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MASSES AND MASS-TO-LIGHT RATIOS OF GALAXIES[1]

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1 INTRODUCTION

Is there more to a galaxy than meets the eye (or can be seen on a photograph)? Many decades ago, Zwicky (1933) and Smith (1936) showed that if the Virgo cluster of galaxies is bound, the total mass must considerably exceed the sum of the masses of the individual member galaxies; i.e. there appeared to be "missing mass" in the cluster. As more data became available, the discrepancy persisted between masses of individual galaxies determined from optical rotation curves and the larger average galaxy mass needed to bind groups and clusters (e.g. Neyman, Page & Scott 1961).

Recently, however, new information has pointed toward larger total masses for individual galaxies, thus decreasing the traditional discrepancy between various methods of mass measurement. Arguing that thin self-gravitating stellar disks are unstable against bar-like modes, Ostriker & Peebles (1973) suggested that the disks of normal spiral galaxies must be imbedded in optically undetected, stabilizing massive halos. Ostriker, Peebles & Yahil (1974) and Einasto, Kaasik & Saar (1974) collected observational evidence in support of the existence of such halos (although Burbidge 1975 used similar data to reach the opposite conclusion). At nearly the same time, high-resolution 21-cm observations of nearby galaxies were showing that H I often extends well beyond the optical

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boundaries of galaxies and that rotation velocities are constant at large galactocentric distances. Simply interpreted, these measurements implied the presence of substantial mass outside the optically visible dimensions of galaxies.

In this review, then, we are especially concerned with the current status of the "missing mass" problem: has it been resolved by new data, or does it linger on essentially unchanged in magnitude? To answer this question we rely on mass-to-light ratios as our primary tool, since they provide a direct intercomparison of galaxy masses measured for many different samples using varied techniques. We further assume that all Doppler shifts are caused by actual velocities of recession, though this view is not universally held (e.g. Arp 1974). Finally, in discussing the possible presence of invisible mass in galaxies, we do not wish to assume any model for its structure or spatial distribution. For this reason, we choose the neutral term "massive envelope" to describe the unseen mass. Although "massive halo" is often used in a similar context, it connotes a more or less smooth and spherical mass distribution, a possibly misleading notion since the present data contain little actual information on the spatial structure of any extended components.

In comparing mass-to-light ratios from different sources, we use a standard system of M/L_B. We define total magnitudes of galaxies on the B_T system of the *Second Reference Catalogue of Bright Galaxies* (de Vaucouleurs, de Vaucouleurs & Corwin 1976, hereafter RC2). We have corrected the B_T magnitudes for internal extinction using the precepts of the RC2, which are nearly independent of type. The resultant luminosities are "face-on" values only; an additional 10–20% increase would be necessary to produce totally absorption-free magnitudes, which, strictly speaking, are those with which the local M/L_B for the solar neighborhood should be compared (Section 2.1). The galactic extinction assumed is $A_B = 0.133(\csc|b|-1)$, close to Sandage's (1973) formulation and in reasonable agreement with the more recent results of Burstein & Heiles (1978). The value of the solar absolute magnitude is here taken to be $+5.48$ in B (Allen 1973) or $+5.37$ in the photographic system used by Holmberg (Stebbins & Kron 1957). Finally, $H_0 = 50$ km s^{-1} Mpc^{-1} is used consistently throughout.

All values of M/L_B used in this paper are corrected to this standard system. Failure to adopt a standard system of M/L_B can easily lead to errors of a factor of two or three in comparisons among mass-to-light ratios from various authors. The standard system adopted here is sensitive to the adopted magnitude and extinction corrections. The resulting uncertainties in the overall scale are ± 30–40%.

2 THE MILKY WAY

Because of the sun's location in the central plane of the galactic disk, the large-scale structure and global mass of the Milky Way are more difficult to determine than those of a nearby external galaxy. However, our position within our own galaxy gives us a bird's eye view of the mass in the solar neighborhood plus a chance to study the dynamics and mass distributions of various subpopulations in the Milky Way.

2.1 *The Solar Neighborhood, a Benchmark in M/L*

Because most of the mass resides in intrinsically faint stars, the stellar mass density can be directly determined only in the immediate neighborhood of the sun. In practice this is accomplished by combining the luminosity function, $\Phi(M)$, with the mass-luminosity relationship for each stellar group to find the stellar density, ρ_s. The faint-star luminosity function now appears to be well determined for $M_V \lesssim +15$ (Wielen 1974, Luyten 1968, 1974). From Gliese's (1969) data, Wielen derives $\rho_s = 0.046\ M_\odot\ \mathrm{pc}^{-3}$ while Luyten (1968) gives $\rho_s = 0.064\ M_\odot\ \mathrm{pc}^{-3}$.

The greatest uncertainty in ρ_s arises from the difficulty of properly applying the mass-luminosity relationship to the observed sample. The mass-luminosity relation (Veeder 1974) and observational data (van de Kamp 1971) are most reliable for main sequence stars, which Wielen finds amount to $0.038\ M_\odot\ \mathrm{pc}^{-3}$. Sources of error include confusion between low-mass stars and more massive, cooled degenerate stars (e.g. Hintzen & Strittmatter 1974) and the interpretation of the drop in $\Phi(M)$ for $M_V \gtrsim +15$. This could be due to a real absence of very low-mass objects. On the other hand, stars with $M \lesssim 0.08\ M_\odot$ are not able to support stable H burning (Graboske & Grossman 1971, Straka 1971) and so might cool sufficiently rapidly to produce an apparent deficiency of low-luminosity stars (Kumar 1969, Greenstein, Neugebauer & Becklin 1970, Hoxie 1970). Joeveer & Einasto (1976) have estimated that this effect requires increasing the contribution of low-mass stars by about $0.02\ M_\odot\ \mathrm{pc}^{-3}$.

The other major mass contribution resides in white dwarfs. Luyten (1975) finds $n_{\mathrm{WD}} \simeq 0.006\ \mathrm{pc}^{-3}$, while Sion & Liebert (1977) obtain $n_{\mathrm{WD}} \gtrsim 0.01\ \mathrm{pc}^{-3}$. For a mean white dwarf mass of $0.7\ M_\odot$ (Wegner 1974, Greenstein et al. 1977), $\rho_{\mathrm{WD}} \gtrsim 0.004$–$0.007\ M_\odot\ \mathrm{pc}^{-3}$, compared to the theoretical prediction of 0.012–$0.03\ M_\odot\ \mathrm{pc}^{-3}$ (Hills 1978), which depends on the age of the disk (Hills took 1.2×10^{10} yr). Hills also shows that the mass in neutron stars is probably negligible.

The stellar mass density near the sun thus most likely lies in the range $0.05 < \rho_s < 0.09\ M_\odot\ \mathrm{pc}^{-3}$. To this must be added the mass in insterstellar matter, which from Savage et al.'s (1977) measurement of the mean density of hydrogen is $\rho_{\mathrm{ISM}} \simeq 0.03\ M_\odot\ \mathrm{pc}^{-3}$. The total density, ρ, then lies between 0.08 and 0.12 $M_\odot\ \mathrm{pc}^{-3}$, consistent with the estimate of $0.09 \pm 0.02\ M_\odot\ \mathrm{pc}^{-3}$ found by Joeveer & Einasto (1976).

The local mass density can also be measured by observing the z density and velocity dispersion for a homogeneous stellar population (Oort 1965). Although straightforward in principle, the accurate measurement of the galactic acceleration gradient perpendicular to the plane has proved elusive. Different determinations are in conflict with each other and in some cases yield nonphysical results (Dessureau & Upgren 1975, Joeveer & Einasto 1976, King 1977). The most likely value of the density found by this method, $\rho_{\mathrm{dyn}} \approx 0.14\ M_\odot\ \mathrm{pc}^{-3}$ (Jones 1976), must be considered uncertain. Thus at present there is no compelling evidence for significant undiscovered mass in the immediate solar vicinity. This result is consistent with a model mass-distribution for the galaxy computed by Ostriker & Caldwell (1979); the model has much unseen mass in an extended halo but very little in the neighborhood of the sun.

To compute the local mass-to-light ratio, we need the local luminosity density, $\mathscr{L}$. This quantity follows directly from $\Phi(M)$, which must now be based on a large volume since rare stars make a significant contribution to the luminosity. The $\Phi(M)$ of Starikova (1960) and McCuskey (1966, Table 8) respectively give $\mathscr{L}_V = 0.049$ and $\mathscr{L}_V = 0.063\ L_\odot\ \mathrm{pc}^{-3}$. A recent study by F. Malagnini (private communication) suggests that the results of Starikova may be preferable, so we adopt $\mathscr{L}_V = 0.055 \pm 0.01\ L_\odot\ \mathrm{pc}^{-3}$. Since Malagnini finds $B-V = 0.62$ for the solar neighborhood, $\mathscr{L}_V \cong \mathscr{L}_B$. For $\rho = 0.09 \pm 0.02$, $M/L_B = 1.1$–2.4. Using $\rho_{\mathrm{dyn}} = 0.15$, we find $M/L_B = 2.3$–3.3. If the light contribution from younger stars (spectral type earlier than G2 on the main sequence) were removed, then the local M/L would be approximately doubled.

2.2 *Mass of the Milky Way*

Historically the mass of the Milky Way has been determined from the rotation curve. This involves two distinct but interrelated observational problems: finding the shape of the rotation curve interior and exterior to the solar radius R_0, usually from 21-cm H I studies (Kerr & Westerhout 1965, Burton 1974), and setting the scale of the rotation curve by estimating the circular velocity at the sun, V_0. The latter measurement is difficult due to lack of a suitable inertial reference frame. One approach is to use extreme Pop II objects as a reference; this technique yields a lower limit, since the amount of rotation of the Pop II spheroid is unknown.

Using this method, Oort (1965) showed that $V_0 \gtrsim 190 \pm 30$ km sec^{-1}, in good agreement with Hartwick & Sargent's (1978) value of 220 km sec^{-1} based on velocities of globular clusters and dwarf spheroidal galaxies. On the other hand, a best-fit solution of 300 km sec^{-1} is obtained from the dynamics of the Local Group (see Section 6.4). Between these two extremes is the officially adopted I.A.U. value of 250 km sec^{-1} for a solar radius of 10 kpc.

A fresh attack on the determination of V_0 has been made recently by Gunn, Knapp & Tremaine (1979). They combine observations of H I interior to the sun with the requirement that the rotation curve join smoothly to their suggested flat rotation curve exterior to the sun. Their reasoning is too complex to detail here, but their preferred value of V_0 is 220 km sec^{-1}. In our opinion the uncertainties are large, but the method does minimally require $V_0 \lesssim 260$, in contrast to $V_0 = 300$ found from Local Group dynamics.

Once V_0 and R_0 are known, the mass in the Milky Way interior to the sun can be obtained by a variety of modelling techniques (see, e.g.,

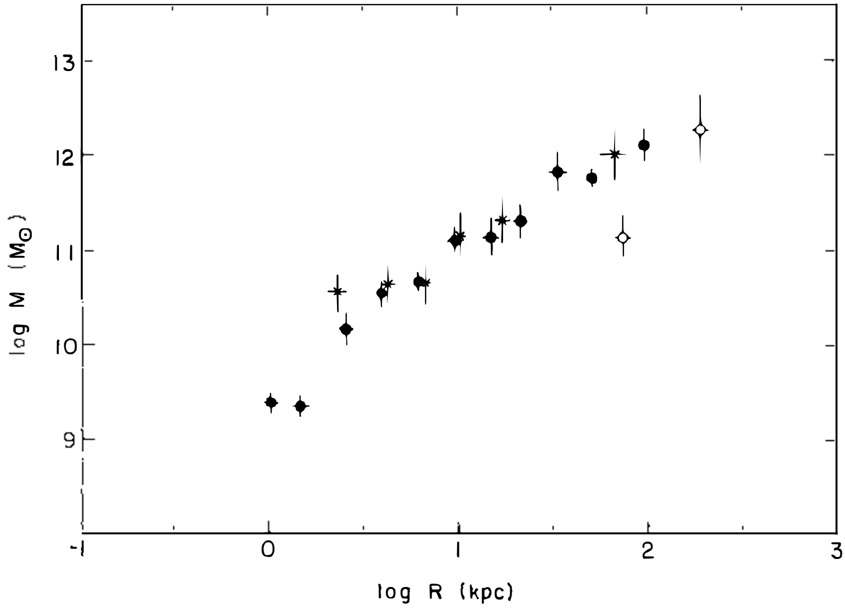

Figure 1 Mass of the Milky Way interior to radius R determined from observations of globular clusters and dwarf spheroidal galaxies; all data have been averaged in radial bins. The vertical bars are the standard error of the mean of each bin. Filled dots are mass measurements from globular cluster tidal radii, stars refer to dynamical mass determinations, and open circles are masses derived from tidal radii of dwarf spheroidal galaxies.

Schmidt 1965). However, these results have lately diminished in significance in the face of mounting evidence for large amounts of nonluminous matter far beyond the sun's orbit. From stellar motions, Fitzgerald et al. (1978) found that the rotation curve stays flat outside the sun for several kiloparsecs. Hartwick & Sargent (1978) analyzed the distribution of radial velocities of globular clusters and nearby dwarf spheroidal galaxies, which they took to be bound to the Milky Way. The outermost sample tests the potential at an effective radius of about 60 kpc and gives an interior mass of $8 \times 10^{11}\ M_{\odot}$ for an isotropic distribution of velocity components.

Finally, using the globular cluster system as a probe of the galactic potential and tidal fields, Webbink (in preparation) has mapped the mass distribution out to a radius of $\sim$100 kpc. Independent estimates of galactic mass were obtained from the tidal radii and radial velocities of 126 globular clusters and 7 dwarf spheroidal companions of the galaxy. The deduced mass distribution of the Milky Way based on radial bin averages is shown in Figure 1. The present tidally-limited radius of the galaxy due to M31 is $\sim$200 kpc. Within this assumed radius, Webbink derives a total mass of $1.4 \pm 0.3 \times 10^{12}\ M_{\odot}$ from tidal effects on globular clusters and $1.4 \pm 0.8 \times 10^{12}\ M_{\odot}$ from globular cluster radial velocities. The consistency between Webbink's two completely independent determinations of the mass distribution is strong empirical evidence for the existence of a dark envelope around the Milky Way.

To obtain M/L_B for the galaxy, we use Sandage & Tammann's (1976) calibration of L_B versus rotation velocity to estimate the luminosity of the galaxy. The result is $2.0 \times 10^{10}\ L_{\odot}$. With Webbink's mass estimate, we obtain $M/L_B \lesssim 70 \pm 20$ on our mass-to-light system. This value is an upper limit, since the mass may not extend as far as the assumed tidal cutoff of 200 kpc.

