade between two parties materially affects another, then a potential function need not exist. This point merits unpacking. At a bazaar (a pure exchange economy), two people trade only if each benefits. Those not party to the trade are unaffected. Thus, any trade increases total utility (happiness), so a potential function exists and trade must eventually stop. The same logic applies to coordination games (Page 1997) and anticoordination games (Page 2001), such as choosing when to go shopping and when to go to the gym. In contrast, in international relations, politics, or business, an alliance may benefit the two parties that form it, but may harm others. Total utility may decrease. We, therefore, have no guarantee that the process will stop because the process of alliance formation is not universally utility increasing. Instead of an equilibrium, the system may produce a pattern or, more likely, complexity.

DYNAMICS

As mentioned above, in a complex system, dynamics occur at both the micro and macro levels. In some cases the macro dynamics are best explained in reference to the micro-level behaviors, but in other cases they are not (Shalizi et al. 2004). These micro-level actions are often rule based and adaptive. In a complex systems model of dating or mate selection, the agents may follow rules that respond to local and global information. Those rules would produce patterns. As agents learn those patterns, they adjust their behavior accordingly. The result may be an equilibrium or, more often, yet another pattern. Over time, the system might well produce a sequence of nonstationarity patterns, i.e., complexity (LeBaron 2001).

This idea of patterns building upon patterns contrasts with the dynamic, stochastic, general, equilibrium (DSGE) models predominant in macroeconomics. In these models, the economy is assumed to be in an equilibrium. Each agent solves for that equilibrium and assumes that everyone else does as well. Dynamics arise due to shocks and to the responses on the part of agents to achieve the new equilibrium. These models can be seen as a sequence of paths to equilibrium. In a complex systems model, the agents would not possess such teleological prowess. They would base their behavior on past patterns. To the extent they might anticipate the future, they need not do so perfectly.

If one knows the rules that agents follow, one can, in some cases, prove that there exists an equilibrium by appealing to a fixed-point theorem. However, the existence of an equilibrium does not guarantee that a collection of learning agents will attain that equilibrium (Epstein 2007). A mantra among complex systems thinkers is, "If you didn't grow it, you didn't show it." Further, a system may have multiple equilibria; which, if any, is attained can depend on the initial state of the system, how individuals learn (Golman & Page 2009), or how they are connected (Jackson 2010).

To be more precise, complex systems models exhibit sensitivity to initial conditions and path dependence. This insight has implications for the practice of sociological research. Consider the Music Lab experiments by Salganik et al. (2006). In these experiments, individuals could sample and then download music. In one treatment, subjects could not see what music others had downloaded. In a second treatment, they could, and as a result, people downloaded the same music as others—a positive feedback. The positive feedbacks produce a longer-tailed distribution. More relevant to this discussion, they also produce path dependence: Final outcomes are contingent on the outcomes that occur along the way (Page 2006).

Path dependence should be of interest to sociologists for both normative and positive reasons. In the Music Lab experiments, the most downloaded songs may have benefitted as much (or more) from luck as from skill. Similarly, an academic paper with 500 citations may owe its success as much to positive feedbacks as to quality, so may sales of any product in which consumers based their choices on the actions of others (Denrell & Liu 2012).

Path dependence also has implications for positive analysis. The contingency of outcomes creates a problem for what might be called end-of-process regression analysis. Ignoring dynamics and considering only end-of-process data may lead to incorrect inferences. Some of the features of songs or papers that correlate with more downloads may have causal effects, but other features might be idiosyncratic. A complexity perspective forces one to explore the extent of outcome contingency, of how much outcomes depend on the dynamic behavior in the system.

Positive feedbacks are just one of many possible features of a system that can influence dynamics and outcomes. Nontrivial dynamics arise for a variety of reasons. They can arise from population-level effects. For example, in a pairing model initial pairings may be nonrepresentative of the population. Those unmatched items will not be representative of the initial population, changing the environment for the remaining individuals (Xie et al. 2015).

