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BLACK HOLE MODELS FOR ACTIVE GALACTIC NUCLEI

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1. INTRODUCTION

It is now 20 years since active galactic nuclei (AGNs) became widely acknowledged as an important astrophysical phenomenon (33, 109). Over the entire subsequent period, one of the few statements to command general agreement has been that the power supply is primarily gravitational: the whole bestiary of models involving dense star clusters, supermassive stars, or black holes at least have this feature in common. Systems dependent on gravitational energy have something else in common: they all undergo an inexorable runaway as the central potential well gets deeper and deeper. According to conventional physics, the almost inevitable endpoint of any dense star cluster or supermassive star will be the collapse of a large fraction of its total mass to a black hole. This is the "bottom line" of Figure 1. Such arguments suggest that massive black holes should exist in the nuclei of all galaxies that have ever experienced a violently active phase. Furthermore, physical processes involving black holes offer a more efficient power supply than any of the "precursor" objects depicted in Figure 1. So massive black holes may not merely be the defunct remnants of violent activity; they may also participate in its most spectacular manifestations.

Considerations such as these have shifted the emphasis of theoretical work away from dense star clusters and supermassive stars and have motivated fuller (or at least less perfunctory) investigations of how black holes might generate the power in quasars, radio galaxies, and related objects. All of the evolutionary tracks in Figure 1 deserve more study: none can be dismissed as irrelevant to the AGN phenomenon. The present review is nevertheless focused on black hole models. Moreover, its scope is even more restricted: I am primarily concerned here with what goes on close to the black hole—in the region where the gravitational potential is not merely

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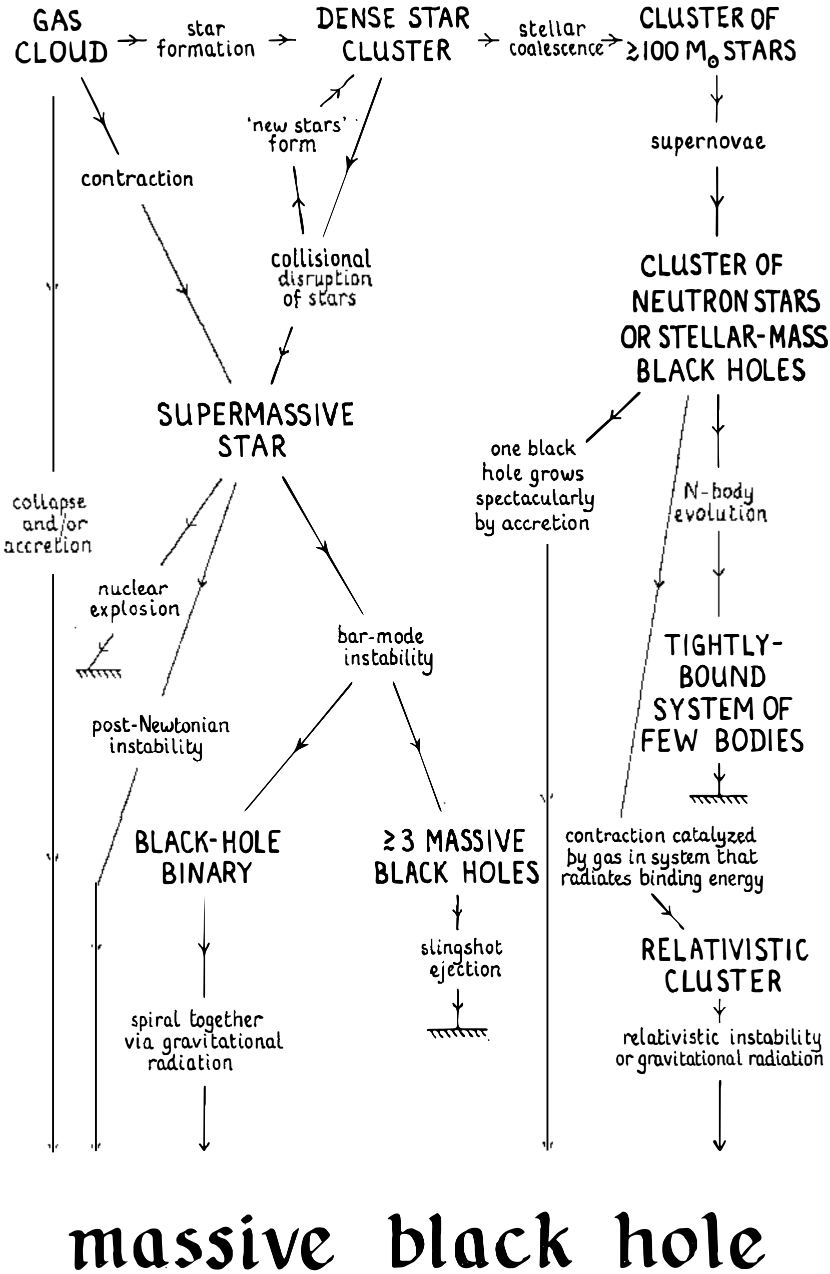


Figure 1 Schematic diagram [reproduced from Rees (106)] showing possible routes for runaway evolution in active galactic nuclei.

"(1/r)," but where intrinsically relativistic features can also be significant. Although this is where the power output is concentrated, many conspicuous manifestations of AGNs—the emission lines, the radio components, etc.—involve some reprocessing of this energy on larger scales. For this reason (and also because of space limitations), little is said here about phenomenology: I merely discuss some physical processes and simple idealized models that have been advanced as ingredients of AGNs.

Two obvious generic features of active galactic nuclei are (*a*) the production of continuum emission, which in some cases at least must be nonthermal (probably synchrotron); and (*b*) the expulsion of energy in two oppositely directed beams. The activity is manifested on many scales—up to several megaparsecs in the case of the giant radio sources. It is a tenable hypothesis, however—and one implicitly adopted here—that the central prime mover is qualitatively similar in all of the most highly active nuclei, and that the wide differences observed reflect "environmental" factors on larger scales (where the primary energy output can be reprocessed) and perhaps orientation effects as well.

2. INFERENCES INSENSITIVE TO DETAILED MODEL

2.1 *Some Fiducial Numbers*

Before focusing on specific properties of black holes, it is interesting to consider some general features of compact ultraluminous sources. Certain order-of-magnitude quantities are involved in any model.