3 MASS-TO-LIGHT RATIOS OF SPIRAL GALAXIES

The rotation of spiral nebulae was first noticed by Wolf (1914) and Slipher (1914) fully a decade before astronomers discovered the true nature of galaxies. Pease (1916, 1918) made the first measurements of what we now call the stellar "rotation curve" in the nuclear regions of M31 and the Sombrero galaxy (M104). These observations required truly heroic dedication; both exposures lasted 80 hours spread out over a period of 3 months!

From these modest beginnings, the study of rotation curves has since matured to become our single most powerful tool for determining mass distributions inside galaxies. For many decades, optical spectroscopists dominated the field by measuring velocities of H II regions. Lately, it has been discovered that neutral atomic hydrogen extends outward past the optically bright regions of most spiral galaxies, and the 21-cm line is now being fully exploited to follow the dynamics to radii far beyond the last observable H II region.

The fundamental theory of inferring mass distributions from rotation curves has been discussed by Burbidge & Burbidge (1975), de Vaucouleurs & Freeman (1973), Freeman (1975), and Schmidt (1965), among others. Brosche, Einasto & Rümmel (1974) give a bibliography of dynamical measurements for all galaxies complete through May 1973, and van der Kruit (1978) reviews recent observational results.

3.1 *Observed Rotation Curves*

Roberts (1975a) has dramatically illustrated the difficulty of using the older optical rotation curves to probe the outer mass distributions of spiral galaxies. A convenient measure of the optical extent of a galaxy is the Holmberg radius (Holmberg 1958), which is the major-axis radius at a surface brightness of 26.5 photographic mag arcsec^{-2}. Roberts found that the median extent of optical rotation curves published up to 1975 was only 0.3 Holmberg radii. These curves typically showed a steep rise in rotational velocity near the nucleus, then a short section of leveling off [e.g. NGC 157 (Burbidge, Burbidge & Prendergast 1961)]. There the data ended, usually because the surface brightness of the galaxy was so low that further measurements were impossible.

Until recently it was customary to assume that in such cases the turnover of the rotation curve had been reached and that the rotational velocity declined smoothly past the turnover radius, eventually to reach the Keplerian falloff, $V \propto R^{-1/2}$, at large R. Considerable theoretical machinery was constructed to model the curve and deduce the mass distribution and total mass from the measured points. Burbidge & Burbidge (1975) give a comprehensive description of these techniques; Bosma (1978) provides a useful additional discussion of Toomre (1963) disk models. A simple and commonly used approximation is based on Brandt's (1960) parametrization of the rotation curve:

$$V_{\mathrm{rot}}(R) = \frac{V_{\max}(R/R_{\max})}{\left(1/3 + 2/3\left(\dfrac{R}{R_{\max}}\right)^{n}\right)^{3/2n}} \tag{1}$$

where $V_{\max}$ is the maximum rotational velocity, $R_{\max}$ is the radius at which the maximum rotational velocity occurs, and n is a shape parameter which determines how rapidly the curve reaches a Keplerian falloff. The value of n is determined by fitting the curve up to the last

Table 1 Galaxies with extended rotation curves

Object	Type[a]	Distance[b]	L_B[c] ($10^9\ L_\odot$)	Corrected Holmberg radius[d]	Rotation velocity[e] (km s^{-1})	Mass[f] ($10^9\ M_\odot$)	M/L_B	Source[g]
N3626	RSA(rs)0$^+$	21.1	16.2	11.3	233	142	8.8	1
N4324	SA(r)0$^+$	23.0	10.1	9.0	163	55	5.5	1
N3593	SA(s)0/a:	11.8	5.9	10.8	108	29	4.9	1
N3623	SAB(rs)a	11.8	27.0	15.8	212	94	3.5	1
N4378	RSA(s)a	48.6	45.0	30	280	540	11.9	2
N4594	SA(s)a	18.2	10.5*	21	350	590	5.6	3,4
M81	SA(s)ab	3.6	20.3	15.8	217	172	8.5	2
N4151	PSAB(rs)ab	19.6	23.0	19.2	146	95	4.1	5,1
N4698	SA(s)ab	23.0	28.0	19.1	246	270	9.6	1
N4736	RSA(r)ab	6.8	22.0	14.2	180	107	4.9	6
N4826	RSA(rs)ab	6.8	15.6	10.7	169	71	4.5	1
M31	SA(s)b	0.69	20.0	15.6	233	196	7.6	7,8,9
N891	SA b	14.4	31.0	21.7	225	255	8.2	10
N2590	Sb	95.9	105.0	34	265	560	5.4	2
N2841	SA(r)b	12.0	28.0	16.9	273	290	10.6	10
N3627	SAB(s)b	11.8	36.0	20	195	176	4.9	1
N4501	SA(rs)b	23.0	79.0	27	295	550	7.0	1
N4565	SA b	18.4	71.0	36	250	520	7.3	1,10
N5383	SB(rs)b	47.0	55.0	29	215	300	5.5	10
M51	SA(s)bcp	12.0	66.0	23	203	220	3.3	11,12
N801	Sbc-c	119.0	240.0	51	220	570	2.4	2
N1620	SA(rs)bc	68.5	92.0	32	240	430	4.7	2
N3145	SB(rs)bc	68.3	105.0	34	250	500	4.7	2
N4258	SAB(s)bc	6.8	25.0	20	200	180	7.3	13
N4527	SAB(s)bc	23.0	35.0	20	186	160	4.5	1
N4536	SAB(rs)bc	23.0	44.0	25	188	210	4.8	1
N5055	SA(rs)bc	12.0	50.0	25	199	230	4.5	10
N7331	SA(s)bc	21.0	85.0	33	227	390	4.6	1,10
N7541	SB(rs)bcp	57.5	70.0	30	230	370	5.2	2
N7664	Sbc-c	74.2	74.0	40	200	370	5.0	[redacted]
N2998	SAB(rs)c	95.6	149.0	43	210	440	3.0	[redacted]
N3198	SB(rs)c	13.5	15.9	19.5	140	89	5.6	10
N3672	SA(s)c	33.1	48.0	22	180	165	3.4	2
N5033	SA(s)c	21.0	48.0	33	209	330	6.8	10
N5907	SA c	17.2	36.0	25	235	320	9.0	10
M33	SA(s)cd	0.72	3.3	7.8	90	14.6	4.5	14,15
M83	SAB(s)cd	6.3	37.0	12.3	170	83	2.3	16
M101	SAB(rs)cd	8.0	53.0	32	194	280	5.3	17,18
N672	SB(s)cd	10.8	7.7	14.5	129	56	7.2	1
N2403	SAB(s)cd	3.6	7.6	10.7	130	42	5.5	19
N4244	SA(s)cd	6.8	8.1	11.5	106	30	3.7	10
N4517	SA(s)cd:	23.0	53.0	29	158	169	3.2	[redacted]
N4559	SAB(rs)cd	18.4	50.0	26	132	106	2.1	[redacted]
N925	SAB(s)d	14.4	25.0	26	122	89	3.5	[redacted]
N4631	SB(s)d	14.0	63.0	27	150	143	2.3	1,20
N4236	SB(s)dm	3.6	2.9	10.8	72	12.8	4.4	19

Table 1 (*continued*)

Object	Type[a]	Distance[b]	L_B[c] ($10^9 L_\odot$)	Corrected Holmberg radius[d]	Rotation velocity[e] (km s^{-1})	Mass[f] ($10^9 M_\odot$)	M/L_B	Source[g]
SMC	SB(s)m	0.065	0.67	2.9	40	1.05	1.6	21
N3109	SB(s)m	2.6	1.57	5.2	40	1.93	1.2	22
N4656	SB(s)mp	14.0	26.0	21	84	40	1.4	1
I1727	SB(s)m	10.8	3.9	13.5	74	17.0	4.3	1
N6822	IB(s)m	0.70	0.18	2.0	15:	0.10	0.58	21

[a] De Vaucouleurs type from RC2 (estimate when not available).
[b] Distance based on group membership (Sandage & Tammann 1975, de Vaucouleurs 1975) and mean group velocity. Radial velocity of galaxy used if not a group member.
[c] B luminosity based on B_T from RC2; corrections for internal absorption from RC2; galactic absorption correction $A_B = 0.133\,[\csc(b)\text{-}1]$; $M_B(\odot) = 5.48$ mag.
[d] Radius on Holmberg's system corrected for inclination using diameter correction from RC2.
[e] Adopted rotation velocity at corrected Holmberg radius.
[f] Mass within Holmberg radius assuming spherical distribution.
[g] Sources for rotation velocity:

1. Krumm & Salpeter, private communication
2. Rubin, Ford & Thonnard 1978
3. Faber et al. 1977
4. Schweizer 1978
5. Bosma, Ekers & Lequeux 1977
6. Bosma, van der Hulst & Sullivan 1977
7. Emerson 1976
8. Newton & Emerson 1977
9. Roberts & Whitehurst 1975
10. Bosma 1978
11. Shane 1975
12. Segalowitz 1976
13. van Albada & Shane 1976
14. Warner, Wright & Baldwin 1973
15. Rogstad, Wright & Lockhart 1976
16. Rogstad, Lockhart & Wright 1974
17. Allen 1975
18. Rogstad & Shostak 1971
19. Shostak 1973
20. Weliachew, Sancisi & Guélin 1978
21. Tully et al. 1978
22. Huchtmeier 1975

measured point. If it is *assumed* that the velocity beyond this radius is adequately approximated by the Brandt model, the total mass of the galaxy, M_T, is $(3/2)^{3/n}\, V_{max}^2 R_{max}/G$.

In the context of extragalactic astronomy a decade ago, the Brandt model and its relatives were a logical way to model the outer regions of a galaxy. After all, the light was falling off rapidly at the last measured point, and in several galaxies the rotation curve also seemed to be falling appreciably as well [e.g. NGC 5055 (Burbidge, Burbidge & Prendergast 1960)]. However, workers at that time were well aware that a convenient extrapolation was being used which might not represent reality. "One does not know how much the tail wags the dog," cautioned Burbidge and Burbidge.

Radio 21-cm observations, which in many galaxies now extend well past the Holmberg radius, do not confirm this extrapolation. The radio rotation curves remain flat to the limit of observation, in some cases beyond 50 kpc, indicating much larger total masses than given by the Brandt formula. This result was strongly hinted at in early observations by Shostak & Rogstad (1973), Rogstad et al. (1973, 1974), and Seielstad & Wright (1973). Further evidence came from observations of M31 by

Roberts (Roberts 1975a, Roberts & Whitehurst 1975) and from an early compilation of rotation curves by Huchtmeier (1975). These initial results have been overwhelmingly confirmed by the more recent work of Bosma (1978), Krumm, and Salpeter (Salpeter 1978), and other references summarized in Table 1. At the same time, Rubin, Ford and co-workers (summarized by Rubin, Ford & Thonnard 1978) have pushed optical observations to greater radii by exploiting improvements in spectrograph and image-tube design. A montage of representative modern rotation curves collected by Bosma is shown in Figure 2.

There are now approximately 50 galaxies for which reliable rotation curves exist out to large radii (see Table 1). Very few are seen to turn over at all, and only three (M81, M51, and M101) show significant declines. All three of these galaxies, however, have nearby companions which may well perturb the outer H I. Furthermore, M81 has a large bulge which might produce a turnover in velocity because of its strong central condensation, while the H I in M101 shows strongly asymmetric motions on opposite sides of the major axis. In short, all three of these galaxies might well be atypical objects.

The reality of flat rotation curves has been questioned on several grounds. Doubts have been raised, for example, as to whether the H I is truly in circular motion. There are indeed good reasons to fear that within the inner regions of galaxies, ionized gas is not always in circular orbit. For example, marked asymmetries in the inner rotation curves amounting to ~ 100 km sec^{-1} exist in both M31 (Rubin & Ford 1971, de Harveng & Pellet 1975) and M81 (Goad 1976). Moreover, the emission-line rotation curves of bulge-dominated early-type spirals do not rise nearly as steeply as the light distributions suggest they should. The brightness profiles of such bulges near the nuclei are similar to those of elliptical galaxies (Kormendy 1977a, Burstein 1978, Kormendy & Bruzual 1978), and by analogy we would expect the rotation curve to rise to a sharp maximum within a few arc seconds, provided the mass-to-light ratio is uniform. In the three Sa galaxies with observed rotation curves, NGC 4378 (Rubin et al. 1978), NGC 4594 (Schweizer 1978), and NGC 681 (Burbidge, Burbidge & Prendergast 1965), this steep rise is not observed. NGC 4594 is an especially puzzling case; M/L_B in the nucleus based on the velocity dispersion is 12.6 (Williams 1977) yet is only 0.26 at 1 kpc according to the rotation curve (Schweizer 1978). Since the spectrum and colors (S. Faber, unpublished) give no hint of a significant change in the mass-to-light ratio of the stellar population, we seriously doubt that the ionized gas is in circular motion. Einasto (1972) has expressed a similar opinion that observed velocities in the ionized gas in the bulge of M31 are too low to reflect true rotation. Lack of adequate spatial resolution and noncircular motions due to bar-like distortions (Bosma 1978) are additional complications affecting rotation curves near nuclei.

For these reasons, rotation curves in the inner regions of galaxies might not be useful indicators of the mass distribution. We prefer to concentrate here on the outer regions, where noncircular motions and lack of angular resolution pose fewer problems.