Interesting dynamics can also arise through niche construction and destruction (Holland 2014). In game theory, agent-based models of repeated play of the prisoners' dilemma produces epochs of cooperation as agents develop ways of identifying other cooperators. This produces a cooperative niche in the space of behavior rules (Axelrod 1997). Such niches may survive for many periods until an agent figures out how to exploit the cooperators, destroying the niche and producing an epoch of defection.

AGGREGATION

The second set of contributions concerns aggregation. Complex systems often produce non-intuitive, even paradoxical, aggregate results. Scholars have developed a variety of models that help unpack some of the mysteries of aggregation. These models demonstrate how simple rules can produce complexity (Wolfram 2002) and how sophisticated micro-level rules can produce aggregate-level statistical equilibria.

This latter phenomenon occurs in the El Farol Bar problem (Arthur 1994). In this model, each agent must decide whether to attend a weekly event or stay home. Each agent would prefer to attend, but not if too many others do. Over time, agents learn rules for whether to attend on the basis of the sequence of past attendance figures. On average, the optimal number of people attend. A time series of weekly attendance looks like a random variation near the optimal value, even though at the micro level it consists of an elaborate ecology of evolving rules. This disconnect between micro and macro can also be seen in real financial markets in which brilliant individuals and teams formulate elaborate strategies but the net result is a pattern of prices that is nearly random (LeBaron 2001).

To describe some of what has been learned about aggregation in complex systems, I first discuss how the attributes of complex systems contribute to the creation of complexity through aggregation. I then describe how changing levels of diversity, connectedness, interdependence, and the rate of learning result in more or less complexity at the aggregate level. Last, I turn to the question of how aggregation can produce self-organization and emergence.

Aggregation and Complexity: Tuning the Dials

In exploring aggregation, many "what if" questions arise. What if we increase this parameter? What if we make agents more adaptive, more diverse, or less connected? To answer these questions, traditional methods must be expanded. In an equilibrium model, one subjects the model to comparative statics analysis by changing parameters and evaluating the change in the equilibrium: What happens to economic growth if we lower taxes, to crime if we increase punishment, to voter turnout if candidates go negative, and so on? Complex systems models do not necessarily produce equilibria, so a standard comparative statics analysis is often not possible. One cannot compare equilibria if equilibria do not exist.

Equally problematic, one cannot just expand the classes of outcomes and talk about comparative dynamics, comparative patterns, and comparative randomness because changing a variable may change the class of outcome. A system in equilibrium may become complex when a parameter changes. Such a class of outcome changes are more common than one might think. From the study of chaos and nonlinear dynamics, we know that systems can move from having a single equilibrium to having two equilibria (a bifurcation) or from an equilibrium state to a periodic orbit or to randomness (a phase transition).

These changes in the class of outcome can occur abruptly at critical values, or tipping points. Tipping points can be contextual (i.e., small changes in a parameter that affect the behavior of the system) or direct (i.e., actions that cause the system to head toward a particular state or class of outcomes) (Lamberson & Page 2012). A contextual tip occurs when a population becomes sufficiently irate for an uprising to be likely. A direct tip is the action that sets the uprising in motion. After a contextual tip occurs, the change is unavoidable. Ascribing that unavoidable change to a particular action may miss the point (Omerod 2012, Watts 2011).

That said, it is not as though changes in the core attributes of complex systems—diversity, connectedness, interdependence, and rate of adaptation—produce arbitrary outcomes, or that complex systems are impossible to understand. To the contrary, many general patterns exist. Most notably, complexity does not occur at the extremes. Complexity occurs in an in-between region (Miller & Page 2007).

To see why this is the case, imagine four dials in which each dial represents one attribute of a complex system, and then imagine tuning those dials. Let us first increase connectedness, holding the other attributes constant. Imagine a collection of banks that are not at all connected, such as was the case perhaps a hundred years ago. If a bank fails, there exists no systems-level effect of note. The bank just fails. Now let us increase connectivity and assume that banks borrow and loan money to banks to which they are connected. Now, if a bank fails, it will no longer be able to pay off the money it borrowed from other banks. This could cause those banks to fail and cause a cascade. Thus, increasing connectivity leads to an increased likelihood of large events and also increased complexity—as each bank operates within its niche.

Now, let us increase connectivity even more. If every bank is connected to every other bank, a single bank failure will be absorbed by the system. Moreover, each bank operates in an identical context—each is connected to every other. The system is not complex.