A central mass M has a gravitational radius

$$r_g = \frac{GM}{c^2} = 1.5 \times 10^{13}\, M_8 \text{ cm}, \qquad 1.$$

where M_8 is the mass in units of $10^8\, M_\odot$. The characteristic minimum time scale for variability is

$$r_g/c \simeq 500\, M_8 \text{ s}. \qquad 2.$$

A characteristic luminosity is the "Eddington limit," at which radiation pressure on free electrons balances gravity:

$$L_E = \frac{4\pi G M m_p c}{\sigma_T} \simeq 1.3 \times 10^{46}\, M_8 \text{ erg s}^{-1}. \qquad 3.$$

Related to this is another time scale (112):

$$t_E = \frac{\sigma_T c}{4\pi G m_p} \simeq 4 \times 10^8 \text{ yr}. \qquad 4.$$

This is the time it would take an object to radiate its entire rest mass if its luminosity were L_E. The characteristic blackbody temperature if luminosity L_E is emitted from radius r_g is

$$T_E \simeq 5 \times 10^5 M_8^{-1/4}. \tag{5}$$

We can further define a characteristic magnetic field, whose energy density is comparable with that of the radiation. Its value is

$$B_E \sim 4 \times 10^4 M_8^{-1/2} \text{ G}. \tag{6}$$

The expected field strengths induced by accretion flows can be of this order. The corresponding cyclotron frequency is

$$\nu_{cE} \simeq 10^{11} M_8^{\ 1/2} \text{ Hz}. \tag{7}$$

The Compton cooling time scale for a relativistic electron of Lorentz factor γ_e (equivalent to the synchrotron lifetime in the field B_E) is

$$t_{cE} \simeq (m_e/m_p)\gamma_e^{-1}(r_g/c) \simeq 0.3\, \gamma_e^{-1} M_8 \text{ s}. \tag{8}$$

The photon density n_γ within the source volume is $\sim(L/r^2c)/\langle h\nu\rangle$. If a luminosity fL_E emerges in photons with $h\nu \approx m_e c^2$, which can interact (with a cross section $\sim\sigma_T$) to produce electron-positron pairs (40), then these photons will interact before escaping if

$$f > (m_e/m_p)(L/L_E)^{-1}(r/r_g). \tag{9}$$

Several inferences now follow about the radiation processes, given only the assumption that a primary flux with $L \simeq L_E$ is generated within radii a few times r_g:

1. Thermal radiation from *optically thick* material would be in the far-ultraviolet or soft X-ray region; if, however, thermal gas in the region were hot enough to emit X-rays, reabsorption would be unimportant.
2. If the bulk of the luminosity L were synchrotron radiation in a field $B \simeq B_E$ (Equation 6), then the self-absorption turnover would be (99)

   $$\nu_{sE} = 2 \times 10^{14} M_8^{-5/14} \tag{10}$$

   (i.e. typically in the infrared). No significant *radio* emission can come directly from $r \simeq r_g$ unless some coherent process operates at $\nu \simeq \nu_{cE}$. Synchrotron emission at $\sim\nu_{sE}$ would require electrons with $\gamma_e \simeq 40\, M_8^{1/14}$.
3. The synchrotron or inverse Compton lifetimes of relativistic electrons is $\ll (r_g/c)$ under these conditions, so in any model involving such mechanisms, the radiating particles must be injected or repeatedly reaccelerated at many sites distributed through the source volume.

4. If a substantial fraction of the radiation were generated as gamma rays with energies $\gtrsim 1$ Mev, then electron-positron pairs would inevitably be produced.

This last point is less familiar than the previous three, and so it may merit some elaboration. Photons with energies above 0.5 Mev will experience an optical depth to pair production that exceeds unity whenever (Lf/r) exceeds a value equivalent to $\sim 5 \times 10^{29}$ erg s^{-1} cm^{-1}. Moreover, the annihilation rate constant for these pairs is $\sim \sigma_T c$ if they are subrelativistic, and smaller by $\sim \gamma_e^2$ if they are ultrarelativistic (104). This has the important consequence that a compact source that produces gamma rays (either thermally or nonthermally) at a steady rate satisfying (9.) will shroud itself within an optically thick "false photosphere" of electron-positron pairs, which scatters and Comptonizes all lower-energy photons (58).

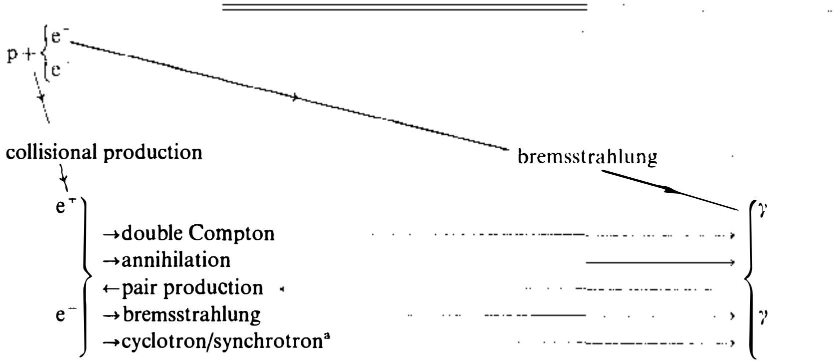
2.2 *Processes in Ultrahot Thermal Plasma*

The only quantities entering into the above discussion have been essentially those involving the electromagnetic energy densities. We now consider the physical conditions in plasma near a collapsed object. If thermal plasma can radiate efficiently enough, it can cool (even at $r \simeq r_g$) to the relatively modest temperature T_E (Equation 5). However, two-body cooling processes can be inefficient at low densities; for this reason, and also because the energy available in the relativistically deep potential well may amount to 100 Mev ion^{-1}, the plasmas in AGNs may get hotter than those familiarly encountered elsewhere (even by astrophysicists).

At ion temperatures up to, say, $kT_i = 100$ Mev the ions are of course nonrelativistic, but the thermal electrons may be relativistic. The main distinctive effects arise because the time scale for establishing electron-ion equipartition via two-body processes, or even for setting up a Maxwellian distribution among the electrons themselves, may exceed the time scale for radiative cooling via the same two-body effects. Moreover, other cooling processes may hold the electron temperature to $\lesssim 1$ Mev even if the ions are much hotter. Detailed discussions of these various time scales are given by Gould (54–56) and Stepney (121).