Even at large radii, however, the interpretation of these observations

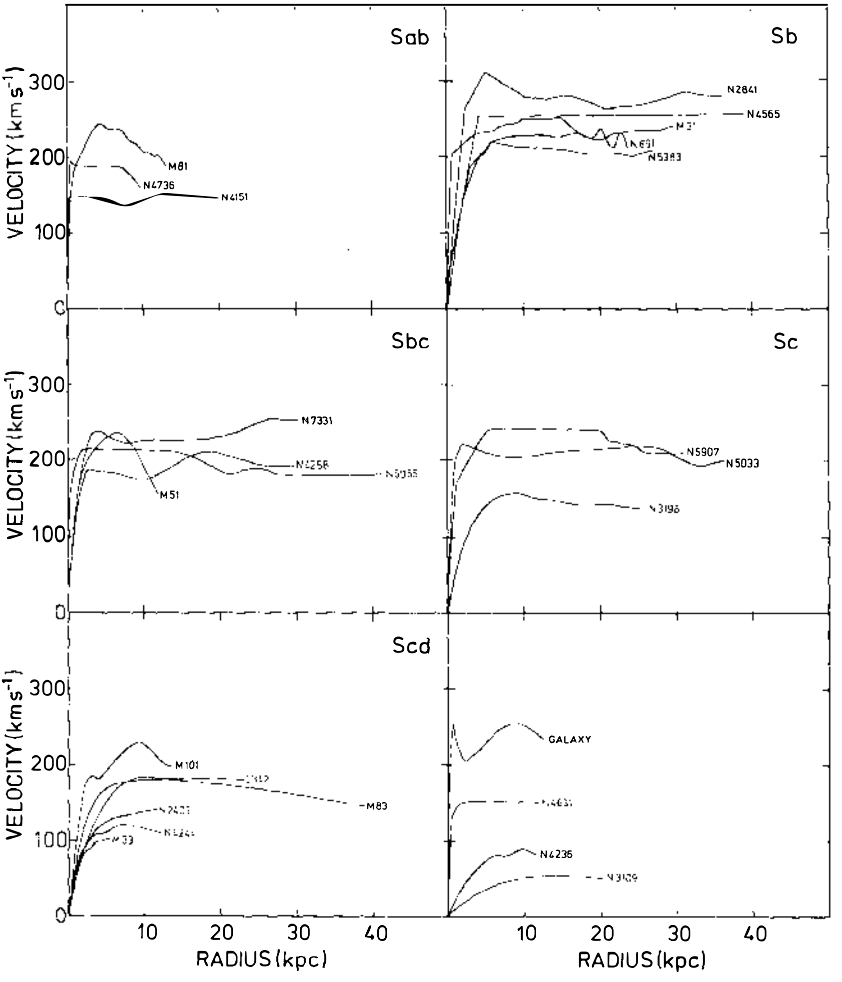

Figure 2 Rotation curves of 25 galaxies of various morphological types from Bosma (1978).

is a subtle matter. For example, sidelobes on radio telescopes could produce a fictitious flat rotation curve due to spillover from the bright H I at smaller radii. The initial results of Krumm and Salpeter (Salpeter 1978) were criticized for this reason by Sancisi (1978), but F. Briggs, N. Krumm, and E. Salpeter (in preparation) have since carefully calibrated the sidelobes of the Arecibo dish, and the final data should be free from this effect. Moreover, it is hard to see how sidelobes could affect the entire body of available data, since 21-cm measurements have been with many different instruments, single dishes as well as interferometers. The fact that flat rotation curves are also measured with optical techniques further strengthens this conclusion.

For all these reasons, it seems most unlikely that flat rotation curves are merely an artifact of observational errors. Even so, various dynamical arguments have been advanced which question the conventional identification with local circular velocity. For example, the outermost H I layer in many spirals is significantly warped out of the main plane of the galaxy (e.g. Rogstad, Lockhart & Wright 1974, Sancisi 1976). These warps might be accompanied by motions which mimic a flat rotation curve along the major axis if the inclination of the warp were properly arranged. However, for several of the galaxies in Table 1, warps have been fully modelled using the entire information available in two dimensions, and a flat rotation curve still persists. Further, the sheer bulk of the data is beginning to tell: it is hard to see how warps with random projection factors could conspire to produce a flat rotation curve in so many different galaxies.

It has also been suggested that H I at large radii might represent recent infall and not yet be in dynamical equilibrium. However, with few exceptions (e.g. M101) the velocities on opposite sides of the galaxy are reasonably symmetric, and circular motions (with a possible warp) satisfactorily fit the observations leaving relatively small residuals (Bosma 1978). Furthermore, because the flat portions in many galaxies extend over a large fraction of the observable radius, one would be forced to conclude that a large portion of the gas is out of equilibrium.

In summary, we feel that no generally valid alternative explanation has been put forward for these flat rotation curves and that the observations and their implications must therefore be taken very seriously.

For an assumed spherical mass distribution and $V_{\rm rot}$ constant with radius, the mass within radius R increases linearly with radius and the surface mass density declines as R^{-1}. Since the surface brightness of spirals declines exponentially (Freeman 1970, Schweizer 1976), this simple model predicts a strong increase in the local mass-to-light ratio projected on the sky as long as the rotation curve stays flat. Bosma (1978) finds that the precise form of this increase depends rather sensitively on the nature of the mass model assumed, whether spherical or disk. However, the increase in local M/L cannot be made to disappear completely by varying the model. Bosma (1978) and Roberts & Whitehurst (1975) have obtained local values of M/L_B of 100–300 in seven spirals at the outer limits of the observations, much larger than M/L_B for the stellar population in the solar neighborhood (Section 2.1).

Despite the complications mentioned above, mass determination using rotation curves is a relatively simple procedure. There are none of the statistical projection factors and group membership decisions which plague the analysis of binary and group motions. Based on present data and standard interpretations, it seems relatively certain that dark material is being detected.

The amount of extra mass actually implied by these rotation curves is itself relatively trivial; galaxy masses on average have perhaps doubled over the older optical estimates, not enough to satisfy the mass discrepancy for groups and clusters, as we shall see. Nevertheless, flat rotation curves have profound implications for the problem of missing mass, first because the detection of unseen matter is relatively secure and second because at least some of the missing mass seems to be associated with individual galaxies themselves.

3.2 *Mass-to-Light Ratios*

The discovery of flat rotation curves has thrown out any hopes we might have had of estimating the total masses of galaxies based on an extrapolation of their rotation curves. Such an extrapolation would necessarily involve an assumption as to how far the flat rotation curves continue, which is something one certainly would not want to guess at this time. The notion of total masses based on internal motions still appears frequently in today's literature and is one we would like to discourage; the derivation invariably involves assumptions which are unjustified, and the results can be misleading.

If we confine ourselves to what is actually measured, we are led inevitably to the concept of mass and mass-to-light ratio within a specified radius. Ideally one would like to use a radius related to some natural length-scale for the galaxy, for example, the *e*-folding length for exponential disks (α^{-1}). This is not realizable at present because α^{-1} is known for too few galaxies. As a practical necessity, we adopt an isophotal radius, even though this radius is systematically smaller for systems of low surface brightness, such as late-type spirals and irregulars. The Holmberg radius $R_{\rm HO}$ seems a good choice because it is comparable in size to the extent of presently available rotation curves and is easily derived by transformation from the large body of diameter data in the *Second Reference Catalogue.*

Table 1 collects information on galaxies available at present with published rotation curves which extend to at least 0.5 Holmberg radii and for which inclination corrections can be reliably estimated. The great majority have velocity curves extending nearly to R_{HO} or beyond. For estimating the mass within R_{HO}, elaborate techniques which exploit every bump and wiggle in the rotation curve seem to us unnecessary. For reasons discussed earlier, the inner sections of the measured curves may not contain useful information and furthermore do not strongly influence the total mass determination. Moreover, if one is looking for trends in mass and M/L with Hubble type, simplicity is a virtue. Insofar as possible, one must avoid the use of assumptions which vary with type since such procedures may themselves introduce spurious trends.

For all these reasons, it seems justified simply to assume that the mass is spherically distributed within R_{HO} and to calculate the mass at R_{HO} as $M_{HO} = R_{HO}V_{HO}^2/G$, where V_{HO} is the observed velocity of rotation at the

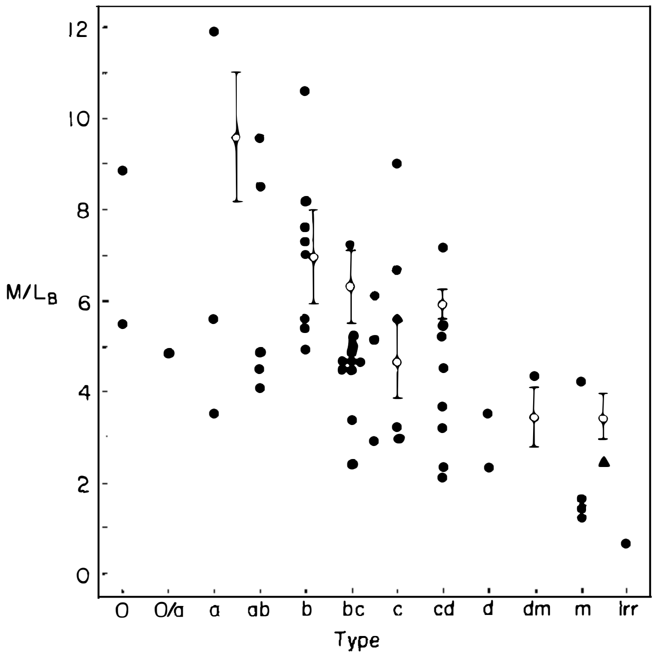

Figure 3 Blue mass-to-light ratios within the Holmberg radius versus morphological type. Black dots: individual galaxies in Table 1. Open circles: logarithmic means from single-dish measurements by Dickel & Rood (1978). Triangle: late-type DDO irregulars from Fisher & Tully (1975).

Holmberg radius. The assumption of a spherical distribution of material has some theoretical justification (see Section 8) but is basically unproven at this time. The use of highly flattened spheroids would yield masses roughly 35% smaller.

The resultant values of M/L_B are computed on the system of luminosities described in Section 1 and are plotted in Figure 3 versus morphological type. A trend is apparent in that later types seem to have lower M/L_B than early types. This conclusion is weak, however, because the number of late-type spirals in the sample is small.

We have therefore used single dish 21-cm profiles to verify this trend. Single-dish observations are useful here because the steep-sided H I profile yields a velocity width that is closely related to the velocity of rotation in the galaxy. Many workers have converted such velocity widths into total masses using the Brandt approximation (Roberts 1969, S. Peterson 1978, Balkowski 1973, Dickel & Rood 1978). We have assumed instead a flat rotation curve and calculated the mass within the Holmberg radius, as described above.

Our results are based on the extensive observations of Dickel and Rood for 121 disk galaxies. Comparison of 21-cm line widths with rotational velocities at R_{HO} for 16 galaxies in common with Table 1 indicates that $\langle V^2 \rangle$ of Dickel and Rood is 25% larger than $\langle V_{rot}^2 \rangle$ from Table 1. We applied this correction to the single-dish masses. The results for M/L_B (logarithmic means) based on this sample are shown as open circles in Figure 3, where the error bars reflect only the formal error in the mean, not the possible systematic errors. These data confirm the trend with type suggested by the individual rotation curves.

To further strengthen the data on the latest types, we used the indicative masses of Fisher & Tully (1975) for DDO irregular galaxies, adjusted to our system. The result is plotted as a triangle in Figure 3, and agrees with the low M/L_B for late types found from the other two methods. Recent data by Shostak (1978) (not plotted) also support the correlation in Figure 3 (see also Nordsieck 1973).

In summary, a trend in M/L_B within the Holmberg radius seems fairly well established. Whether or not a trend exists in *total* mass-to-light ratio cannot be determined from rotational velocities at the present time.

An estimate for S0's from Section 4 is also included. For completeness, M/L_V and M/L_K are also given, transformed from M/L_B using the data described in the notes. Interestingly, the trend in M/L with morphological type seen with B luminosities disappears when K magnitudes are used, and M/L_K is approximately constant. This is strong confirmation of the prediction of this effect made on quite different quasi-theoretical grounds by Aaronson, Huchra & Mould (1979).

The observed trend in M/L_B with type is generally what one would expect from variations in stellar content along the Hubble sequence. For example, Larson & Tinsley (1978) have found that M/L_B is well correlated with $B-V$ for model stellar populations over a wide range of ages. In their work, the mass refers to stars alone. To compare with their results, we must correct the observed M/L_B for mass due to gas. To do this, we have made the crude assumption that Robert's (1975b) global values of M_{HI}/L_B versus Hubble type apply also within the Holmberg radius and have neglected any contribution by molecular hydrogen.

The result is the column labeled M^*/L_B in Table 2, to be compared with Larson and Tinsley's model M^*/L_B, also in Table 2. Exact agreement is not expected, since the amount of material in evolved degenerate stars in the model is quite uncertain. Furthermore any matter in dark envelopes is not included in the Larson-Tinsley model. Nevertheless, within a scaling factor, the models appear to represent the total range of M^*/L_B rather well. The detailed agreement is not quite so good, however; the strong change in color of spirals from type Sa to Sd would suggest a noticeable decrease in M^*/L_B whereas the data show only a slight decrease.

Finally, we note that M/L_B for spirals within the Holmberg radius is $\sim$4–6, not much greater than the local M/L_B for the solar neighborhood, which is $\sim$1–3 (Section 2). This comparison indicates that unseen matter does not strongly dominate the mass within R_{HO}. This result is consistent with the Ostriker-Caldwell model of the Milky Way; their model contains only 25% dark matter within 20 kpc (roughly the Holmberg radius for our galaxy).

Table 2 Mass-to-light ratios within the Holmberg radius[a]

Type	M/L_B	M/L_V[b]	M/L_K[c]	M^*/L_B[d]	Model M^*/L_B[e]
$S0^-$	10:[f]	7.6:	1.4:	10:	5.3
$S0^+$-Sa	6.2 ± 1.1	5.4	1.1	6.1	3.5
Sab-bc	6.5 ± 0.5	6.1	1.2	6.3	2.8
Sbc-Sc	4.7 ± 0.4	5.0	1.1	4.4	1.6
Scd-Sd	3.9 ± 0.6	4.5	1.4	3.5	0.80
Sdm-Irr	1.7 ± 0.6	2.0	0.9	0.9	0.80

[a] R_{HO} corrected for inclination according to RC2.
[b] $(B-V)_\odot = 0.65$ (Allen 1973). Mean $B-V$ for galaxies from de Vaucouleurs & de Vaucouleurs (1972).
[c] $(V-K)_\odot = 1.42$ (Allen 1973). Mean $V-K$ for galaxies from Aaronson (1978).
[d] Includes mass of stars only in M^*/L_B. See text for details.
[e] Model estimates of M^*/L_B by Larson & Tinsley (1978).
[f] From Section 4.

4 MASS-TO-LIGHT RATIOS OF E AND S0 GALAXIES

Three basic methods can be used to determine the mass and mass-to-light ratio of a spheroidal stellar system. The first of these utilizes the global virial theorem, extensively discussed by Poveda (1958):

$$\tfrac{1}{2}d^2I/dt^2 = 2T+\Omega, \tag{2}$$

where I is the moment of inertia, T the kinetic energy, and Ω the gravitational potential energy (Limber 1959). For a galaxy in equilibrium, the left-hand side vanishes. Let us assume further that the galaxy is spherical and nonrotating and that the kinetic energy of each star per unit mass is independent of mass. Then

$$M\langle V^2\rangle+\Omega = 0, \tag{3}$$

where the potential energy Ω is given by

$$\Omega = -G\int_0^R \frac{M(r)dM}{r}; \tag{4}$$

M, R are the total mass and radius, $M(r)$ is the mass contained within a sphere of radius r, and $\langle V^2\rangle$ is the mass-weighted average of the square of the space velocities of the stars relative to the center of mass of the galaxy.