These effects are evident in **Figure 3*a*,*b***. It shows the probability of large bank failures as a function of the degree of interconnectedness in a model of financial interdependence written by Elliot et al. (2014). As interconnectedness increases, failures become more likely, but as connectedness increases, the likelihood drops off.

Relatedly, a variety of models consider agents playing fixed strategies in the game prisoners' dilemma, in which better strategies grow faster in the population (replicator dynamics). When these agents are placed in a one-dimensional network, nothing interesting happens. In models

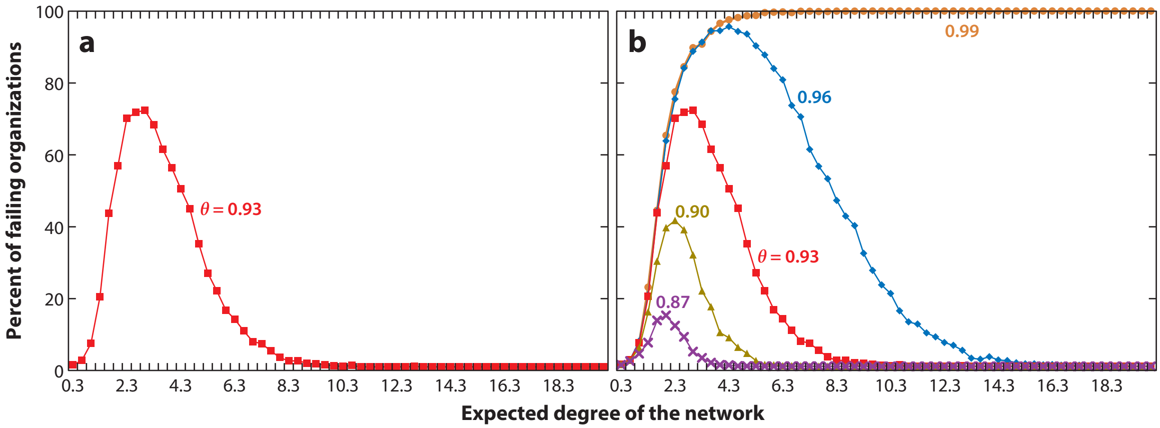

Figure 3

Bank failures as a function of interdependence. (*a*) Effects of diversification: the percentage of organizations failing as a function of expected degree for $\theta = 0.93$ ($c = 0.5$, $n = 100$). (*b*) Effects of diversification for several failure thresholds: percentage of organizations failing as a function of expected degree for various levels of θ ($c = 0.5$, $n = 100$). Figure and caption reproduced with permission from Elliot et al. 2014.

with three or more dimensions, one strategy comes to dominate; however, in two dimensions the resulting dynamics are complex. This result, shown in computer simulations, has been replicated biologically (Kerr et al. 2002). Again, complexity happens in the in-between region.

Next, we can tune the diversity dial. If each agent is identical, the system will not be complex. As we increase diversity, and by this I mean diversity of behavior, then the system may well become complex. But if each agent behaves in a unique way, then the micro-level behavior will be random, so that at the system level a law of large numbers logic will apply and the result will be a statistical equilibrium. At the micro level, the system may be complex in that it will be hard to define, but it will not produce complexity.

The third dial corresponds to interdependence. The effects of changing interdependence can perhaps best be seen in the context of social interaction models (Durlauf & Ioannides 2010). Consider a set of agents arranged on a rectangular grid who can choose one of two actions, up (+1) and down (−1). In an application of the model, these actions might represent choosing whether or not to adopt a given behavior, e.g., working hard in school, doing drugs, or they might represent a choice among two technologies. To start the model, each agent begins with an initial action—either up or down. Assume the two actions are equally likely.