COMPTONIZATION If photons of energy $h\nu$ are scattered by electrons with temperature T_e such that $kT_e \gg h\nu$, then there is a systematic mean gain (67, 125) in photon energy of $(\delta\nu/\nu) \simeq (kT_e/m_e c^2)$ until, after many scatterings, a Wien law is established. If soft photons are injected in an optically thick ($\tau_T > 1$) source, then the emergent spectrum depends essentially on the parameter $y = \tau_T^2(kT_e/m_e c^2)$: if $y \ll 1$, nothing much happens; if $y \gg 1$, a Wien law is set up; but in the intermediate case when $y \simeq 1$, the emergent spectrum has an approximate power-law form. When $kT \simeq m_e c^2$, the

Table 1 Main production/annihilation processes for electron energies >0.5 Mev



[a] If magnetic field is present.

energy change in each scattering is too large for a diffusion approximation to be valid, and Monte Carlo methods are needed (57).

PAIR PRODUCTION EFFECTS When the electron energies on the tail of the Maxwellian distribution exceed a threshold of 0.5 Mev, collisional processes can create not only gamma rays but also e^+-e^- pairs. These pairs then themselves contribute to the cooling and opacity; the physical conditions must therefore be computed self-consistently, with pairs taken into account (22). Discussions have been given by several authors (22, 40, 69, 70, 132).

The main production/annihilation processes are summarized in Table 1. Further high-energy processes can operate above 50 Mev (46). The fullest discussions of thermal balance in relativistic plasma that take pairs into account are due to Lightman and collaborators (7, 69, 70) and Svensson (122–124). There is a maximum possible equilibrium temperature, of order 10 Mev; but if the heat input is raised beyond a certain value, the increment in pair density is so great that the temperature falls again toward 1 Mev. Note that to extend the usual cooling function $\Lambda(T_e)$ into the temperature range where pair production is important, one must specify the column density $n_i r$ of the source as a second parameter (n_i being the ion density). When $n_i r \ll 1$, the dominant pair production is via e-p collisions; but for sources of higher column density, relation (9.) may be fulfilled, and more pairs come from $\gamma+\gamma$ encounters.

2.3 *Cyclotron/Synchrotron and Inverse Compton Cooling*

Suppose that the magnetic energy is q times the rest mass density of the plasma: we might expect $q \lesssim kT_i/m_p c^2$ for accretion flows. The ratio of the cyclotron cooling time (neglecting reabsorption) to the bremsstrahlung time for a subrelativistic electron is $\alpha_f(m_e/m_p)q^{-1}(kT_e/m_ec^2)^{-1/2}$, which is $\ll 1$ for a plasma with $kT_i \gtrsim 1$ Mev with an equipartition field; for ultrarelativistic electrons the dominance of synchrotron losses over bremsstrahlung is even greater. Analogously, Compton losses can be very important: indeed, in any source where Thomson scattering on electrons (or positrons) yields $\tau_T > 1$, the requirement that the Compton y-parameter be $\lesssim 1$ implies that the electrons or positrons must be mostly subrelativistic.

The conventional distinction between thermal and nonthermal particles becomes somewhat blurred in these contexts where two-body coupling processes cannot necessarily maintain a Maxwellian distribution. Various acceleration mechanisms (relativistic shocks, reconnection, etc.) may, moreover, boost some small fraction of the particles to high γ: such mechanisms operate in many contexts in high-energy astrophysics and should be even more efficient in an environment where the bulk velocities and Alfvén speeds are both $\sim c$. These particles would then emit synchrotron or inverse Compton radiation. Such acceleration would be "impulsive," in the sense that its time scale is $\ll r_g/c$. The accelerating force would be eE, where E ($\lesssim B$) is the electric field "felt" by the charge. There is then a characteristic peak energy attainable by such processes (39), namely that for which the radiative drag due to synchrotron and inverse Compton emission equals eB. For $B = B_E$ (Equation 6), this yields $\gamma_{drag} = 4 \times 10^5\ M_8^{1/4}$. For acceleration along straight fieldlines, synchrotron losses are evaded, and the terminal energy could be $\sim B_E r_g$ (corresponding to $\gamma_e = 3 \times 10^{14}\ M_8^{1/2}$) if linear acceleration operated over the whole scale of the source. Such limiting energies have emerged from specific studies of accretion disk electrodynamics (34, 72). However, inverse Compton losses cannot be evaded in this way, and they would set a limit not much greater than γ_{drag}. (Individual *ions*, not subject to radiative losses, could in principle get more energetic than electrons.) The parameter γ_{drag} scales as $B^{-1/2}$, and electrons with this energy emit synchrotron photons with $h\nu \simeq \alpha_f^{-1} m_e c^2$ (i.e. 60 Mev) (58, 99). Inverse Compton radiation from the same electrons could of course have photon energies right up to $\gamma_{drag} m_e c^2$. There is thus no reason why a (power-law?) spectrum should not extend up to the gamma-ray band.

3. RADIAL ACCRETION FLOWS

The plasma around black holes will be in some dynamical state—participating in an accretion flow, or perhaps in a wind or jet. Realistically, it would probably be very inhomogeneous: a "snapshot" might reveal many dense filaments at $T \simeq T_E$, embedded in ultrahot thermal plasma filling most of the volume, as well as localized sites where ultrarelativistic electrons are being accelerated. But it is a basic prerequisite for such modeling to know how the various cooling and microphysical time scales compare with the dynamical time at a radius r ($\gtrsim r_g$). The latter can be written as

$$t_{\text{inflow}} = \alpha^{-1}(r_g/c)(r/r_g)^{3/2}. \tag{11}$$

The parameter α, equal to one for free-fall, is introduced explicitly at this stage because the numbers all scale straightforwardly to cases (with $\alpha < 1$) where the inflow is impeded by rotation or by pressure gradients. (In deriving these characteristic numbers, we approximate the flow as spherically symmetric: although this is roughly true for thick tori, further geometrical factors obviously enter for thin disks.)

If accretion with efficiency ε provides the power, the value of $\dot{M}$ needed to supply a luminosity L can be written as $\dot{m}\dot{M}_E = (L/L_E)\varepsilon^{-1}$, where $\dot{M}_E = L_E/c^2$. The particle density at radius r corresponding to an inflow rate $\dot{m}$ is

$$n \simeq 10^{11}\dot{m}\alpha^{-1}M_8^{-1}(r/r_g)^{-3/2}\ \text{cm}^{-3}. \tag{12}$$

Another quantity of interest is the Thomson optical depth at radius r, which is

$$\tau_T \simeq \dot{m}\alpha^{-1}(r/r_g)^{-1/2}. \tag{13}$$

The "trapping radius," within which an accretion flow would advect photons inward faster than they could diffuse outward [i.e. within which $\tau_T > (c/v_{\text{inflow}})$] is

$$r_{\text{trap}} = \dot{m}r_g. \tag{14}$$

Note that this depends only on $\dot{m}$ and not on α.