Equations (3) and (4) involve theoretical parameters which are far removed from observed quantities. $\langle V^2\rangle$ for example is usually estimated from the observed line-of-sight velocity dispersion (σ) in the nucleus by asssuming that σ^2 is constant throughout the galaxy, an assumption usually not supported by any observational data. The estimate of the total potential energy Ω is likewise subject to great uncertainty. It is generally assumed that the light distribution is an adequate tracer of the mass and that the luminosity profiles of most ellipticals are similar and are adequately described by empirical expressions, such as the $R^{1/4}$ law of de Vaucouleurs (1948) (see also Young 1976). The integral in (4) is then $\Omega = -0.33\ GM^2R_e$, where R_e is the isophotal radius containing half the light (and mass). This approach assumes that the outer structure of the galaxy obeys de Vaucouleur's law. In fact, there seem to be significant departures from de Vaucouleur's law in the outer profiles of elliptical galaxies which correlate with environment (Kormendy 1977b, Strom & Strom 1978). Furthermore, the total light in the envelopes of some cD ellipticals shows no sign of converging to a finite value (Oemler

1976, Carter 1977), making the determination of R_e operationally impossible. Finally, if ellipticals contain appreciable amounts of dark material which is more extended than the luminous material, R_e as determined for the stars alone may have no connection with the true mass distribution of the galaxy.

The virial theorem thus leads to uncertain results basically because it treats the whole galaxy, including the poorly understood outer regions. To circumvent this difficulty, King (King & Minkowski 1972 and in preparation) has devised a second method to determine M/L; this method is based on stellar hydrodynamical equations applied to the core only. The observational data required include the central surface brightness, core radius (the point where surface brightness drops to $\frac{1}{2}$ of central value), and core line-of-sight velocity dispersion. From these one determines the core density and core mass-to-light ratio. Total mass is not derived. The method assumes only that the nuclear velocity distribution is Gaussian and isotropic with constant σ over the core region, in agreement with the properties of model star clusters whose cores closely resemble the nuclear regions of elliptical galaxies (King 1966). Young et al. (1978) and Sargent et al. (1978) have developed a similar formalism which is applicable to regions outside the core.

King's formula contains an explicit correction for rotational motion based on the observed ellipticity. However, several studies (e.g. Bertola & Capaccioli 1975, Illingworth 1977, C. Peterson 1978) have shown that even flattened ellipticals rotate very slowly and are almost completely pressure supported; rotational corrections should therefore be small. Binney (1976) and Miller (1978) have presented alternative models for elliptical galaxies having anisotropic velocity dispersions. These models imply a correction to our assumption of an isotropic velocity distribution in the core, but the effect should again be small.

The last method for determining M/L in E and S0 galaxies is the most straightforward: find a test particle in circular motion about the spheroidal component. This approach is applicable to the stellar disks of S0 galaxies and to gas in orbit about an elliptical [NGC 4278 is apparently such a galaxy (Knapp, Kerr & Williams 1978)]. For S0 disks seen directly edge-on, the observed rotational velocity must be increased by 30–40% to correct for stars at large spatial radii projected along the line-of-sight (Bertola & Cappaccioli 1977, 1978).

Burbidge & Burbidge (1975) summarized the results on M/L_B in early-type galaxies through 1969. Their mean value was 19.7, for a variety of objects and techniques. King & Minkowski (1972) reported values of 7–20 for luminous elliptical galaxies based on King's method applied to the cores and utilizing Minkowski's velocity dispersions.

Since this early work, the trend in M/L_B has been generally downward owing to two factors: remeasurements of velocity dispersions significantly smaller than earlier values, and the general adoption of core analyses in place of the global virial theorem. It is not easy, however, to summarize these recent results because there is still significant disagreement between various groups as to the correct measurement of σ. The two largest sets of data available are those of Faber & Jackson (1976) (FJ) and Sargent, Young and co-workers (SY) (Sargent et al. 1977, 1978, Young et al. 1978). Although it seemed initially that the values of FJ exceeded those of SY by 28%, new measurements (S. Faber, unpublished, Schechter & Gunn 1979) now make it seem likely that the two systems agree within 10%. In comparison to these results, however, the measurements of Williams (1977), Morton and co-workers (Morton & Chevalier 1972, 1973, Morton, Andereck & Bernard 1977), and de Vaucouleurs (1974) average about 35% smaller. This comparison is quite uncertain, however, because the number of objects in common is in all cases very small.

If these systematic differences in σ are taken into account, one finds that the agreement between investigators is good, with M/L_B typically 5–10. Sargent et al. (1977) found substantially higher M/L_B for elliptical galaxies but did so by applying the virial theorem to the entire galaxy assuming constant σ. Since σ was available only for the core, we believe that an analysis based only on the measured core quantities is preferable.

Determinations of velocity dispersions for normal elliptical galaxies have revealed two possible regularities. Both FJ and Sargent et al. (1977) found that σ increased with total galaxy luminosity approximately as $L_B^{1/4}$. For a sample of ellipticals with core radii and central surface brightness determined by I. R. King (unpublished), FJ derived power-law correlations between luminosity, core radius, and central surface brightness (see also Kormendy 1977b). Using these correlations plus the $L^{1/4}$ law for velocity dispersions, FJ found that M/L_B increased with luminosity as $L^{1/2}$. Schechter & Gunn (1979) found no such correlation, based on a published subset of King's data (King 1978). However, the published data were not corrected for seeing effects, unlike the unpublished list used by FJ. This difference apparently accounts for the discrepancy.

Using a method rather different from those above, Ford et al. (1977) have estimated the mass of M32 from motions of planetary nebulae far from the nucleus. Their result is $4.3 \times 10^8\ M_\odot$, from which $M/L_B = 1.8$. This result would seem to be consistent with the possibility noted above that M/L_B is smaller in less luminous ellipticals.

The nucleus of M87 differs significantly from other ellipticals in having a bright central luminosity spike and a rapid decline in σ just outside the spike (Young et al. 1978, Sargent et al. 1978). A mass with very

Table 3 Mass-to-light ratios in early-type galaxies

Core values:

Object	M/L_B	Source
Mean luminous E	8.5	Faber & Jackson 1976
M31 bulge outside nucleus	8.5	Photometry from Light et al. 1974; $\sigma = 150$
NGC 4473, inside 5 kpc	5.4	Young et al. 1978

Other values:

Object	Type	Distance (Mpc)	R^a (kpc)	R/R_{HO}	$V_{rot}{}^b$	$M/L_B{}^c$	Source
NGC 128	S0p	90.0	17.4	0.4	250	7.2	Bertola & Cappaccioli 1977
NGC 3115	$S0^-$	9.5	4.6	0.3	350	12.3	Rubin et al. 1976
NGC 3636	$RSA(rs)0^+$	21.1	11.3	1.0	233	8.8	Table 1
NGC 4278	E	14.4	16.8	1.9	195	19.8	Knapp et al. 1978
NGC 4324	$SA(r)0^+$	23.0	9.0	1.0	163	5.5	Table 1
NGC 4762	$SB(r)0^0$	23.0	7.8	0.3	210	5.1	Bertola & Cappaccioli 1978

[a] Radial extent of observed rotation curve.
[b] Assumed velocity of rotation, corrected when necessary for projection effects.
[c] M/L_B within radius R, assuming mass spherically distributed.

high M/L, perhaps even a black hole, apparently exists in the middle of the core. Since very few elliptical galaxies have been studied with such high spatial resolution, it is not known whether such cases are common.

Recent results for M/L_B among early-type galaxies (corrected to the M/L system of the preceding section) appear in Table 3. The first group contains objects for which nuclear values of M/L_B have been measured using King's method or a related treatment. The mean value obtained by Faber and Jackson is 8.5 for 10 galaxies, while the value of 8.5 for the inner bulge of M31 is new in this review. The second group consists of galaxies for which the circular rotation of test particles can be measured. The mass and M/L_B within radius R have been computed assuming a spherically symmetric mass distribution as in the preceding section.

Taken at face value, these data suggest that there is no gross increase in M/L_B from the core to the Holmberg radius. This conclusion is supported by the tendency of velocity dispersions to decrease away from the nucleus in M32 (Ford et al. 1977), NGC 3379 and NGC 4472 (FJ), and NGC 4486 (Sargent et al. 1978), leading to constant M/L in the inner regions. On the other hand, σ does not declinc with radius in NGC 4473 (Young et al. 1978). Schechter and Gunn find that σ is basically constant in 12 more ellipticals, but their measurements extend to only a few core radii.

In summary, rotation curve data indicate that M/L_B within the Holmberg radius is approximately 10 for S0's. This number is entered in Table 2. No comparable estimate for E's can be given at this time owing to inadequate data on the velocity dispersions away from the nuclei.

Of great importance is the question whether a strong increase in M/L occurs beyond the Holmberg radius, as seems to be the case with spirals. The evidence on this point is fragmentary but highly suggestive of dark envelopes around early-type galaxies as well. NGC 4278 is the only galaxy in Table 1 for which rotation measurements extend beyond R_{HO}, and its M/L_B seems significantly higher than the others. Faber et al. (1977) found no decrease in the velocity dispersion in the halo of cD galaxy Abell 401 at a radius of 44 kpc, but the accuracy of the measurement was not high. Dressler & Rose (1979) have detected an actual increase in σ out to 100 kpc in the halo of the cD galaxy Abell 2029 with much better data, implying a strong increase in the local mass-to-light ratio. Finally, we mention the novel mass determination of M87 based on the assumption that the X-ray emission centered on M87 is due to thermal bremsstrahlung from isothermal gas in hydrostatic equilibrium within the potential well of the galaxy (Bahcall & Sarazin 1977, Mathews 1978). Mathews finds in this case that the total mass of M87 exceeds 10^{13} $M_\odot$ and the total M/L_B is several hundred.

All these data point strongly to the existence of dark matter around at least some elliptical galaxies. With the recent increased availability of efficient two-dimensional detectors for spectroscopy, additional information on the dynamics of the outer regions of elliptical galaxies should soon be forthcoming. Ultimately, radial velocities of globular clusters will be used to probe the structure of spheroidal systems at very large radii, but these observations seem to lie just beyond the capabilities of present equipment.

5 MASS-TO-LIGHT RATIOS OF BINARY GALAXIES

The derivation of masses of binary galaxies rests on several important assumptions. Let us first consider the simplest case of circular orbits. Assume that the galaxies are gravitationally bound, they interact as point masses, the distribution of orbital inclinations and phases is random, there is no intergalactic matter, and there is no dynamical interaction with matter outside the binary. A full discussion of this case is given by Page (1961), for example, but we present the rudiments here. Let M_T be the mass of each component (the two masses are assumed equal), R the spatial separation, and V the total orbital velocity. Then $M_T = V^2R/2G$. Owing to projection effects, V and R are unobservable. One measures instead Δv, the radial velocity difference, and r_p, the projected separation. These are given by $\Delta v = V \cos\phi \cos\psi$, and $r_p = R\cos\phi$, where ϕ is the angle between the spatial separation R and the plane of the sky and ψ is the angle between the orbital velocity V and the plane determined by the two galaxies and the observer. Let us define an "indicative mass":

$$F(M_T) \equiv \frac{\Delta v^2 r_p}{2G} = M_T \cos^3\phi \cos^2\psi. \tag{5}$$

$F(M_T)$ is always less than or equal to the true mass according to the projection factor $\cos^3\phi\cos^2\psi \equiv F_p(\phi,\psi)$. To obtain the true mass in the simplest possible way, we can average over some appropriate statistical sample to derive a mean projection factor. Then, in principle, for a large enough statistical sample,

$$\langle M_T\rangle = \frac{\langle F(M_T)\rangle}{\langle F_p(\phi,\psi)\rangle}. \tag{6}$$

Page (1952) assumed that for a collection of circular orbits, the angles ϕ and ψ are randomly oriented, whence $\langle F_p\rangle = 3\pi/32 = 0.295$. This assumption is unrealistic because binary galaxies are selected and analyzed on the basis of their projected separation, r_p. Once r_p is specified, ϕ is no longer a random variable. The distribution of true spatial separations combines with the observed r_p to make certain values of ϕ more likely than others.

Depending upon the true distribution of spatial separations, $\langle F_p \rangle$ can therefore be a strong function of r_p. Page (1961) later amended his treatment to include this dependence. As a model for the spatial distribution function, he used

$$D(R) = K\left[1 - \left(\frac{R}{R_{\max}}\right)^3\right], \tag{7}$$

an expression empirically derived by Holmberg (1954). This distribution produces a marked increase in $\langle F_p \rangle$ as r_p approaches $R_{\max}$, the maximum spatial separation of binary galaxies. Such a variation in $\langle F_p \rangle$, if real, is quite inconvenient. It means we cannot compare indicative mass values $F(M_T)$ at different projected separations without first correcting them for trends in $\langle F_p \rangle$. Comparisons between independent data sets thus become much more cumbersome.

An important breakthrough in binary galaxy studies has been the realization that Equation (7) is not correct and that apparently $D(R)$ is a power law over the observable range 20 to 500 kpc. Turner (1976a,b) and S. Peterson (1978), both working from samples of binary galaxies chosen by means of well-determined selection criteria, find that $D(r) \propto R^{-\gamma}$, where $\gamma = 0.5 \pm 0.1$. Since a power law has no scale-length, $\langle F_p \rangle$ is independent of r_p and is given by (Peterson 1978):

$$\langle F_p \rangle = 4 \frac{\Gamma^2\left(\frac{\gamma+4}{2}\right)}{\Gamma(\gamma+4)} \times \frac{\Gamma(\gamma+1)}{\Gamma^2\left(\frac{\gamma+1}{2}\right)}. \tag{8}$$

Fortunately, the sensitivity to γ is not great; over the range $\gamma = 0.0$ to $\gamma = 1.0$, $\langle F_p \rangle$ increases from 0.212 to 0.295. The fact that the projection factor is independent of radius for a power law makes the analysis much more transparent since radial trends in mass and M/L_B can now be determined immediately from trends in indicative mass.

The use of a mean correction factor $\langle F_p \rangle$ to rectify the observed values of $F(M_T)$ is unwise for a number of reasons. First, regardless of what criteria are employed to select the sample of binaries, spurious pairs will creep in. Such pairs tend to have large values of Δv^2 and hence of $F(M_T)$. Thus $\langle M_T \rangle$ for the sample is sensitive to the adopted cutoff in Δv. To avoid this difficulty, one should use a method based on the entire distribution $F(M_T)$ and which weights the observations more or less equally.

Second, the use of a simple mean ignores all information contained in the shape of the distribution $F(M_T)$ and, more particularly, in the bivariate probability density $p(\Delta v, r_p)$. As pointed out by Noerdlinger (1975), $p(\Delta v, r_p)$ is sensitive to the orbital eccentricities of galaxies in the sample. Turner (1976b) was able to rule out rather conclusively highly eccentric models for binary orbits on this basis.