In a social interaction model, each agent has a neighborhood of influencers. For the moment think of these either as the four neighbors to the north, south, east, and west (the Ising model) or as the entire grid (the Curie-Weiss model). These influencers can cause an agent to change its action. Assume the following behavioral rule: With some probability s (for social pressure) an agent chooses the action that matches the action of a majority of its neighbors. With probability $(1 - s)$, it chooses randomly. If s is small, then the system will have, on average, close to half of the agents choosing each action. As the amount of social pressure (s) increases, the system tips so that rather than having one equilibria, the system has two: one with a large number of agents choosing up and one with a large number choosing down. At this critical value, the model produces complex dynamics and patterns. Once again, complexity occurs in the intermediate range of values. Low social pressure results in no complexity (equal numbers of each action) and high social pressure leads to homogeneity. In between lies complexity.

Finally, and briefly as the general logic should now be clear, consider changing the rate of adaption or learning. If agents follow fixed rules, then some system can produce complex dynamics (Wolfram 2002), but only for well-chosen sets of rules. If the adaptivity dial is set infinitely high, then it is "as if" the agents can rationally expect what will happen, and the agents will always take the optimal action. The result will be similar to economic models that assume rational expectations: an equilibrium. In the middle, when agents adapt and learn at more moderate rates, they produce aggregate patterns. They then learn those patterns and create new patterns.

Aggregation: Self-Organization and Emergence

Aggregation within complex systems can produce patterns, functionalities, and properties that do not or cannot exist at the micro level. This is often called emergence. Consciousness in the brain, culture in a society, and (as mentioned) the wetness of water all exist at the macro level but cannot be maintained by a single constituent part. One molecule of water cannot be wet.

Emergence can happen at multiple levels. Information embedded in DNA and in the environment combines to produce differentiated cells. In turn, combinations of these cells form organs and organ systems. Each level has functionalities that do not exist at the level below it. A similar characterization applies to people, groups, and societies. Each aggregation produces entities that have different capacities and can exhibit distinct sets of properties.

Here I make a distinction between emergence and self-organization. By the former I mean properties and functions, and by the latter I mean patterns or forms. The flocking of birds, segregation by race, and the rhythmic clapping of individuals in an audience are self-organized patterns. Consciousness, culture, and collective cognition are examples of emergence. In each case, a functionality exists at a macro level that did not exist at the micro level (Hutchins 1996).

Self-organization and micro-level rules. Within sociology, an understanding of the concept of self-organization is particularly relevant for explaining and evaluating at macro-level patterns. As known from Schelling's (1971) segregation model, patterns that emerge need not align with micro-level preferences and behaviors. In his model, Schelling (1971) assumed a homogeneous, threshold-based behavior rule for residential relocation—people relocated if less than a fixed fraction of their neighbors belonged to their same racial group or income class. He found that even with a (relatively) tolerant threshold, the system self-organizes into segregated communities.

Here we have a disconnect between the micro and the macro. But that occurs within an extremely stylized model that makes no attempts at realism. Schelling's model assumes a homogeneous threshold rule and two types of agents who exist in equal proportions. He makes no effort to calibrate his model.

Bruch & Mare (2006) consider a more plausible linear behavioral rule with a probabilistic component (in effect a random utility model). They also calibrate their model to preferences derived from the Detroit Area Study. The original analysis by Bruch & Mare revealed far less segregation than Schelling's model did. Bruch & Mare's results were challenged by van de Rijt et al. (2009), who found a coding error, identified parameters under which segregation is even more extreme under linear rules with errors, and showed that if the random component is sufficiently large, segregation decreases.

Subsequent analysis by Bruch & Mare (2012) shows how the extent of segregation depends on the slope of the linear rules and on the size of the error term. Relatedly, Xie & Zhou (2012) show how segregation is attenuated by individual-level heterogeneity because more tolerant individuals hold racially mixed neighborhoods together.

My reasons for this brief and incomplete review are twofold: to classify Schelling's model and its predecessors as examples of self-organization and to highlight the robustness of the main result.

One can change the threshold rule to a linear rule, one can ramp up or down the extent of randomness, and one can make the agents heterogeneous, and for a rather large range of parameters, the system will produce some degree of segregation (Bruch 2014). This occurs because relocation produces two types of positive feedbacks. First, it increases the probability that individuals in the old neighborhood of the same type as the individual who moved will want to move. Second, it increases the probability that individuals of the opposite type in the new neighborhood of the individual who moved will now want to relocate. These positive feedbacks create an amplification that manifests itself in segregation beyond what a linear appraisal of preferences would suggest.