In Figure 2 are shown the ratios of various physically important time scales to t_{inflow} for a radial free-fall with $\dot{m} = 1$, calculated on the assumption that the ions at each radius are at the virial temperature [i.e. $kT_i = m_p c^2(r/r_g)^{-1}$]. This assumption is self-consistent because bremsstrahlung cooling and electron-ion coupling are indeed ineffective for $\dot{m}\alpha^{-2} = 1$. If the magnetic field is close to equipartition, synchrotron cooling is effective for the electrons (except insofar as it is inhibited by self-absorption); Comptonization is important whenever $(kT_e/m_e c^2)$ $\max[\tau_T, \tau_T^2] > 1$. This diagram helps us to understand the detailed results derived for various specific cases.

3.1 *Spherical Accretion*

The specific angular momentum of accreted material is likely to control the flow pattern, especially when close to the hole. Nevertheless, it is worthwhile to start off with the simpler case of spherically symmetric accretion. Some of the quantities derived in this section (for relative time scales, etc.) can, moreover, be straightforwardly scaled to cases where inflow occurs at some fraction α of the free-fall speed.

If the inflow is laminar, then the only energy available for radiation is that derived from PdV work; therefore, any smooth inflow at high Mach

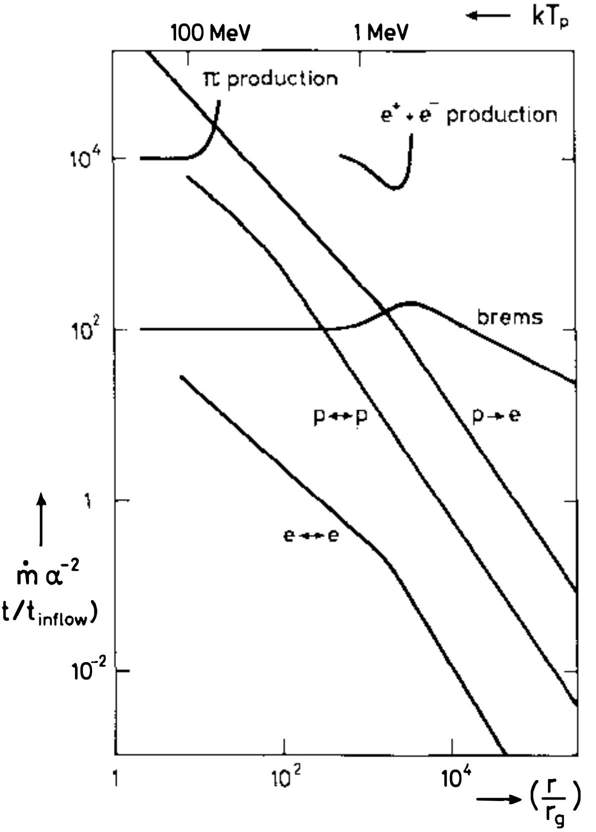


Figure 2 The time scales for various two-body plasma processes are here compared with the inflow time scale for an accretion flow. Processes shown are the self-equilibration time for electrons (e-e) and protons (p-p) (the latter includes nuclear as well as Coulomb effects at > 10 Mev); the time scale for transferring the proton thermal energy to the electrons (p → e); the bremsstrahlung cooling time for the electrons; and the effects of $e^+ + e^-$ and π production. The proton temperature is taken as the virial temperature $[kT = m_p c^2 (r/r_g)^{-1}]$, but the electron temperature is assumed to vary as $r^{-1/2}$ when the electrons are relativistic. The diagram shows that for free-fall accretion at the "critical" rate ($\dot{m} = 1$), two-body cooling processes are inefficient, and the electrons and protons are not thermally coupled by Coulomb interactions when $kT > 1$ Mev. The ratio of the various time scales to t_{inflow} scales as $\dot{m}^{-1}$; for inflow at α times the free-fall speed, the ratio scales as α^2 (for given $\dot{m}$). Data on cross sections are from Stepney (121).

number is certain to be inefficient irrespective of the radiation mechanism. Higher efficiency is possible if the Mach number is maintained at a value of order unity, or if there is internal dissipation (83). However, the fact that the bremsstrahlung cross section is only $\sim \alpha_f \sigma_T$ means that this mechanism alone can never be operative on the free-fall time unless $\dot{m} \gg 1$, in which case (from Equation 14) most of the radiation is swallowed by the hole. Several authors have discussed the important effects of Comptonization. If the only photons are those from bremsstrahlung, then merely a logarithmic factor is gained in the radiative efficiency. However, if the magnetic field is comparable with the value corresponding to full equipartition with bulk kinetic energy, then photons emitted at harmonics of the cyclotron frequency can be Comptonized up to energies such that $h\nu \simeq kT$. The most detailed work on this problem is that of Maraschi and collaborators (42, 80): the calculated spectrum is a power law of slope ~ -1 extending upward from the cyclotron/synchrotron self-absorption turnover to the gamma-ray band.

When a high luminosity L emerges from $r \simeq r_g$, Compton heating or cooling of material at larger r can create important feedback on the flow (45, 92). If the central source emits power $L(\nu)\,d\nu$ at frequencies between ν and $\nu + d\nu$, then Compton processes tend to establish an electron temperature such that

$$kT_e \simeq \tfrac{1}{4} h \langle \nu L(\nu) \rangle / L. \qquad 15.$$

(This formula strictly applies only if $h\nu < m_e c^2$ for all the radiation, and if induced processes can be neglected.) The time scale for this temperature to be established is

$$t_{\mathrm{Comp}} \simeq (m_e/m_p)(L/L_E)^{-1}(r/r_g)^2(r_g/c). \qquad 16.$$

If $t_{\mathrm{Comp}}(r) < t_{\mathrm{inflow}}(r)$, and if no other heating or cooling processes come into play, the consequences depend on whether $kT_e \gtrless kT_{\mathrm{virial}} = m_p c^2 (r/r_g)^{-}$ $T_e < T_{\mathrm{virial}}$, then the inflow must be supersonic, with the pressure support unimportant. Conversely, if there is a range of r where $t_{\mathrm{Comp}} < t_{\mathrm{inflow}}$ but $T_e > T_{\mathrm{virial}}$, steady inflow is impossible: if the flow were constrained to remain spherically symmetrical, "limit cycle" behavior would develop; but in more general geometry, inflow in some directions could coexist with outflow in others (18, 19).