Finally, the distribution of $F(M_T)$ is highly skewed, owing to the skewness of $F_p(\phi,\psi)$ itself. $F(M_T)$ peaks strongly at zero, and for circular orbits the median value of $F(M_T)$ is only 11% of the value of $\langle M_T \rangle$ ultimately derived. Peterson argues that such a skewed distribution is not well represented by its mean.

The use of a simple mean projection factor nevertheless provides a quick and moderately accurate way to compare data sets from various workers and to estimate the effects of different assumptions concerning the geometry of the orbits. From the work of Turner (1976b), who analyzed his data using both a simple mean and a more elaborate technique (see below), we estimate that use of the simple mean yields results accurate to $\pm 20\%$.

Because of the greater complexity of analyzing noncircular orbits, the general solution analogous to Equation (8) has not yet been derived. If the orbits are eccentric, the angles ϕ and ψ as defined in this section are not applicable. Nevertheless, a mean projection factor still exists, call it $\langle \eta \rangle$, which reduces to $\langle F_p \rangle$ in the circular case. Karachentsev (1970) and Noerdlinger (1975) have calculated values of $\langle \eta \rangle$ for collections of binaries with eccentric orbits. However, in their existing form these expressions share the same drawback as Page's original estimate: they do not take into account the bias introduced into the angle ϕ once r_p is specified. To treat the noncircular case, it would seem best at present to rely on Turner's model simulations of binaries with various eccentricities. These show that, for eccentric orbits, the projection factor is smaller and the derived masses therefore larger. If $\gamma = 0.5$, for example, the minimum value of $\langle \eta \rangle$ is 0.105 for purely radial orbits, compared to 0.261 for circular orbits. Qualitatively this effect is easy to understand: if the orbits are eccentric, one views them preferentially near apogalacticon, where the kinetic energies are lower than average. Therefore the observed $F(M_T)$ are systematically lower than in the circular case.

The success of the binary method clearly depends on being able to identify pairs that are real physical systems. The first discussion of this problem and the resulting catalog of doubles was prepared by Holmberg (1937). Other major lists having historical interest include Vorontsov-

Velyaminov's *Atlas of Interacting Galaxies* (1959), Arp's *Atlas of Peculiar Galaxies* (1966), and Karachentsev's comprehensive list of 603 pairs (Karachentsev 1972). Recent velocity measurements have been published by Karachentsev et al. (1976), Karachentsev (1978), Turner (1976a), and S. Peterson (1978).

In all, there have been four major studies of masses in binary galaxies. Page's work (Page 1961, 1962) was the standard reference for many years. Assuming Holmberg's relation (Equation 7) and circular orbits, he obtained $M/L_B = 0.7 \pm 0.9$ for spiral-spiral pairs and $M/L_B = 46 \pm 46$ for pairs containing ellipticals and/or S0's (corrected to $H_0 = 50$ km sec^{-1} Mpc^{-1}). The value for spirals is much smaller than the average of ~ 5 within the Holmberg radius derived in Section 2, while the value for early-type galaxies is significantly larger than our estimates for single galaxies presented in Section 3.

Turner (1976a,b) introduced a new standard of rigor into the study of binary galaxies by selecting a binary sample according to well-defined criteria and using, instead of the traditional mean projection correction $\langle \eta \rangle$, a rank-sum test that essentially compared the observed frequency distribution $p(\Delta v, r_p)$ with simulated versions of $p(\Delta v, r_p)$ for various orbital eccentricities. He was also able to model extended spherical halos surrounding the galaxies. Using the shape of the distribution $p(\Delta v, r_p)$, he showed that binary orbits cannot be highly eccentric and that if massive halos are present, they must have radii $\lesssim 100$ kpc.

S. Peterson (1978) applied a rank-sum procedure to the distribution $F(M_T)$. His analysis was confined to the case of circular orbits, but his radial velocities, many of which came from 21-cm measurements, were more accurate than Turner's largely optical velocities. His selection criteria also differed from Turner's: his binaries were less isolated but extended to much larger radial separations because no outer angular cutoff was employed.

In Table 4 Turner's and Peterson's results for spiral-spiral pairs are compared. The original values of M/L_B given by these authors have been corrected to our system. For reference, Table 4 also includes median values of Δv and r_p for the Turner and Peterson samples. We have also reduced the results of Karachenstev (1977, 1978) to our system and included them in Table 4. Unfortunately only fragmentary accounts of Karachentsev's work were available to us, and we have had to combine results from several different papers. Details appear in the footnotes. We regret any inaccuracies introduced by this procedure.

Inspection of Table 4 shows that values of M/L_B derived by various authors differ substantially. Is this discrepancy due to differences in the data sets or to differences in statistical treatment? To answer this ques-

Table 4 Binary mass-to-light ratios in spiral-spiral pairs

	M/L_B	Median r_p[a]	Median Δv[b]
Turner 1976a,b:		50 kpc	105 km s^{-1}
$\varepsilon = 0.00$	17 ± 4		
$\varepsilon = 0.67$	20 ± 5		
$\varepsilon = 0.74$	24 ± 7		
Small halo[c]	33 ± 10		
S. Peterson 1978:		110 kpc	125 km s^{-1}
$\varepsilon = 0$; all objects	32 ± 11		
$\varepsilon = 0$; $r_p > 112$ kpc	37 ± 14		
Karachentsev:		27 kpc[d]	125 km s^{-1}[e]
$\varepsilon = 0$; $\eta = 0.295$	5.2 ± 2.3[d]		
$\varepsilon = 0$; $\eta = 0.261$	5.9 ± 2.7[f]		

[a] Projected separation.
[b] Only systems with $\Delta v \leqq 750$ km s^{-1} included.
[c] M/L_B for total galaxy, including dark halo, is given. Radius of halo = 100 kpc.
[d] Karachentsev 1977.
[e] Karachentsev 1978.
[f] Transformed from line above using $\eta = 0.261$.

tion, it is helpful to consider just those values based on the assumption of point masses, circular orbits, and $\gamma = 0.5$. (We have provided an additional entry for Karachentsev's data which converts his results to this case.) We then obtain M/L_B(Turner) = 17 ± 4, M/L_B(Peterson) = 32 ± 11, and M/L_B (Karachentsev) = 5.9 ± 2.7, values which still differ by more than the observational errors. Therefore the differences must be due to the data themselves.

Histograms of Δv (uncorrected for observational errors) for all three samples are quite similar. The median values of Δv (for $\Delta v \leqq 750$ km sec^{-1}) in Table 4 confirm this fact. Thus, the observed differences in the Δv distributions do not seem large enough to account for the discrepancy in M/L_B (see below).

On the other hand, the three samples differ markedly in their distribution of projected separation, r_p. Evidently Karachentsev's sample primarily includes rather close pairs, a conclusion supported by the high percentage of interacting galaxies in his sample (Karachentsev 1977). Turner's sample is intermediate, while Peterson's, chosen without any arbitrary cutoff in angular separation, contains many very wide binaries.

It seems to us that these results are consistent with an increase in M/L_B with radius, as would be expected from massive envelopes. This view is supported by the fact that M/L_B for Peterson's outermost pairs (with $r_p > 112$ kpc) is in good agreement with M/L_B for Turner's sample if massive halos having radii of 100 kpc are assumed. Peterson furthermore finds that M_T increases linearly with r_p out to ~ 100 kpc and then levels off, whereas Δv is constant out to ~ 100 kpc, and then begins to decline. Although of limited statistical significance (Peterson's radial bins overlap), this behavior is also consistent with the existence of massive envelopes having a limited extent. On the other hand, the same data also show that although M_T increases with r_p out to 100 kpc, M/L_B appears to be approximately constant from 20 kpc to 500 kpc, a result which Peterson takes as strong evidence *against* massive envelopes.

In our opinion the data are not yet strong enough to take any of these radial trends too seriously, and the global result must be given highest weight. Taken as a whole, the binary data of Turner and Peterson imply that $M/L_B \approx 35$ at large separations. As the average value for spirals within the Holmberg radius is only ~ 5, this result would seem to argue convincingly for additional mass beyond R_{HO}.

Before continuing, it is of interest to inquire why Page with essentially similar data obtained a much lower value of M/L_B for spiral-spiral pairs. According to Peterson, the difference is due to Page's weighting scheme, which set weights inversely proportional to the square of the variance in $(\Delta v)^2$ due to observational error. Small observed values of Δv therefore are given very high weight, which acts to reduce the calculated M_T and hence M/L_B. Page's method also gives highest weight to pairs with small separations (large $1/r_p$). If M/L_B does increase with radius, this effect would further shift M/L_B to systematically smaller values. Both Turner and Peterson have subjected their data to Page's scheme of analysis and confirm the fact that the method yields spuriously low values.

The high M/L_B values determined from binary galaxies are subject to several potential sources of uncertainty. The first arises from observational errors in the velocity differences. Let us suppose that there were no actual increase in M/L_B beyond R_{HO}. Then velocities would decline approximately as $r_p^{-1/2}$, and at a distance of 100 kpc, we would predict orbital velocity differences of roughly 140 km s^{-1}. The observed velocity difference Δv is further reduced by the projection factor $\cos \phi \cos \psi$, the mean value of which is roughly 0.46 (for circular orbits). The expected Δv on this hypothesis is therefore only ~ 65 km s^{-1}. Since this is comparable to the precision of typical optical velocities, it has therefore been argued that existing data are biased against low mass-to-light ratios.

Although simple, this argument approaches the problem in backwards fashion. The proper question is whether the full width of the observed histogram of Δv's is essentially all due to observational error, for only if this is the case can we substantially reduce the measured values of M/L_B.

Peterson's 21-cm velocities are most useful in answering this question because of their high accuracy, typically better than ± 20 km s^{-1}. M/L_B for this subsample (30 pairs) is actually slightly larger than for his sample as a whole. Furthermore, the Δv distribution for pairs with 21-cm velocities is very similar to that for pairs having at least one optical velocity. Both these tests indicate that the optical velocities are substantially correct. L. Schweizer (in preparation) and Karachentsev (1978) are currently collecting new, highly accurate velocities for binary galaxies which should fully resolve this question. For the moment, however, we are inclined to believe that the velocities are not at fault.

The second problem which might affect the results is contamination by spurious pairs. Turner was able to show conclusively that his sample is not appreciably contaminated by objects in the distant foreground or background. However, a great many of Turner's and Peterson's binaries are members of small groups of galaxies, in which the problem of contamination by foreground and background group members could be serious. Statistical estimates of the frequency of spurious pairs made to date are unsatisfactory because they do not include a probable spatial correlation between the target galaxy and contaminating galaxies. Furthermore, the relative velocities of group members, typically a few hundred km s^{-1}, are just in the range where much of the information in the Δv distribution resides.

Yahil (1977) has pointed out a disturbing fact which may be related to contamination problems. He has searched for a positive correlation between $F(M_T)$ and the combined luminosity of the pair. Even though variations in $\langle \eta \rangle$ produce a large spread in $F(M_T)$, Yahil predicts that there should be a marked correlation between $F(M_T)$ and luminosity, provided binaries have uniform M/L_B. For Turner's sample, no correlation is found, indicating that M/L_B must vary over at least one order of magnitude. The correlations between $F(M_T)$ and luminosity for the Peterson and Karachentsev samples appear similar, supporting this conclusion. To Yahil, this result suggests that the large-scale distribution of matter in the universe is not strongly coupled to the distribution of luminous matter, and that the concept of mass-to-light ratio is not useful on scales much larger than 10 kpc. Alternatively, one might conclude that the lack of correlation is caused by errors in $F(M_T)$ introduced by the inclusion of spurious pairs. Yet the observed distribution of radial separations, $D(R)$, suggests that the majority of pairs must be real. If they were chance alignments, $D(R)$ would increase roughly as R for small separations, whereas the observed distribution is peaked near zero, suggesting real physical association.

To investigate the contamination problem further, it would be extremely useful to compile a binary sample having significantly more stringent isolation criteria than those used heretofore. At the very least, one might test whether M/L_B is noticeably smaller for those binaries in existing samples having only distant neighbors but sizeable spatial separations. Note that an analysis confined to just those binaries that show obvious signs of interaction will not help to test the existence of dark material, since such pairs have separations not much larger than the Holmberg radius. They therefore should have rather small M/L_B.

For the moment we continue to assume that the masses for spiral-spiral pairs as measured by binary galaxies are real, but the exact value of the mass-to-light ratio remains somewhat doubtful until the problem of contamination is conclusively cleared up.

Turning now to binary mass determinations for early-type galaxies, we recall Page's finding that E and S0 galaxies have much larger M/L_B than spirals. Both Turner and Peterson obtained a similar result, although their measured differences are smaller: Turner finds the ratio to be 2.0 ± 0.5, while Peterson obtains 1.7 with larger errors.

Actually it seems more probable that most of the mixed E-spiral pairs identified to date are not in fact physically bound to one another. The evidence for this assertion can be found in Figure 4, which presents distributions of Δv for Turner and Peterson's data. For simplicity let us

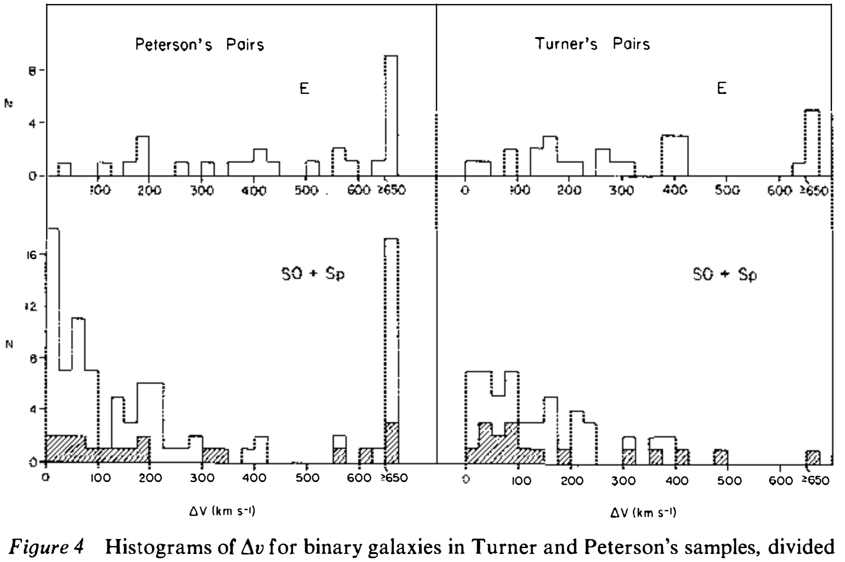

Figure 4 Histograms of Δv for binary galaxies in Turner and Peterson's samples, divided according to morphological type. Lower histograms represent pure spiral or S0 pairs; upper histograms represent pairs in which at least one member is an E. Shaded area refers to spiral pairs in which at least one member is an S0.

assume circular orbits, although our conclusion does not rest on this assumption. For a sample of binary galaxies that are real physical pairs, we expect that the distribution of Δv's will always be peaked at zero owing to the effect of the projection factor $\cos \phi \cos \psi$. This prediction is verified for spiral-spiral pairs (lower histograms). But the upper distributions, in which at least one member is an E galaxy, are nearly flat, with little or no peak at zero. The Kolmogorov-Smirnov test (Hollander & Wolfe 1973) shows quantitatively that the E and spiral distributions are unlikely to be drawn from the same population. The probability for the Turner sample is only 8% and that for the Peterson sample is only 3%. This test therefore confirms the fact that the samples really are quite different.