Emergence Versus Self-Organization

Racial segregation is self-organization because it is a pattern and not a new function, such as consciousness, which is emergent. The distinction between self-organization and emergence is blurrier than what might be thought because patterns can also have functionalities. The small-world property (Watts 2004) of social networks can be considered emergent. Small-world networks have a low average path length, colloquially known as the six degrees of separation phenomenon. People do not build networks with the goal of connecting to the musician Wynton Marsalis in five steps, but their networks allow them to do so, and that feature occurs without anyone trying to make it so.

Emergence in social systems takes many forms. Here I focus on two forms: systemic robustness and group-level phenomena. Many complex systems, e.g., our brain, our bodies, ecosystems, economies, political systems (Bednar 2008), and the Internet, among others, exhibit remarkable robustness. This occurs even though the primary incentives of the agents in each of these systems are not systemic robustness, but individual success. This could be chalked up to intelligent system design. But few social systems other than political systems have designers, and even those designers have limited control.[3] Ecosystems have no blueprint, nor does the brain.

In some systems, robustness emerges because the parts adapt. Agents possess or evolve repertoires that enable them to respond to internal dynamics and external stimuli (Jen 2005). If each type of agent remains viable, then so too does the system. Adaptive parts imply a robust system. The behavioral plasticity of the parts finds echo in assemblage theory, in which components can move between systems (DeLanda 2006). Thus, assemblages can perform a variety of functions as opposed to one specific function. Note that the aggregate system changes as a result of the adaptations, so it should not be seen as a fixed entity. That is why complexity scientists distinguish robustness from stability. Stable systems return to the previous equilibria. Robust systems keep adapting, but perhaps in different ways.

The flip side of robustness is, of course, systemic risk (see Centeno et al. 2015). Just as robustness can be an emergent property, so too can the potential for large events. As mentioned above, complex systems often produce an abundance of both very small event sizes and, on occasion, large event sizes. The lack of a central planner should be a cause for concern. A complex system is not guaranteed robust.

Next, consider emergent group-level functionalities. In social systems, groups can be considered an aggregate, and properties of groups can differ from and exceed those of individuals (Sawyer 2005). Just as a brain can perform operations that far exceed the capabilities of any one neuron, so too can groups perform tasks that no one person could achieve. Hutchins (1996) provides an

[3]The literature on design focuses on creating systems with efficient equilibria and not robust outcomes (Page 2012, Reiter 1977).

account of how no single person guides a ship into port. That ability is a property of a network of individuals who communicate. The same might be said of the ability to run any organization or of the functioning of society writ large. No one runs the economy. Its functioning or failure is an emergent property of individual actions aggregating.

Emergent functionalities of groups occur at multiple levels of aggregation in a variety of contexts. An otherworldly performance by a jazz trio, a brilliant play by a sports team, and a breathtaking staging of Shakespeare are emergent. They are not properties of the parts. Their existence depends on the connections between individuals and on the communication that occurs along those connections (Sawyer 2005).

DISTRIBUTIONS

Within a complex system, the core entities are often diverse. Thus, an analysis of a complex system focuses both on characterizing that diversity and on considering its implications. I first take up distributions and then discuss the implications of the levels of diversity.

Even if all the agents start out identical, they will often learn to take different actions, so diversity levels will be endogenous. Further, because complex systems often produce paths of outcomes and not equilibria, one must often compare distributions of outcomes as opposed to equilibria when making normative and positive assessments of causal effects. Most importantly, the interactions in a complex system imply that distributions are often not normal (Weaver 1948). Therefore, to understand a complex system, one needs to know more than just means and variances. One needs to also analyze the full distribution, whether it be of behaviors, outcomes, or durations.

In many cases, these distributions are long tailed. They have more small events and more large events than would occur for a normally distributed variable. One explanation for the preponderance of long tails lies in the fact that complex systems include both positive and negative feedbacks. Positive feedbacks produce longer tails for an obvious reason: More begets more. Negative feedbacks produce long tails for a different reason. Keep in mind that long tails are relative to overall variation. By reducing fluctuations, negative feedbacks lower variance and increase the relative lengths of tails. Imagine a system in which either negative or positive feedbacks predominate. When the former predominate, the system produces large events. When the latter predominate, the system produces many more small events. In each case, the distributions of outcomes are (relatively) long tailed.