A characteristic feature of the region where $kT_i \gg m_e c^2$ is that the electron-ion coupling time is so long that equality of the electron and ion temperatures is not guaranteed. For low $\dot{m}$, the collisional mean free paths for each species may exceed r (see Figure 2), though even a very weak magnetic field would suffice to make the inflow fluidlike. However, if there were no such field at all, then each electron or ion could orbit the hole many times between collisions (a situation resembling stellar dynamics around a massive central object): the net inflow velocity would be $\ll c(r/r_g)^{-1/2}$, and the density (and hence the radiative efficiency) would be higher than for the fluidlike free-fall solution with the same value of $\dot{M}$ (85).

Material infalling toward a collapsed object obviously eventually encounters the relativistic domain (51). It is therefore necessary to take note of what general relativity tells us about black holes; this is done in the next section.

4. BLACK HOLES ACCORDING TO GENERAL RELATIVITY

The physics of dense star clusters and of supermassive objects are complex and poorly understood. In contrast, the final state of such systems—if gravitational collapse indeed occurs—is comparatively simple, at least if we accept general relativity. According to the so-called no-hair theorems, the endpoint of a gravitational collapse, however messy and asymmetrical it may have been, is a standardized black hole characterized by just two parameters—mass and spin—and described exactly by the Kerr metric. If the collapse occurred in a violent or sudden way, it would take several dynamical time scales for the hole to settle down; during that period, gravitational waves would be emitted. But the final state would still be the Kerr solution, unless the material left behind constituted a strong perturbation. [The perturbation due to the infalling material in steady accretion flows is a negligible perturbation by a wide margin of order $(r_g/c)/t_E$.]

The expected spin of the hole—an important influence on its observable manifestations—depends on the route by which it formed (see Figure 1). A precursor spinning fast enough to be significantly flattened by rotational effects when its radius was $\gg r_g$ would probably have more specific angular momentum than the critical value GM/c. A massive black hole that forms "in one go" is thus likely to have been fed with as much spin as it can accept and to end up near the top of the range of angular momentum permitted by the Kerr metric; the same is true for holes that grow by gradual accretion of infalling galactic gas (11) (though the expectation is less clear if they grow by tidal disruption of stars). We should therefore take full cognizance of the distinctive properties of spinning black holes.

4.1 *The Kerr Metric*

The Kerr metric changes its character, and the event horizon disappears, if the specific angular momentum $J = J_{\max} = GM/c$. The so-called cosmic censorship hypothesis would then require that holes always form with

$J < J_{max}$. The Kerr solution then has a critical radius called the static limit, within which particles must corotate with the hole, though they can still escape. This arises because the frame-dragging is so strong that even light cones necessarily point in the ϕ direction. This critical surface, with equatorial radius

$$r_{stat} = r_g\{1 + [1+(J/J_{max})^2 \cos^2 \theta]^{1/2}\}, \tag{17.}$$

is not the event horizon itself; the latter occurs at a smaller radius. The region between the event horizon ($r = r_{EH}$) and the static limit is called the "ergosphere," because one can in principle extract energy from it via a process first proposed by Penrose (96): a particle entering the ergosphere can split in two in such a way that one fragment falls into the hole, but the other leaves the ergosphere with more energy than the original particle. The extra energy comes from the hole itself. A Kerr hole can be considered to have two kinds of mass-energy: a fraction associated with its spin, which can be extracted via the Penrose process, and an "irreducible" mass (14, 41). The fraction that can in principle be extracted is

$$1-2^{-1/2}\{1+[1-(J/J_{max})^2]^{1/2}\}^{1/2}, \tag{18.}$$

which is 29% for a maximally rotating hole. The above limit is an instance of a general theorem in black hole physics, according to which the area of the event horizon (a quantity analogous to entropy) can never decrease: a Kerr hole has smaller surface area than a Schwarzschild hole of the same mass. There have been various attempts to incorporate Penrose-style energy extraction into a realistic astrophysical model (64, 100). Those mechanisms that involve particle collisions or scattering operate only for a special subset of trajectories (14), and they would be swamped by accompanying processes. However, a process involving *electromagnetic* effects—the Blandford-Znajek mechanism (29)—seems more promising (and is discussed further below and in Section 5).

ORBITS The binding energy per unit mass for a circular orbit of radius r around a Schwarzschild hole (with $J = 0$) is

$$c^2\left\{1-\left[\frac{r-2r_g}{(r^2-3rr_g)^{1/2}}\right]\right\}. \tag{19.}$$

For $r \gg r_g$ this reduces to $GM/2r$, which is just the Newtonian binding energy. However, the binding energy has a maximum of $0.057c$ for an orbit at $r_{min} = 6r_g$, with angular momentum $\mathscr{L}_{min} = 2\sqrt{3}r_gc$. Circular orbits closer in than this have more angular momentum and are less tightly bound (as for orbits in classical theory when the effective force law is $\propto r^{-n}$, with $n > 3$): the orbits have zero binding energy for $r = 4r_g$ (with corresponding angular momentum $\mathscr{L}_0 = 4r_g c$); and for $r = 3r_g$, the expression (19.) goes to infinity, which implies that photons can move in circular orbits at this radius. In the Kerr metric, the behavior of orbits depends on their orientation with respect to the hole and on whether they are corotating or counterrotating (14). For corotating equatorial orbits, the innermost stable orbit moves inward (as compared with the Schwarzschild case); it becomes more tightly bound, with a smaller $\mathscr{L}_{min}$. For $(J/J_{max}) > 0.94$, r_{min} actually lies within the ergosphere. As $J \rightarrow J_{max}$ the stable corotating orbits extend inward toward $r = r_g$, and their binding energy approaches $(1-3^{-1/2})c = 0.42c$. These numbers determine the maximum theoretical efficiency of accretion disks.

4.2 *Three Astrophysically Important Relativistic Effects*

THE MINIMUM ANGULAR MOMENTUM An important inference from the above is that there are no stationary bound orbits whose angular momentum is less than a definite threshold value: particles whose angular momentum is too low plunge directly into the hole. This qualitative feature of the orbits means that *no stationary axisymmetric flow pattern* can extend too close to the rotation axis of a black hole (even well away from the equatorial plane)—no such constraint arises for flows around an object with a "hard" surface. Many authors have suggested that the resultant "funnels" play a role in the initial bifurcation and collimation of jets.