This disparity between E's and spirals was first noticed by K. C. Freeman and T. S. Van Albada (in preparation) for Turner's pairs. The existence of the same trend in Peterson's data, which is an essentially independent sample, is strong confirmation that the effect is real. We conclude that few if any of the pairs containing elliptical galaxies are physical associations. Virtually all the E pairs are members of groups or clusters, and it seems likely that they are due to chance superpositions of cluster members.

The great majority of these E pairs are mixed, that is, only one member is an E galaxy. Very few are EE pairs, and their small number does not allow us to test whether they, in contrast to the mixed pairs, are physically associated. One conclusion seems probable, however. Although luminous ellipticals and spirals are quite commonly associated with one another in groups, close associations in binaries are rare. This fact might be an important clue to processes which determine the Hubble type.

Histograms for S0 pairs are shown for comparison in Figure 4 as the hatched areas. The S0 distributions apparently resemble those of spirals more closely, so that S0-spiral binaries probably exist. It is this fact which is responsible for the queer nature of the E pairs having escaped discovery before now: since E and S0 pairs have traditionally been lumped together, the peculiar histogram of the ellipticals was diluted by the more normal one for the S0's.

If these mixed E pairs are indeed not physical associations, there exists at present virtually no reliable information on masses of early-type galaxies in binary systems. Jenner (1974) studied the motions of the companions of cD galaxies, but only ten pairs were included. Using his mean mass to obtain M/L_B on our system, we find $M/L_B \sim 50$ at 60 kpc spatial separation. However, if one system with extremely large Δv is omitted, M/L_B drops to only ~ 15. Smart (1973) obtained results consistent with Jenner's.

We found in Section 3 that data on M/L for S0 galaxies within R_{HO} are scanty, while those for ellipticals are noncxistent. The binary data are likewise fragmentary for these early morphological types. The results of Jenner and Smart, however, suggest that the mass-to-light ratio of E and S0 galaxies are broadly similar to those of later-type spirals.

Although the binary data seem to imply the existence of dark matter, we encounter a possible problem when trying to estimate the extent of the dark envelopes from these data. Such an estimate can be made in two ways. First, we have the results of Turner's model simulations, which ruled out envelopes larger than 100 kpc in extent. Second, we have the estimates of global M/L_B from Turner's halo model and also from Peterson's widely-spaced pairs. These both yield $M/L_B \gtrsim 35$ at large separations (the value is a lower limit because both estimates assume circular orbits). According to the usual version of the massive halo hypothesis, $M/L_B(R) \propto R$. Since the average value of M/L_B within R_{HO} is ~ 5 for spirals, the extent of the envelopes must be greater than or equal to $\sim 7\ R_{HO}$, or $\gtrsim 150$ kpc.

These estimates are in fair agreement with one another, but are marginally at variance with the conclusions of White & Sharp (1977), who pointed out that spherical halos around close binaries must interpenetrate strongly and that the effects of dynamical friction will be severe. In fact, two binaries ought to merge completely within an orbital period if their distance of closest approach is less than three times the half-mass radii ($r_{1/2}$) of the halos. For Turner's sample, this implies that the mean value of $r_{1/2}$ is less than 58 kpc, and hence that r, the outer boundary of the halo, is less than 116 kpc for the usual halo model. This limit must be reduced even further if the orbits are appreciably eccentric.

Even though this limit is barely consistent with Turner's estimate, many individual binaries must have true spatial separations much smaller than 100 kpc, and their envelopes should interpenetrate strongly. How are they then able to persist? Perhaps the outer envelope radius varies widely from galaxy to galaxy. Close pairs might then simply be those objects with initially small envelopes, the others having already merged long ago. This reasoning would suggest that the observed radial distribution function for binaries, $D(R)$, is strongly determined by the initial distribution function for the envelope radii themselves and that the binaries we see today are just those which were able to survive over a long period of time. In this regard, we recall the suggestion that merged galaxies become ellipticals (Toomre 1977, White 1978); this effect might then explain the rarity of E binaries.

As White and Sharp point out, the dynamics of binary galaxies, if analyzed from this more general point of view, might well place severe constraints on the distribution of unseen matter. N. Krumm (private communication) has emphasized the advantages of studying interacting pairs because of the information they afford in disentangling projection effects, which make the study of ordinary binaries so difficult. Tidal tails in interacting galaxies might have significantly different shapes if the gravitational effect of dark envelopes were included. In short, dynamical modelling of binary galaxies including the effects of halos seems a fruitful area for observer and theorist alike in the near future.

In summary, for spatial separations greater than 100 kpc, the binary data indicate $M/L_B \approx 35$–50, where the higher value applies if the orbits have moderate eccentricity ($\varepsilon \approx 0.7$).

6 DYNAMICS OF SMALL GROUPS OF GALAXIES

Over the past decade, our knowledge of the statistics of galaxy clustering has increased enormously, due in large part to the pioneering analysis of galaxy positions by Peebles and co-workers [see *Proceedings of I.A.U. Symposium No. 79, The Large Scale Structure of the Universe* (Longair & Einasto 1978) for discussions and references to earlier work]. The great majority of galaxies are apparently located in groups (Gott & Turner 1977, Soneira & Peebles 1977, Gregory & Thompson 1978). The definitive determination of group masses would therefore yield a representative estimate of the mass associated with galaxies in the universe as a whole.

Studies of galaxy clustering have demonstrated quite clearly that clusters exist on all scale sizes from the traditional great clusters like Coma down to binaries. Dynamically speaking, then, there exists a continuum, and there is no obvious dividing line between clusters and smaller groups. In practice, however, analyses of great clusters and small groups are unlike in a number of ways. Because the membership sample in groups is usually very small, the procedures for statistical mass determination and error estimate differ substantially from those in great clusters, where techniques based on large samples are more appropriate. Furthermore, the contrast of the cores of rich clusters against the background is much larger than for small groups, and the treatment of foreground-background contamination is therefore different. Finally, there are a number of dynamical processes more likely to occur in large clusters, where the densities are large and the crossing times short. Foremost among these are dynamical friction and tidal stripping, both of which can significantly rearrange the distribution of matter inside the cluster. For these reasons, we reserve the discussion of large clusters to Section 7, and treat here only the analysis of small groups.

Earlier work on groups of galaxies culminated in de Vaucouleurs' (1975) monumental listing of nearby associations, a systematic survey of volume density enhancements within 35 Mpc. Membership determination was based on absolute magnitude indicators as well as on redshifts. Sandage & Tammann (1975) presented a catalog selected in similar fashion. Despite the use of all available criteria in addition to redshift, the assignment of galaxies to groups remained a very difficult question and one without any obvious statistical solution. A major new development has been the identification of groups via a surface-density criterion only (Turner & Gott 1976). In this technique, one accepts the inevitable contamination by foreground and background galaxies as the price one must pay for a statistically well-defined and unbiased sample. However, one then must find a satisfactory method for dealing with the contamination problem.

6.1 *Crossing Times*

It has been obvious for a long while that the motions of galaxies in groups and clusters, if simply interpreted, imply the existence of a substantial amount of mass in addition to that traditionally associated with individual galaxies (Burbidge & Burbidge 1975). However, this conclusion depends entirely on the assumption that the groups are at least bound, if not in virial equilibrium. It is therefore necessary at the outset to decide on the nature of small groups: bound or unbound density enhancements? A useful concept here is the crossing time (Field & Saslaw 1971). If crossing times are short compared to the Hubble time, the groups must be bound; otherwise they would have dispersed long ago. It has sometimes been argued that groups may be unbound and "exploding" (e.g. Ambartsumian 1961), but the fact that the great majority of galaxies are group members makes this hypothesis unattractive as a general explanation.

One may choose various definitions of the velocity and radius in defining the crossing time, and these choices make significant and systematic differences in the final values. For example, one may use as R the mean harmonic radius, $R_{\rm VT}$, of the group (Limber 1959). $R_{\rm VT}$ is the characteristic radius used in the virial theorem. Taking $V_{\rm VT}$ equal to the square root of the mass-weighted rms space velocity with respect to the center of mass, one has $t_{\rm VT} = R_{\rm VT}/V_{\rm VT}$ for the virial crossing time. Using this definition of t, Turner & Sargent (1974) concluded that a large number of de Vaucouleurs' groups have long crossing times and are therefore not bound. However, Jackson (1975) demonstrated that $t_{\rm VT}$ is a poor estimator of the crossing time, yielding values that are systematically too large. He suggested instead a moment-of-inertia crossing time based on the moment-of-inertia radius. Rood & Dickel (1978a) have used the linear crossing time

$$t_{\mathrm{L}} = \frac{2}{\pi} \frac{\langle r \rangle}{\langle V \rangle}, \tag{9}$$

where $\langle r \rangle$ is the average projected radial distance of group members from the center of mass and $\langle V \rangle$ is the average of the absolute value of the radial velocities with respect to the center of mass. Gott & Turner (1977) have adopted a similar definition of t_{L}. With these new definitions of crossing time, all three studies conclude that virtually all the groups identified by Sandage, Tammann, and de Vaucouleurs (STV) and Turner and Gott (TG) have crossing times significantly less than H_0^{-1}.

There are a number of effects which combine to bias these crossing times to systematically small values, notably the inclusion of nonmembers and the existence of binaries and subclusters, both of which increase the mean projected velocity. Even after all reasonable nonmembers are removed, however, the crossing times are still short, and it seems safe to assume that the great majority of these groups are bound density enhancements after all.

However, it is not clear that these groups have had time to reach virial equilibrium. Gott & Turner (1977) conclude that the median TG group is just now entering the virialized regime. Their calculation rests on necessarily rough dynamical estimates, however, which might not apply accurately to groups with only a few members. Given all the uncertainties, it seems possible that many of the looser groups have not virialized. We must therefore keep in mind that in some cases masses may be as much as a factor of two smaller than those obtained from the virial theorem.

6.2 *Mass-to-Light Ratios*

Limber (1959) derived the virial theorem as applied to small groups while Materne (1974) examined the treatment of observational errors. A number of uncertainties plague the analysis, as discussed by Aarseth & Saslaw (1972). Potential problems include observational errors, extrapolation to include the luminosity of faint members, and incomplete data. Furthermore, the virial theorem is based on the time-averaged energies, whereas we observe the group at just one moment in its history, when it may be out of equilibrium. Finally, we must employ projection corrections to convert the radial velocity dispersion and angular separations to their three-dimensional values. These mean projection corrections may be valid averages over a long period of time but are incorrect at any given moment. Derivation of the mean correction factors also involves assumptions as to the character of the motions, in particular that the projection corrections in velocity and radius are uncorrelated. This assumption is untrue for binaries, for example, and the standard virial formulation can substantially over- or underestimate binary masses, depending on the eccentricity of the orbits.

Taken together, these are serious difficulties. However, they are of minor importance compared to the twin problems of group membership and group definition. The group catalogs referred to above differ substantially in their membership assignments; these differences introduce uncertainties of a factor of 3–4 in the resultant mass-to-light ratios (see below).

As a result of a heightened awareness of these sources of error, a new consensus is emerging that the earlier piecemeal approach which attacked groups one at a time is unlikely to succeed. Since the analysis of any one group is subject to large statistical uncertainty, discussions of group properties must be based on a representative sample. On the observational side, we require accurate redshifts for a large, magnitude-limited sample of galaxies. Although not complete as yet, the available redshift sample is growing. Recent lengthy lists include redshifts in the RC2, velocities of galaxies in Gott-Turner groups measured by Kirshner (1977), accurate 21-cm redshifts for spiral galaxies (Dickel & Rood 1978, Shostak 1978, S. Peterson 1978, Thuan & Seitzer 1979), and many new optical redshifts by Sandage (1978). Several lengthy unpublished lists also exist (Tully & Fisher 1978, G. Knapp and W. L. W. Sargent, private communication, M. S. Roberts and co-workers, unpublished). A complete program to obtain a magnitude-limited sample of redshifts has been initiated by Davis and Huchra (Huchra 1978), but the final results are still a few years away. Taking an alternative approach, Gregory & Thompson (1978) have collected a redshift sample complete to a very deep limiting magnitude over a small region of sky.

On the theoretical side, we strongly believe that it will never be possible to assign individual galaxies to groups or fields in a definitive way. Any approach which relies solely on such group assignments is inevitably subject to insurmountable bias and cannot possibly yield reliable results. New theoretical methods are required which either avoid this bias altogether or correct for it in a statistical way.

Before discussing recent work along these lines, we summarize the results of traditional virial analyses of small groups. As was noted in previous sections, it is of paramount importance to employ a consistent system of masses and luminosities when comparing mass-to-light ratios derived by various workers. For example, the magnitude correction factor for Gott-Turner M/L_B's to our system is 0.50, and for Rood-Dickel values is 1.25. The local luminosity density on our system is $\sim 1.0 \times 10^8$

$L_\odot$ Mpc^{-3} (Gott & Turner 1976, Davis, Geller & Huchra 1978). With this value, $M/L_{\rm crit}$, the mass-to-light ratio for a universe having critical density, is $\sim$700. In such a universe, $\rho/\rho_{\rm crit}(\equiv \Omega)$ is unity.

We confine ourselves to those investigations in which a sizeable number of groups have been treated simultaneously in homogeneous fashion. Results are collected in Table 5. Consider first the TG sample. The median M/L_B for the original TG groups including bogus members is 70. Gott & Turner (1977) later presented a list of revised groups, culled of obvious nonmembers. The median M/L_B for this sample is 30, a significantly lower value. Thus although the median M/L_B for the unculled groups is of theoretical interest because of its value as an unbiased estimator of *some* statistical property of the ensemble, by itself it has no obvious connection with the true M/L_B of the sample. In order to obtain M/L_B from the TG catalog alone, one must resort to the usual strategy of choosing members, accepting the inevitable bias therein. The great value of the unculled TG catalog is that it is an unbiased sample which can properly be compared with numerical simulations of galaxy clustering. In this role it appears to be quite powerful (see below).