Whereas normal distributions arise because of a central limit theorem logic, long-tailed distributions can arise for any number of reasons. Here, I describe four processes that can produce long tails. The first process is multiplicative effects. If random shocks are multiplied together rather than added, then the resulting distribution will be lognormal. Many complex systems models include replicator dynamics, in which the number of types in a population grows proportional to that type's relative fitness. This is an example of a multiplicative effect and thus will also produce a long tail.[4]

Although a lognormal distribution has a longer tail than normal, many complex systems produce an even longer-tailed distribution known as a power law.[5] Power laws can be produced by several models, three of which seem most germane to sociology (Newman 2005).

The first model, preferential attachment, captures the Matthew effect (Merton 1968), in which the rich get richer rendering the poor relatively poorer. In a preferential attachment model, new

[4]Nearly all ecosystems exhibit long-tailed species abundance distributions (Hubbell 2001).

[5]In a power-law distribution, the frequency of an event of size *x* can be written as *x* to a negative power.

entities arrive sequentially. Imagine that the first arrival creates a type. For all other arrivals assume that with a probability close to 1, a new arrival chooses an existing type and with the remaining small probability, it creates a new type. If an individual's probability of choosing a particular type is proportional to that type's relative population (more begets more), then the resulting distribution will be a power law.[6]

As an example, if the probability that a person moves to a city or works for a firm is proportional to the city's population or the firm's size, then city sizes and firm sizes will be distributed according to a power law. To a first approximation, both are power-law distributed, as are academic paper citation counts, website links, industries within cities, and book sales (Bettencourt & West 2010, Newman 2003).[7] Here we see more evidence of the transdisciplinary promise of complexity theory: Cities, the Internet, and citations are all covered by the same model. The nonelephants share a feature.

The second model that produces power laws relies on a phenomenon called self-organized criticality (Bak 1996, Hidalgo et al. 2014). Imagine dropping grains of sand on a table until a pile forms. The pile is a critical state; i.e., when you add a grain of sand, there exists a possibility of an avalanche, a large event. Some models suggest that traffic may also evolve to a critical state (Nagel & Paczuski 1995). Drivers may choose to enter freeways and, once on the freeway, to leave gaps between adjacent cars so that the entire system is poised to produce large events. Some scholars argue that the same logic might apply to financial networks, international political agreements, electric power grids, or even the social fabric of diverse societies (Centeno et al. 2015, Goldin & Mariathasan 2014). These are intuitions worth exploring. Complex systems models provide the tools to do so.

Notice that preferential attachment produces power laws in populations, e.g., cities, firms, number of citations, and that self-organized criticality produces a power-law distribution in the magnitude of events, e.g., avalanches, market collapses, traffic jams, and war deaths. The third model, random-walk return times, explains power-law distributions in duration. Recall that in a complex systems model, we often begin with agents that are assigned types. Imagine that the number of agents of each type follows a random walk; that is, in each period it is as likely to increase by 1 as it is to decrease by 1. It can then be shown that the distribution of duration over types will be a power law (Newman 2005).

The preponderance of long-tailed distributions in complex systems is surprising. The standard assumption in most cases is a normal distribution. Any deviations from normal would have required explanation. Complex systems models imply that, in many cases, long-tailed distributions should be the outcome; thus, any size distribution that is not long tailed must be explained. In the sociological context, one might then be forced to demonstrate whether an approximately normal distribution should occur (by central limit theorem thinking) or whether it is the result of structural constraints imposed by society that prevents a long tail.

DIVERSITY AND STRUCTURE

The final set of findings from the complexity sciences relevant to sociology concerns the functional properties of diversity and structure. Diversity (distinct types) and variation (distinctions within a type) play a larger role in sociology than in economics and political science, although not to the

[6]Recall from the discussion of the Music Lab experiments that even if we can predict the distribution of outcomes, path dependence implies that we will not be able to predict with much accuracy where a given type will lie in that distribution.