LENSE-THIRRING PRECESSION An orbit around a spinning (Kerr) hole that does not lie in the equatorial plane precesses around the hole's spin axis with an angular velocity [discussed by Bardeen & Petterson (13)] of

$$\omega_{BP} \sim 2(r/r_g)^{-3}(c/r_g)(J/J_{max}). \qquad 20.$$

This precession has a time scale longer than the orbital period by a factor of $\sim(r/r_g)^{3/2}(J/J_{max})^{-1}$. However, if material spirals slowly inward (at a rate controlled by viscosity) in a time much exceeding the orbital time, then the effects of this precession can mount up. The important consequence follows that the flow pattern near a black hole, within the radius where $2\pi/\omega_{BP}$ is less than the inflow time, can be axisymmetric with respect to the hole irrespective of the infalling material's original angular momentum vector. The Lense-Thirring precession, an inherently relativistic effect, thus guarantees that a wide class of flow patterns near black holes will be axisymmetric—an important simplification of the problem.

ELECTROMAGNETIC PROPERTIES OF BLACK HOLES Interactions of black holes with magnetic fields imposed on their surroundings can have important astrophysical effects. When a hole forms from collapsing magnetized material, the magnetic field outside the horizon decays ("redshifts away") on the collapse time scale r_g/c. But if, for instance, an external electric field were applied to a Schwarzschild hole, then after transients had decayed, a modified field distribution would be established where the electric field appeared to cross the horizon normally. The event horizon (or "surface") of the hole thus behaves in some respects like a conductor (47, 76, 134). It does not have *perfect* conductivity, however: if it did, electromagnetic flux would never be able to penetrate the horizon. Comparing the decay time scale for transients around a black hole (r_g/c) with the time scale ($r_g^2/4\pi\sigma$) appropriate to a sphere of radius r_g and conductivity σ, we can associate a surface resistivity of 377 Ω with the horizon. This analogy can be put on a more rigorous basis (134), and the "resistance" of a black hole is found to be $Z_H \simeq 100$ ohms. More generally, a *Kerr* black hole behaves like a *spinning* conductor. A simple discussion (98) valid for $J \ll J_{max}$ shows that a hole embedded in a uniform magnetic field B_0 would acquire a quadrupole distribution of electric charge

$$-\Omega^H \frac{r_{EH}}{8\pi c} B_0(3\cos^2\theta - 1), \qquad 21.$$

where $\Omega^H = (J/J_{max})c/2r_{EH}$ is the effective angular velocity of the hole. The corresponding poloidal electric field in a nonrotating frame is

$$E_\theta = -\frac{2\Omega^H r_{EH}}{c} B_0 \sin 2\theta. \qquad 22.$$

Just as in a classical "unipolar inductor," power can be extracted by allowing a current flow between a spinning hole's equator and poles. The maximum electric potential drop is $\sim B_0 r_g(J/J_{max})$, where B_0 is the imposed field. This can be very large, as it is when a similar argument is applied to spinning magnetized neutron stars in conventional models for pulsars.

For the fiducial field strength B_E (Equation 6), this emf is

$$\sim m_e c^2 r_g (v_{cE}/c)^{-1}(J/J_{max}) \simeq 3 \times 10^{15} M_8^{1/2}(J/J_{max}) m_e c^2. \qquad 23.$$

A single test charge introduced into this electromagnetic field will extract from the hole an energy of this order. However, the magnetosphere is unlikely just to contain a few "test charges"; indeed, the bare minimum charge density needed to modify the imposed field is

$$3 \times 10^{-4} M_8^{-3/2}\ \mathrm{cm}^{-3} \qquad 24.$$

(cf. Equation 12), and pair production generates far more charges than this (see Section 5). Just as in pulsars (8), a realistic magnetospheric current system and plasma distribution, though very hard to calculate, is likely to "short-out" the electric field. A relevant parameter is then Ω^F, the angular velocity of the field lines at large distance from the hole. This is related to the ratio of the effective resistance Z_∞ to the resistance of the hole Z_H:

$$(\Omega^H - \Omega^F)/\Omega^F = Z_\infty/Z_H. \qquad 25.$$

In the charge-starved limit, corresponding to infinite resistance at infinity, $\Omega^F = 0$. The "matched" case when $Z_\infty = Z_F$ ($\Omega^F = \frac{1}{2}\Omega^H$) corresponds to the maximum power extraction for a given B_0. This power is of order

$$B_0^2 r_g^2 (J/J_{max})^2 c. \qquad 26.$$

The efficiency in this case is lower than when $Z_\infty \rightarrow \infty$ (zero power), in the sense that half of the power is dissipated in the hole, and raises its irreducible mass; nevertheless, 9.2% of the rest energy could be extracted while slowing down a hole that started off with $J = J_{max}$.

Electromagnetic extraction of energy from black holes seems a realistic and important possibility. Its astrophysical context is discussed in Section 5.

SUMMARY The results of this section can be summarized by saying that three distinctively relativistic features of black holes are important in models for galactic nuclei:

1. There is a definite lower limit to the angular momentum of any stably orbiting material.
2. The Lense-Thirring precession enforces axisymmetry on any inward-spiraling flow pattern near the hole; consequently, any directed outflow initiated in the relativistic domain will be aligned with the hole's spin axis and will squirt in a constant direction (irrespective of the provenance of the infalling gas), except insofar as precession or accretion processes can reorient the hole's spin (105).
3. A rotating hole's latent spin energy can be tapped by externally applied magnetic fields; this can provide a power source far exceeding that from the accretion process itself.

5. ACCRETION FLOWS WITH ANGULAR MOMENTUM

5.1 *Origin of Infalling Matter*

The accreted material could fall in from the body of the galaxy (gas expelled from ordinary stars via stellar winds and supernovae); it could even come from intergalactic clouds captured by the galaxy. [Relevant here is the evidence that galaxies are more likely to be active if they are interacting with a neighbor (10, 43), and that quasars may be in interacting galaxies (62).] Alternatively, the gas supply may originate in the central parts of the galaxy: e.g. (*a*) debris from stars tidally disrupted by the hole (60, 61); (*b*) debris from stellar collisions in a compact star cluster around the hole (52); or (*c*) a positive feedback process whereby stars are induced to lose mass (and thereby provide further fuel) by irradiation from a luminous central source (82).