Rood & Dickel (1978a) have determined M/L's for the culled TG groups and for STV groups, all required to have at least three members with measured velocities. Their median value for the TG groups is 40, not significantly different from the Gott-Turner value despite the inclusion of 40% more radial velocities. The median value for the STV groups, however, is 140. According to Rood and Dickel, this difference is due to different definitions of what constitutes a group: many of the STV groups break up into subgroups using the TG prescription. This is one

Table 5 Mass-to-light ratios of small groups

Source	Median M/L_B	No. groups
Turner & Gott 1976:		
All groups, unculled	70	39
Culled groups	30	48
Rood & Dickel 1978a:		
Turner-Gott groups, culled	40	29
Sandage-Tammann-de Vaucouleurs groups	140	63
Materne & Tammann 1974	$\sim$260	14
Tammann & Kraan 1978	$\sim$40[a]	7
Tully & Fisher 1978	$\sim$40	9

[a] Mean value.

more illustration of the extent to which the final value of M/L depends on how the groups are defined.

Table 5 also includes M/L_B values from Materne & Tammann (1974), Tammann & Kraan (1978), and Tully & Fisher (1978), all based on a selection of nearby, rather poor groups. The values in the table are very approximate because these authors did not give M/L_B directly. Thus, M/L_B had to be inferred from M_{VT}/M_L, the ratio of virial mass to luminous mass.

Gott & Turner and Rood & Dickel have emphasized the existence of missing mass outside of galaxies, while the remaining three authors have minimized its importance. Yet as Table 5 indicates, the observational data seem to be similar in all cases. The disagreements among authors stem in part from differences in adopted magnitude conventions, and have been removed by placing all values of M/L_B on a common system, as in Table 5. To a large extent, however, the disagreement is philosophical: Gott & Turner and Rood & Dickel have placed greatest weight on the median values, which appear to support the existence of unseen matter. The remaining authors instead have emphasized those groups with small M/L_B and have criticized the rest as being contaminated or not in equilibrium.

Derived M/L's have also been questioned because of observational errors in the radial velocities (Karachentsev 1978, Materne & Tammann 1975, Tully & Fisher 1978). Group velocity dispersions are in many cases less than 100 km s^{-1} (Tammann & Kraan 1978), conceivably too small to be accurately measured using conventional optical velocities, which often have errors of the same magnitude. However, we believe that velocity uncertainties are not likely to grossly inflate the median M/L_B for the following reasons: first, the velocities used by Rood & Dickel are substantially more accurate than those used by Gott & Turner, yet their median M/L_B is slightly higher: second, an increasingly large number of velocities are very accurate 21-cm redshifts and 21-cm checks of optical velocities in the mean show good agreement (Rood & Dickel 1976); and third, the really high values of M/L_B are virtually all associated with groups having large velocity dispersions of several hundred km s^{-1}, much greater than the measuring errors. The radial velocities are therefore not a likely source of error. Nevertheless, we are still left with the membership question, which cannot be resolved in any convincing way on a group-by-group basis.

Attempting to break this deadlock, Aarseth and co-workers (Aarseth, Gott & Turner 1979, Turner et al. 1979) have recently introduced a new method based on N-body simulations of galaxy clustering. The resultant models are based on a variety of initial conditions, obtained by varying the mass-to-light ratio for galaxies, initial density fluctuation spectrum, starting redshift, and peculiar velocities of galaxies. Group catalogs are constructed for the model universes in a manner identical to that of the original TG catalog, and the two sets of group properties are compared. Since no culling is performed in either case, contamination enters equally in both analyses. By comparing models and data in exactly parallel fashion, one ought to obtain a useful measurement of M/L_B for groups free of bias. Turner et al. point out that an N-body model with Ω equal to 0.1 ($M/L_B = 70$) exhibits group membership characteristics very similar to those of the real universe. In particular, the distribution of galaxies in redshift space is quite different from the true spatial distribution, and group assignments based on radial velocities would often be in error.

For the same model, Turner and collaborators have emphasized that the median M/L_B for the simulated unculled groups agrees well with the true (i.e. model) M/L_B, both being equal to 70. However, this good agreement is largely a coincidence. The binary galaxies in this model have highly eccentric orbits, and their mass is badly underestimated as a result. The median M/L_B for binaries is only 23, considerably less than the model value of 70. On the other hand the groups with three or more members are strongly affected by contamination. Their median M/L_B is 200, much larger than the model value. The two errors combine fortuitously so as to make the median M/L_B of all groups together equal to the true value of the model. Moreover, nearly all TG groups in the real universe have three or more members. Therefore, if the model can be taken as a guide, it corroborates our previous conclusion that contamination in the real unculled TG groups is serious and significantly biases the median M/L_B to higher values. Furthermore, if we restrict our attention to groups with three or more members, the spread in the model mass-to-light ratios is significantly smaller than is observed in the real universe. This result supports the suggestion of Rood & Dickel (1978b) that there exists an intrinsic spread in M/L_B among groups which is larger than can be attributed to the various sources of error.

These model experiments are in their infancy and can surely be improved. Future calculations should include a realistic mass spectrum for galaxies, the effect of massive envelopes on the interactions of galaxies, and a more complete examination of initial conditions. The method of comparison between the real and simulated data might also be refined. Nevertheless, as an unbiased statistical approach to the problem, the present computations represent an original and promising line of attack which should be vigorously pursued.

6.3 *Compact Groups*

Because of possible interpenetrations of nonluminous massive envelopes in compact groups of galaxies, these groups provide a severe challenge to any theory of gravitational encounters between galaxies. Rose (1979) and Rose & Graham (1979) have recently reviewed the various hypotheses for the origin of such groups. A possible problem with these groups is their short lifetimes; if the member galaxies really do have massive envelopes, the galaxies should coalesce rapidly due to dynamical friction. Alternatively, perhaps these compact groups survive just because their galaxies have abnormally small envelopes.

Reviews of compact groups have been given by Karachentsev (1966) and by Burbidge & Sargent (1971); Rose (1977) has conducted a new survey for such groups. The problem of discrepant redshifts has long plagued the dynamical study of compact groups. Rose (1977) suggests that they can be explained simply on the basis of chance projections, but Nottale & Moles (1978) feel that discrepant cases are too frequent to be explained in this way.

For many of the groups, however, the velocity dispersion appears normal, and the virial theorem should apply. Regardless of the origin of these compact configurations, one would predict small mass-to-light ratios for those groups because the spatial separations between the member galaxies are too small to sample the gravitational potential of extended, nonluminous envelopes. This prediction appears to be consistent with the available information. The median M/L_B for the compact groups listed by Burbidge & Sargent is $\leqq 30$ (an exact value cannot be specified because the magnitude system was not fully described), while Rose & Graham find values of only 2.3 and 12.2 for two southern compact groups.

6.4 *The Local Group*

The seminal paper on Local Group dynamics was written by Kahn & Woltjer (1959). Lynden-Bell & Lin (1977) and Yahil, Tammann & Sandage (1977) have recently given thorough rediscussions of the problem, while de Vaucouleurs, Peters & Corwin (1977) have given a new solution for the solar motion relative to Local Group galaxies.

The dynamical analysis of the Local Group is deeply entwined with the determination of the velocity of the sun's motion about the galactic center. The Milky Way and M31 together dominate the kinetic and potential energies of the Local Group to the extent that the problem becomes in essence an ordinary two-body interaction. The velocity of M31 relative to the galaxy is therefore essential to a knowledge of their mutual orbit. By sheer bad luck, the apparent radial velocity of M31 with respect to the sun is in large measure simply a reflection of the sun's motion about the galactic center. We may take comfort in the fact that the coincidence is lessening with time: in 40 million years or so the two problems will be geometrically independent. For the present, however, we must struggle to disentangle the two motions.

The magnitude of the sun's orbital velocity is a matter of dispute, but the values widely discussed range between 220 km s^{-1} (Section 2.2) and 300 km s^{-1} (Lynden-Bell & Lin 1977, Yahil et al. 1977). Using these values to correct the radial velocity of M31 to the galactocentric value, we obtain -125 km s^{-1} and -60 km s^{-1}, both negative. This is the key point: no matter how large a rotational velocity for the sun is assumed, within reasonable limits, we cannot convert the apparent approaching motion of M31 into one of recession. Barring a theoretically implausible "slingshot" effect in which M31 or the Milky Way caromed off some third galaxy in the past, the velocity of approach of the two galaxies must arise from their mutual gravitational interaction. Hence, there must have been time for the orbital motion to "turn around" during the lifetime of the universe, and this requirement in turn makes the galaxies considerably more massive than mere boundedness of the orbit would imply. We review this argument in some detail because, even though it was outlined quite clearly in Kahn & Woltjer's original paper (see also Peebles 1971), one still sees it ignored today in favor of estimates based on energy considerations alone. This omission seems hardly reasonable given the lack of any other convincing theory for the motion of approach of the two galaxies.

We assume that the orbital motion is radial and that the orbital time is 2×10^{10} yr. Using convenient formulae given by Gunn (1974), we calculate the total mass of M31 plus the Milky Way to be $\geqq 2.9 \times 10^{12}$ $M_\odot$ and $\geqq 1.1 \times 10^{12}$ $M_\odot$ for local circular velocities of 220 and 300 km s^{-1}, respectively. The luminosity of M31 is 2.7×10^{10} $L_\odot$ on our system, and that of the Milky Way is 2.0×10^{10} $L_\bullet$ (Section 2.2). We then obtain mass-to-light ratios of $\geqq 60$ and $\geqq 25$ respectively, values quite typical of small groups and binary galaxies. These values are lower limits because the assumption of radial motion yields the minimum possible mass. These results are quite consistent with the value of $M/L_B \lesssim 70$ for the Milky Way alone found in Section 2.2.

We have so far neglected a second consideration. The orbital solution described above must also be consistent with the observed motion of the sun with respect to the center of the mass of the Local Group, which is assumed to be the center of mass of M31 and the Milky Way. Using this additional constraint, one obtains best-fit solutions of the solar orbital velocity close to 300 km s^{-1} (Lynden-Bell & Lin 1977, Yahil et al. 1977), although 220 km s^{-1} is still within the 90% probability contour. A more accurate value for the solar motion relative to Local Group galaxies would help greatly to narrow these possibilities, but it probably will never be forthcoming—there are simply too few Local Group members to serve as referents.

6.5 *Conclusion*

It is still impossible to give a definitive value for the mass-to-light ratios of small groups of galaxies because the remaining uncertainties are large. As a temporary expedient, one can use the numerical simulation of Aarseth et al. to estimate the effect of contamination on the median M/L_B for unculled TG groups. This is the only bias-free method available at present. We employ the model with $\Omega = 0.1$, including only those simulated groups with three or more members. The median M/L_B for this subsample is 200, compared to the true value of 70 for the model as a whole. The median M/L_B for observed unculled TG groups having three or more members is 80. Assuming the simulated universe is a fair match to the real one, we apply the correction factor 70/200 to this value, obtaining 30 as the best unbiased estimate for the median M/L_B of small groups. This value is to be compared with values of 30 and 40 for the culled TG groups found by Gott & Turner and Rood & Dickel, and 90 for both the TG and STV groups having the best membership assignments (Rood & Dickel 1978a). These values probably encompass the allowable range. Though still very uncertain, these estimates seem distinctly higher than M/L_B within R_{HO} for individual spirals (~ 5) yet very compatible with the range of 35–50 estimated for binaries in the previous section. The mass-to-light ratios in groups therefore do not imply the existence of additional dark matter beyond that sampled by the orbits of wide binary galaxies.

7 GALAXY MASSES FROM CLUSTER MEMBERSHIP

With the recognition of galaxy clusters as dynamical entities, it was realized that the internal kinematics could be used to measure average masses of galaxies. Zwicky (1933) and Smith (1936) applied the virial theorem to the Virgo cluster using the small sample of radial velocities then available and found that the virial mass exceeded by a factor of several hundred that expected from the sum of the masses of individual galaxies. In a classic paper, Zwicky (1937) derived an M/L_B of 500 for the

Coma cluster which he compared with the $M/L_B \simeq 3$ for the solar neighborhood. Thus the "missing mass," or more properly "invisible mass," problem was born. Since then, our understanding of the structure and properties of clusters of galaxies has grown enormously (see reviews by Abell 1975, van den Bergh 1977, Bahcall 1977). However, the problem of invisible mass in the great regular clusters has not abated.

7.1 *The Virial Theorem*

In applying the virial theorem to large clusters, it is assumed that the cluster is in a stationary state. The kinetic energy is obtained from an appropriately weighted determination of the space velocity dispersion V^2 (Rood, Rothman & Turnrose 1970; see also Rood & Dickel 1978a). Choosing a proper weighting scheme is not a trivial point; for example, Chincarini & Rood (1977) show that the calculated mass of Abell 194 (hereafter, A194) varies by almost a factor of three among plausible weighting schemes. The issue is whether to weight by observed luminosities or by masses inferred from mean mass-to-light ratios. The latter method introduces great uncertainty since mean M/L's are poorly known for early-type galaxies, which are a significant fraction of the membership in large clusters.

As with small groups, a correction must also be applied to convert the observed projected velocity dispersion σ^2 to V^2, which will in general depend on the types of orbits in the cluster. If the orbits are primarily radial and we determine σ^2 from the cluster core, then V^2 may only slightly exceed σ^2, while if the orbits are nearly circular with an isotropic distribution, $V^2 \simeq 3\,\sigma^2$ (Abell 1977). Thus the kinetic energy term may be uncertain by as much as a factor of 3, although Rood (1970) has suggested that usually $2 < V^2/\sigma^2 < 3$ and that a ratio of 2.1 is most appropriate for the Coma cluster.

In calculating the potential energy, the mass distribution of the cluster is usually taken to be the same as that of the galaxies. Schwarzschild (1954) showed that the number of galaxies per unit area, S, along a strip passing a minimum distance q from the cluster core could be used to find the effective virial radius,

$$R_{\mathrm{VT}} = 2\,\frac{(\int S\,dq)^2}{\int S^2\,dq}. \tag{10}$$

Unfortunately, S is difficult to measure precisely, as it depends on the statistical correction for background galaxies, which is most important near the outer parts of the cluster and can introduce significant errors in R_{VT} (Rood et al. 1972). As an alternative to Equation (10), the observed surface density profile may be inverted to give the space density distribution $\rho(r)$, and the potential integral then can be explicity evaluated (Oemler 1974, Abell 1977). Once R_{VT} has been found, the virial mass of the cluster can be calculated in the usual way as $M_{VT} = V^2 R_{VT}/G$.

In this section we use luminosities on the V system since virtually all cluster work has employed this convention. Furthermore, most cluster photometry is necessarily somewhat less precise than that for nearby, bright galaxies, and thus the derived M/L are more uncertain than those found by other methods. We convert to M/L_B at the end of the section.