[7]See Shalizi (2013) for a critique of the fit of these distributions.

extent that they do in ecology and anthropology, in which they are fundamental. It would not be inaccurate to say that the fundamental question of ecology is, Why do we see the diversity we do?

Sociologists capture diversity both categorically and continuously. Examples of the former include distinctions by socioeconomic class, race, ethnicity, or gender, and examples of the latter include levels of tolerance (Xie & Zhou 2012), ages, or levels of risk aversion. Sociology and demography, in particular, place much more emphasis on diversity than do economics and political science. What complex systems can add to this accounting for and appreciation of difference is a set of insights into the functional contributions of some forms of diversity.

Complex systems research identifies several such contributions of diverse behaviors. They can contribute to the robustness or instability of a system, signal phase transitions, drive innovation by enabling faster systems-level learning, and underpin the phenomenon of collective intelligence (Page 2010). The first functional role of diversity, the contribution to robustness and instability, is perhaps the best understood. Ashby's law of variety explains how diversity enhances robustness. A system with more types can respond to more types of disturbances. You cannot chop down a tree with a screwdriver. You cannot salve a wound with a beggartick.

The feedbacks in a system determine whether diversity can increase robustness. If the feedbacks are either negative or only weakly positive, then diversity will increase robustness, because the most extreme move first and diffuse the dynamics. If the feedbacks are strongly positive, then heterogeneity can produce instability, as one person takes an action, causing another to do so, which in turn causes more to act (as occurs in models of riots, Granovetter 1978). Note that in the racial segregation models described above, even though positive feedbacks exist, they are sufficiently weak such that adding diversity reduces segregation (Xie & Zhou 2012).

The second role of diversity, how increases in variation can signal tips or phase transitions (Scheffer et al. 2009), relies on the following logic: When a system is about to undergo a change, say, from one equilibrium to another, the previous equilibrium becomes less powerful. This is modeled on a landscape as a fitness peak that is less steep. Selection then has less force, flattening the peak. This flattening enables more variation. Variation portends the search for a new peak, i.e., a change to the system.

Diversity can also be linked to innovation. For biological systems Fisher's fundamental theorem states that the rate of adaptation in an evolutionary system is proportional to the amount of variation. The logic here is again straightforward: Greater variety implies more high-fitness and more low-fitness individuals. The former increase the rate of adaptation. In social contexts, diversity, in the form of distinct representations and problem-solving tools, can speed learning by reducing the number of common local optima (Page 2007). When one person gets stuck on a problem, another is likely to find a way out.

Finally, diversity in both sensory apparatuses and models contributes to collective intelligence. Recall that in a complex system individuals are situated locally; they likely have distinct information and experiences. This diversity of situations begets diversity in how the agents interpret their worlds. Diversity therefore begets diversity. The result of that diversity is that the collective can be more accurate than any member in it. It can be shown that a diverse crowd is necessarily more accurate than the average of its members (Page 2007).

Each of these properties, robustness, innovativeness, and collective intelligence, is emergent; thus, they could have been included in the previous section. However, the extent to which they emerge depends on the diversity maintained by the system. And, as already mentioned, too much diversity can prevent structure from emerging. The dial can be turned too far. Thus, complex systems often balance exploration of the new with exploitation of the successful so that they have neither too little nor too much diversity.

Diversity also contributes to dynamics. As a thought experiment, imagine a system composed of a collection of entities of identical size. Suppose that each entity has a value or weight that fluctuates within some fixed percentage of its current value. A time series of the value would be normally distributed with relatively modest fluctuations. Now suppose that the entities have values that are distributed according to a power law, and make the same assumption about entity-level fluctuations. The aggregates exhibit much greater volatility because the averaging will be uneven. The fluctuations of large entities will have substantial effects. This thought experiment may well explain the Great Moderation, the stability of the US economy that began in the mid-1980s and continued for nearly two decades. During that time, the distribution of firm sizes was less long tailed owing to a transition from large manufacturers, such as General Motors, to large retailers, such as Walmart (Carvalho & Gabaix 2013).