The accretion flow pattern depends on the angular momentum of the infalling gas: if this is large and has a steady orientation, then an accretion disk may extend out to very large values of (r/r_g); but the Lense-Thirring effect renders the flow pattern near the hole (where the power is primarily released) insensitive to conditions at large r, provided only that the matter has enough angular momentum to prevent it from falling directly into the hole. Accretion disks have been reviewed by Pringle (101) in a general astronomical context; I summarize here some new developments insofar as they may relate to massive holes in galactic nuclei.

5.2 *Thin Disks*

The simplest hypothesis is that the central object is being fueled steadily via an accretion disk (35, 73, 117). The standard thin disk model assumes that the gas at each radius is in a nearly Keplerian orbit. Slow radial infall occurs as viscosity transfers angular momentum outward. Energy dissipated by the viscous stress is radiated locally at a rate three times the local rate at which gravitational energy is liberated ($GM\dot{M}\, dr/r$ between r and $r+dr$). The factor of 3 arises because viscous stresses transport energy as well as angular momentum outward. This local imbalance is globally rectified in the innermost region of the disk, where the local release of binding energy exceeds the dissipation. For thin disks, slow inflow can be maintained down to the innermost stable orbit; the efficiency then equals the fractional binding energy for this orbit.

A disk has a scale height h normal to the orbital plane such that $(h/r) \simeq c_s/v_{\rm virial}$, where c_s is the internal sound speed, and is "thin" if this is $\ll 1$. One can write

$$(h/r)^2 \simeq (kT_{\rm gas}/m_p c^2)(1 + p_{\rm rad}/p_{\rm gas})(r/r_g). \qquad 27.$$

In this expression, $T_{\rm gas}$ is the gas temperature in the plane of symmetry (which could significantly exceed the surface temperature if the optical depth were very large); the quantity on the right-hand side is essentially the ratio of thermal and gravitational energies. Generally, the vertical support is provided by gas pressure at large r and for low accretion rates (116). Disks with high $\dot{M}$ are strongly radiation dominated in their inner regions: this is more true when the central hole is supermassive than for a stellar-mass hole because [for a given L/L_E, and thus a given (h/r)] the gas pressure per particle, proportional to $T_{\rm gas}$ (cf. Equation 5), scales as $M^{-1/4}$.

The very simplest models for such disks predict a thermal spectrum typically peaking in the ultraviolet (cf. Equation 5); they thus cannot in themselves account for the very broadband radiation from galactic nuclei. But the major uncertainties in the theory of these disks are the interlinked questions of viscosity and magnetic fields. These fields, amplified by shearing motions (49) and possibly by turbulence-driven dynamo action (102, 103), probably provide the main viscosity. Only crude estimates can be made of the resultant α-parameter. Moreover, it is unclear whether the magnetic stresses build up to a fixed fraction of the *total* pressure or only of the *gas* pressure. The argument for the latter view (44, 110, 111) is that large-amplitude density contrasts can be induced as soon as magnetic stresses become competitive with gas pressure, and buoyancy effects then elevate the flux into the disk's "corona," impeding further amplification. This can happen, however, only if the radiation is able to diffuse relative to the gas: in the limit of very large optical depths, the field could be amplified by differential rotation on time scales much shorter than those on which density inhomogeneities could develop. Gas and radiation would then act like a single composite fluid, and only the total pressure would be relevant. The answer to this somewhat confusing (though well-posed) theoretical question makes a big numerical difference to the inward drift time scale; more importantly, it determines whether such a disk would be unstable to the "visco-thermal" instability (101).

Magnetic fields may also have a big effect on the radiation spectrum emerging from a realistic thin disk. Energy transported by magnetic buoyancy into a hot corona could dominate the (approximately blackbody) radiation from the dense part of the disk. Magnetic flares in the corona may accelerate relativistic electrons that radiate nonthermally.

Blandford (24) has emphasized that there is no obvious ultimate repository from the angular momentum of disks in galactic nuclei (whereas the companion star and the orbit serve this role for binary star systems). If the magnetic field were sufficiently well ordered, a coronal wind (rather than outward transfer via viscosity within the disk itself) could be the main sink for the angular momentum of accreted material (23, 26). An alternative resolution of the problem, suggested by Ostriker (91), is that the angular momentum is transferred via dynamical friction to a star cluster in which the disk is embedded.

Most of the recent theoretical work on thin disk structure is aimed primarily at understanding cataclysmic variables, X-ray binaries, etc., but it is relevant also in the galactic nucleus context. In all disks, the thermal balance of the outer parts is likely to be controlled by irradiation (causing photoionization, Compton heating, etc.) from the central region. Even where such disks exist, they could be embedded in hotter quasi-spherical structures. There may thus be no clear demarcation in the real world between thin disks and the toroidal structures to which we next turn.

5.3 *General Structure of Tori or Thick Disks*

Disks become geometrically thick, with $h \simeq r$, if the internal pressure builds up so that $c_s \simeq (GM/r)^{1/2}$. This can happen either because radiation pressure becomes competitive with gravity or because the material is unable to radiate the energy dissipated by viscous friction, which then remains as internal energy. Before discussing the (very different) internal physical conditions in these two kinds of tori, let us consider their general equilibrium structure.

In thick disks, radial pressure gradients cannot be ignored; the angular velocity is therefore not Keplerian and becomes (within certain constraints) a free parameter. Uncertainty about the viscosity is a major stumbling block. This uncertainty is not crucial to many qualitative features of thin disks (e.g. their overall energetics). However, in thick disks one must deal explicitly with shear stresses in two directions. The stresses determine the distributions both of angular momentum and enthalpy, and therefore the shape of the isobars inside the disk; internal circulation patterns may be important for energy transport. There is always a pressure maximum at $r = r_{\rm max}$ in the equatorial plane. Outside $r_{\rm max}$, the angular velocity is sub-Keplerian, but for $r < r_{\rm max}$ it is faster than Keplerian. Such structures around Kerr holes were investigated by Bardeen (12) and by Fishbone & Moncrief (50; see also 36, 37). Recent work, from a more astrophysical viewpoint, has been spearheaded by Abramowicz and colleagues (1–3, 63, 65, 93, 129). They have exploited an important simplifying feature: the shape of a torus depends only on its *surface distribution* of angular momentum. If the angular velocity $\Omega(\mathscr{L})$ is given as a function of angular momentum $\mathscr{L}$, then the surface binding energy U is given implicitly by