We have recomputed virial masses and M/L's in a homogeneous way for seven large clusters. We have taken σ^2 from Yahil & Vidal (1977) and cluster luminosities and virial radii from Oemler's study of galaxy populations in 15 clusters. In order to illustrate a standard virial mass, we have assumed $V^2/\sigma^2 = 3$. The dispersions have been multiplied by $(1+z)^{-1}$ to correct for redshift (Harrison 1974, Faber & Dressler 1977), and cluster distances are based on mean radial velocities. We find the median $M/L_V \approx$ 290, with a range of 165–800. This result is typical of values quoted in the literature (Bahcall 1975) and is substantially larger than M/L's of individual galaxies found in Table 2.

However, there is considerable room to maneuver within the framework of the virial theorem. The ratio M/L could be reduced by a factor of 2 or 3 if our guess concerning velocity isotropy is incorrect. An additional decrease might be obtained by mass-weighting the velocity dispersion and by correcting the luminosities for halos of galaxies and faint cluster members. Chincarini & Rood (1977) show that these considerations can reduce M/L_{pg} for A194 to 36 and that of Coma to 170. It is also possible that irregular clusters such as Hercules are not yet in virial equilibrium. The simple requirement for boundedness would reduce the mass by a factor of two. Hercules would then have a visual mass-to-light ratio of 270, rather typical of other large clusters.

7.2 *Other Methods of Mass Determination*

Unfortunately, the use of the virial theorem to measure cluster masses is sensitive to the structure of the outer parts of the cluster (Rood et al. 1972). An alternative is to consider only the well observed core regions. Zwicky (1937, 1957) found that the number-density profile of the Coma cluster could be fit by a bounded isothermal sphere. Using such a model, Bahcall (1975) derived a mean core radius, r_c, of 0.25 ± 0.04 Mpc for 15 clusters. Dressler (1978a) also found a small dispersion in core radius but his mean value was 0.5 Mpc. Avni & Bahcall (1976) have tested the measuring techniques for r_c on simulated clusters. They found that the existence of cluster cores is real, not an artifact of the data analysis, but that the close fit of Bahcall's clusters to a common isothermal model is due to the fitting procedure used.

If r_c and the core velocity dispersion, σ_c, are known for a cluster, a dynamical model can be used to calculate the central mass density. The King (1966, 1972) models were shown by Rood et al. (1972) to provide a reasonable representation of both the run of surface density and of velocity dispersion with radius in the Coma cluster. With this method, M/L_V values of 280, 350, and 250 have been derived for the cores of Coma (Rood et al. 1972), Perseus (Bahcall 1974), and Virgo (van den Bergh 1977). The value for Coma has been rescaled using a revised core velocity dispersion of 1260 km s^{-1} (Gregory & Tifft 1976).

Core-fitting procedures are subject to operational difficulties which Rood et al. clearly describe: first, the core radius is difficult to determine (e.g. the factor of two difference in mean r_c between Bahcall and Dressler); second, due to small-number statistics, the core velocity dispersion is uncertain and the core luminosity distribution is grainy. The situation is often further complicated by the presence of centrally located luminous galaxies, which make it difficult to measure the core luminosity accurately. To avoid the problem of graininess in the number density, Dressler (1978b) used the smoothed central number density from the model fit, assigned a mean luminosity per galaxy using a Schechter (1976) luminosity function, and applied a 20% correction to convert from isophotal to total luminosities. From a sample of nine rich clusters with velocity dispersions primarily taken from Faber & Dressler (1977), he found a median M/L_V equal to 270. For comparison, he computed a global M/L_V for each cluster using a fit to a de Vaucouleurs' (1948) law (see Young 1976). This approach yielded a median M/L_V equal to 280. The global and core-fitting methods are therefore in good agreement for these clusters. Rood et al. also analyzed the Coma cluster with a global de Vaucouleurs law and found $M/L_V = 200$.

With the increasing sophistication of computer N-body models, it has become possible to custom build a model to represent a specific cluster. White (1976b) extended the N-body program of Aarseth (1969) to produce a realistic dynamical model of the Coma cluster using 700 particles. The model was scaled to Oemler's gravitational radius and the radial velocities of Rood et al. and Gregory (1975). From the model, White (1976b) found $M/L_V = 258 \pm 36$. Melnick, White & Hoessel (1977) have shown that the value of M/L_V is reduced slightly to 207 ± 47 if L_V is corrected to the total light in the central regions of Coma.

All of these analyses of the Coma cluster yield values of M/L_V close to 250. The exception is Abell's (1977) study, which concludes that $M/L_V =$

120. This result is due to two factors. First, the derived mass is small because Abell assumed $V^2/\sigma^2 = 2$ rather than 3. More important, however, the form of the luminosity function chosen by Abell results in a luminosity twice as large as Oemler's. Relative to Oemler, Abell counts more galaxies with $13.5 < m_v < 14.5$. Furthermore, Oemler's faint-end extrapolation adds fewer galaxies than actually counted by Abell. Both effects contribute roughly equally to the net difference, with the excess at the bright end being somewhat more significant. Godwin & Peach (1977) also measured the luminosity function in Coma and found more galaxies with $15 \leqq m_v \leqq 17.5$ than either Abell or Oemler.

It suffices to note that luminosity functions of clusters are still not well known. This problem is especially severe in poorly studied clusters, where one must assume a universal luminosity function to estimate the contribution of faint galaxies. Furthermore, there is good evidence for significant cluster-to-cluster variations in the luminosity function, and luminosity functions may not be amenable to simple analytic forms (Dressler 1978a).

Ignoring uncertainties in the total luminosity for the moment, we emphasize the good agreement between the mass-to-light ratios for Coma found by many different authors using various techniques. Furthermore, these various approaches weight subsets of the data in very different fashions, some emphasizing the core, others the outer regions. It is indeed remarkable that these techniques yield such consistent values.

7.3 *Uncertainties*

Adopting the mass-to-light ratio of Coma as typical of great clusters, we have $M/L_V \approx 250$ and $M/L_B \approx 325$, much larger than those of individual galaxies and significantly larger than those of binaries and small groups. Without doubt, the single aspect of the data that contributes most to the large M/L in clusters is their large velocity dispersions. We must therefore consider the possible effects of contamination and subclustering on this parameter.

To assess contamination, the cluster must be viewed in the context of its environment. The N-body calculations of Aarseth et al. (1979) are appropriate here since they model a representative volume of space rather than an isolated cluster. For their model with $\Omega = 0.1$, the largest cluster has $M/L_B = 320$, much larger than the model value of 70 (Turner et al. 1979). This discrepancy is entirely a result of contamination. Turner et al. point out that substantial contamination is to be expected in rich clusters because of their large angular extent, and on this basis question the belief that the great clusters provide a reliable estimate of masses of galaxies. They also show that because of the high intrinsic velocity dispersions in clusters, noncluster members in a large volume of the Hubble flow can cause confusion. Because of this effect, Turner et al. suggest that velocity sampling alone cannot eliminate contamination. L. Thompson (private communication) has likewise speculated that the traditional Coma sample is contaminated by unbound members of the Coma supercluster.

To assess the severity of this effect, we need better dynamical models of clusters and their environs, plus a very deep and complete survey of radial velocities in several cluster complexes. Unfortunately this material is not yet available. For the present, however, we are inclined to think that contamination is not solely responsible for large cluster dispersions. Our belief rests principally on the core properties of such clusters, where the density contrast is large and the contamination therefore smaller. As we have seen, the core fitting procedures yield mass-to-light ratios just as high as those from global methods. Furthermore, in Coma the velocity dispersion declines markedly outside the core region, whereas one would in general expect the reverse if contamination were significant. Finally, virtually all the galaxies in the Coma core are of early morphological type, a much larger fraction than those either in small groups or in the extended supercluster (Gregory & Thompson 1978). These galaxies therefore appear to be a distinct population physically located in the core itself.

Holmberg (1961) suggested that cluster velocity dispersions might be spuriously inflated through the inclusion of binaries and subclusters. However, since binaries and small groups generally have dispersions $\lesssim 300$ km s^{-1}, this effect cannot produce the large dispersions of several hundred km s^{-1} typical of great clusters.

X-ray observations of clusters may soon provide an independent check on cluster potential and kinetic energies. With the discovery of iron-line emission in clusters of galaxies (Mitchell et al. 1976, Serlemitsos et al. 1977), the probability that cluster X rays originate from thermal bremsstrahlung in a hot intracluster medium (ICM) has greatly increased. If the emission is thermal, the X-ray temperature is determined by the cluster potential. For gas in equilibrium, the X-ray temperature should follow $T \propto V^2$ (or σ^2) (Mushotzky et al. 1978, Jones & Forman 1978). Although the current data are limited, there is a good correlation between the observed X-ray temperatures and σ^2 in X-ray clusters (Mitchell, Ives & Culhane 1977, Jones & Forman 1978, Mushotzky et al. 1978). A strong statement would be premature, but the currently available data are consistent with the relation $V^2 = 3\,\sigma^2$, where σ is the usual line-of-sight velocity dispersion. With more observations, we expect that modelling of the ICM will provide an accurate measurement of cluster potentials and thus of cluster masses.

Finally, the considerable range in M/L_V among large clusters themselves suggests that the amount of unseen matter might vary significantly from cluster to cluster. Rood (1974) and Rood & Dickel (1978a) have emphasized this possibility and noted that the virial mass discrepancy is strongly correlated with the velocity dispersion for both small groups and large clusters.

7.4 *Conclusion*

For the present, we conclude that the available data continue to support the existence of high cluster mass-to-light ratios, with $M/L_B \approx 325$. This value at first sight looks significantly larger than the values of 30–90 which we determined for binaries and small groups. However, part of the discrepancy is due to stellar population differences between the early-type galaxies in clusters and the spirals in small groups. The theoretical and observational data in Table 2 suggest that M/L_B for the stellar population in spirals is only half that in ellipticals and S0's. So, to properly compare small groups with great clusters, we must increase M/L_B for small groups from 30–90 to 60–180. Thus roughly 20% to 50% of the total mass can plausibly be associated with galaxies.

A further small correction must be made for ionized gas. Conventional X-ray measurements of cluster cores suggest that the mass of ionized gas is probably $\sim$10% of the virial mass (Lea et al. 1973, Field 1974, Gull & Northover 1975, Malina et al. 1978), in agreement with the requirements imposed by the detection of microwave diminution by hot gas in clusters (Birkinshaw, Gull & Northover 1978). Adding 10% to our previous total, we can account for roughly 30% to 60% of the virial mass, leaving a net discrepancy of approximately a factor of two. Although this difference is uncomfortably large, real difficulties still remain in the determination of cluster M/L ratios (e.g. the luminosity function). The reality of excess unseen mass in great clusters relative to small groups must therefore still be considered uncertain at the present time.

We note in passing that a controversial detection of large X-ray halos around clusters has just been reported by Forman et al. (1979). While the gas in these halos potentially could bind the cluster as a whole, the halo has little dynamical influence on the core and thus cannot ease the M/L problem in the central regions.

White (1976a, 1977) has shown through N-body models that the dark material in clusters cannot all be attached to individual galaxies, or else a marked degree of radial mass segregation should be observed owing to dynamical friction. Since little segregation is apparent in real clusters, the hidden matter must be distributed rather uniformly throughout the cluster as a whole. Hence, galaxies in clusters cannot have large massive envelopes still attached. Gallagher & Ostriker (1972) and Richstone (1975, 1976) have suggested that high-velocity encounters between galaxies might liberate these envelopes through tidal shocks, thus spreading the dark matter throughout the cluster. The exact interrelationship between the competing processes of tidal disruption and dynamical friction remains to be worked out.

8 CODA

After reviewing all the evidence, it is our opinion that the case for invisible mass in the Universe is very strong and getting stronger. Particularly encouraging is the fact that the mass-to-light ratio for binaries agrees so well with that for small groups. Furthermore, our detailed knowledge of the mass distribution of the Milky Way and Local Group is reassuringly consistent with the mean properties of galaxies and groups elsewhere. In sum, although such questions as observational errors and membership probabilities are not yet completely resolved, we think it likely that the discovery of invisible matter will endure as one of the major conclusions of modern astronomy.

In addition to the dynamical evidence, there are other indirect indications of dark material in galaxies. The most important of these are the stability analyses of cold, self-gravitating axisymmetric disks (e.g. Ostriker & Peebles 1973, Hohl 1976, Miller 1978), which show them to be susceptible to bar-formation if not stabilized by a hot dynamical component. This hot component may or may not be related to massive envelopes.

Although present data give us little information on the shape of massive envelopes, further study of the outermost hydrogen in spirals may tell us more about this question. For example, the apparent lifetime of warps in many spirals poses severe theoretical difficulties as long as it is assumed that disks are self-gravitating (e.g. Binney 1978, Bosma 1978). This problem would not arise if the warps existed within the potential of a nearly spherical massive envelope. The precession of the warp due to the torque of the disk would then be much smaller, and the warp would be very long-lived. Alternatively, Binney (1978) has suggested that a warp might actually be driven by a triaxial dark halo. Finally, z-motions of H I far from the nucleus can be used to measure the space density of matter in the plane and thus to set limits on the flattening of the envelope.

Despite the general lack of observational evidence on the shapes of massive envelopes, there exists a strong consensus among theorists that they cannot be very flat. It is widely suggested that the large radial extent of the dark material relative to the luminous matter is due to the dissipationless collapse of the invisible matter. If so, a thin disk is unlikely, and we would more plausibly expect a thickened mass distribution spheroidal or triaxial in shape.

Suggestions as to the identity of the unseen matter include massive neutrinos (Cowsik & McClelland 1972, Gunn et al. 1978), faint stars (Ostriker, Peebles & Yahil 1974), black holes (Truran & Cameron 1971), and comets (Tinsley & Cameron 1974). Many attempts have been made to detect luminous matter in the halos of edge-on galaxies (e.g. Freeman, Carrick & Craft 1975, Gallagher & Hudson 1976, Hegyi & Gerber 1977, Kormendy & Bruzual 1978, Spinrad et al. 1978). Although faint luminosity has been found in some cases, it can plausibly be identified with the normal spheroidal stellar component.

Further progress in the study of unseen matter will continue to be made by mapping the gravitational potential using all observable test particles. Massive envelopes may well have a significant effect on the shapes and velocities of bridges and tails created in tidal encounters. The embarrassingly short theoretical lifetimes of binary galaxies and compact groups require careful consideration, as does the hypothesized stripping of extended halos and subsequent redistribution of the dark matter during the collapse of dense clusters. Most important, we need to know whether luminosity is a good indicator of mass density over scales greater than a few kiloparsecs. If a sizeable fraction of the mass in the universe is uncorrelated with the visible light, our dynamical analyses might be greatly in error. For example, our basic model for the formation of a group or cluster as a dissipationless collapsc of noninteracting mass points might need serious revision. It is to be hoped that the systematic redshift surveys now in progress, coupled with more realistic theoretical simulations of galaxy interactions, will eventually yield definitive answers to these and related questions.

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