Just as diversity plays a functional role in complex systems, so too does structure. A substantial literature in sociology demonstrates how an individual's position within society contributes to power, influence, and social capital (Burt 1992, 2005). Complex systems models often analyze the effect of the entire structure (Omerod 2012). For example, Golub & Jackson (2012) find that the speed of learning depends on the extent of homophily. In their influential paper, Centola & Macy (2007) find that long ties, which were intuitively thought to speed diffusion, can have the opposite effect when the innovations require reinforcement. In a similar vein, Lloyd-Smith et al. (2005) show that superspreaders, or highly connected people, can cause a disease to spread much more quickly than predicted by models that assume random mixing.

Diversity and structure also interact. The recent work of Padgett & Powell (2012) describes in detail the interdependent functional aspects of network structure and diversity. The authors explain how diverse actors construct networks of relations and how these relationships create niches. The niches in turn create opportunities for new types of agents. To oversimplify a rich, empirically embedded collection of models and case studies, Padgett & Powell demonstrate how people create networks and networks create people. Diversity can therefore be understood in the context of structure and structure can be understood partly as a result of diversity.

The crux of Padgett & Powell's logic is that diverse agents embedded in a structure can produce functional substructures, namely autocatalytic sets of behaviors. These sets can loosely be thought of as virtuous circles of activities, which can be seen as emergent functionalities. Autocatalytic interactions exist across multiple networks—social, political, and economic. These linked networks lead to innovation by making beneficial connections more likely, as actors linked in one domain tend to have common interests in related domains.

DISCUSSION

In this article, I have described how complex systems research provides models, ideas, and insights that have direct bearing on topics of interest to mainstream sociologists and how sociology might benefit from a deeper engagement with research on complexity. Rather than reiterate points made in the main body of the article, I conclude with three observations.

First, the relationship between sociology and complexity is path dependent. Complexity science did not exist when sociology was formed. This means that the core ideas from complexity science can occupy a central place in sociology only if sociologists discovered them first. In many cases, such as in paradoxes of aggregation and the importance of network structure, that is true. Imagine, though, that the timing had been different, that complexity science had been explicated in full (this, by the way, has yet to occur) prior to the creation of sociology. Complex systems might then be considered foundational to the study of sociology given the resonance between the two disciplines.

Second, the availability of big data means that sociologists can at times link complex systems models to data. There now exist network-level data and distributional data that would have been impractical to gather even a decade ago. Third, social interactions that take place virtually can be monitored. One can also perform experiments over the Internet that resemble or even mimic real-world interactions (see Watts 2011 for an in-depth review). This observation links to the previous one: Big data is increasingly part of the real world. Activities such as shopping, dating, voicing opinions, exploring ideas, and having fun used to take place primarily in the physical realm. Big data was impossible to gather. Because many of these activities now take place virtually, they can be captured to create big data.

None of what has been described should imply that the absorption of complexity models and ideas will be easy. Empirically based complex systems models require new methodological approaches. First, their micro-level assumptions on behavior must fit what the individuals do. This might seem like fitting assumptions, and to some extent it is, but it is more proper to think of this as estimating the lower level of the aggregation diagram. Second, the macro-level predictions of the model must also be consistent with the empirically estimated micro-level behaviors. Finally, the macro-level predictions must be supported, i.e., not rejected by the data.

In other words, (*a*) the micro-level assumptions in the model must fit micro-level data, (*b*) the micro-level behaviors must produce the macro-level properties (here one often uses an agent-based model), and (*c*) the macro-level properties that emerge in the model must fit macro-level data. Those are high hurdles. Those sociologists who have succeeded at all three tasks (see Bruch & Mare 2006, 2012; Centola & Macy 2007) have been, to no one's surprise, acknowledged by the discipline.

Finally, the science of complexity, though far from complete, has on offer a large and growing set of useful models and insights. They come from a variety of disciplines and cover everything, from the role of network structure to the functional values of diversity. The idea that models from physics, physiology, and ecology could inform sociology (May et al. 2008), though not controversial, points toward a path not often followed. But it should be followed. By engaging and applying a richer set of models, sociologists expand the set of lenses through which they see the social world and improve the discipline. In sum, sociologists should listen. The nonelephants have something to say.

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