$$dU/U = \Omega \, d\mathscr{L}/(c^2 - \mathscr{L}\Omega^2). \qquad 28.$$

A simple special case is that for which $\mathscr{L}$ is the same everywhere. The binding energy is then constant over the whole surface of the torus; there is thus, for each value of $\mathscr{L}$, a family of such tori, parametrized by the surface binding energy U. As U tends to zero, the tori "puff up," and the part of the surface close to the rotation axis acquires a paraboloidal shape. The gravitational field is essentially Newtonian throughout most of the volume, but relativistic effects come in near the hole if $\mathscr{L} \simeq \mathscr{L}_{\rm min}$, the angular momentum of the smallest stable orbit. For $\mathscr{L}$ in the range $\mathscr{L}_{\rm min} < \mathscr{L} < \mathscr{L}_0$, special significance attaches to the torus for which U exactly equals the binding energy of the (unstable) orbit of angular momentum $\mathscr{L}$. There is then a cusplike inner edge, across which material can spill over into the hole (just as material leaves a star that just fills its Roche lobe in a binary system). This particular relation between U and $\mathscr{L}$ would approximately prevail at the inner edge of any torus where quasi-steady accretion is going on (see Figure 3 and caption).

More generally, one can consider (99) tori where Ω goes as some power of $\mathscr{L}$. Such tori exist in all cases where the increase of angular momentum with Ω is slower than Keplerian. The funnels tend to be conical rather than paraboloidal if the rotation law is nearer to Keplerian; they extend closer to $r = r_g$ when the black hole is rapidly rotating.

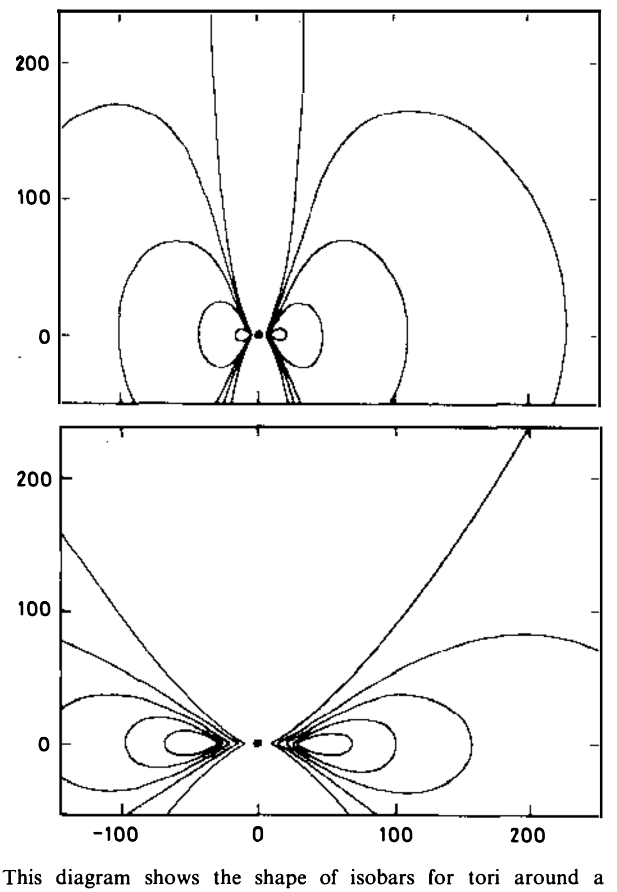


Figure 3 This diagram shows the shape of isobars for tori around a nonrotating (Schwarzschild) hole. The upper picture shows the case $\mathscr{L}$ = constant; in the lower picture, the angular momentum law is $\Omega \propto \mathscr{L}^{-4}$ (i.e. less different from Keplerian), and the funnels are less narrow. For a given rotation law, narrower funnels (extending inward to smaller r) are possible if the hole is rapidly rotating ($J \simeq J_{max}$). The units of length are r_g [from Phinney (99)].

Accretion flows where high internal pressures guarantee $h \simeq r$ [from (27.)] could resemble such tori if the viscosity parameter were low enough that the flow was essentially circular, and provided also that the configuration were stable (though there is frankly no firm basis for confidence in either of these requirements).

A generic feature of accretion tori is that they are less efficient—in the sense that they liberate less energy per gram of infalling matter—than thin disks. The efficiency is given by the binding energy of the material at the cusp; this depends on the angular momentum profile (via Equation 28), but for an $\mathscr{L}$ = constant torus of outer radius r_0, it is $(r_0/r_g)^{-1}$, which implies very low efficiency for large tori.

In any torus with $r_0 \gg r_g$ and a strongly sub-Keperlian rotation law, rotation is unimportant (gravity being essentially balanced by pressure gradients, and the isobars almost spherical) except near the funnel along the rotation axis. To avoid convective instability, the density must fall off with radius at least as steeply as the isentropic laws

$$n \propto r^{-3} \qquad 29.$$

for $\gamma = 4/3$ (e.g. radiation pressure support), and

$$n \propto r^{-3/2} \qquad 30.$$

for $\gamma = 5/3$ (e.g. ion pressure support).

The two very different cases of radiation-supported and ion-supported tori may incorporate elements of a valid model for some classes of galactic nuclei. I discuss them here in turn, and then (in Section 6) I consider another question: whether the "funnels" in such flow patterns are important in collimating the outflowing jet material.

The foregoing discussion begs the question of whether these tori are stable and whether stability requirements narrow down the possible forms for $\Omega(\mathscr{L})$. Local instabilities can arise from unfavorable entropy and angular momentum gradients (66, 115). These presumably evolve to create marginally stable convection zones, as in a star. Dynamically important magnetic fields may induce further instabilities. Moreover, tori may be seriously threatened by nonaxisymmetric instabilities. Papaloizou & Pringle (94) recently demonstrated that an $\mathscr{L}$ = constant toroidal configuration marginally stable to axisymmetric instabilities possesses global, nonaxisymmetric dynamical instabilities, which would operate on a dynamic time scale. It is not clear to what extent more general angular momentum distributions are similarly vulnerable, but it may turn out that funnel regions where pressure gradients are balanced by centrifugal effects rather than by gravity are *never* dynamically stable.

5.4 *Radiation-Supported Tori*

A thick structure can be supported by radiation pressure only if it radiates at $L \simeq L_E$. Indeed, in any configuration supported in this way, not only the *total* luminosity but its *distribution over the surface* is determined by the form of the isobars. Tori with long narrow funnels have the property that their total luminosity can exceed L_E by a logarithmic factor (118). More interestingly, most of this radiation escapes along the funnel, where centrifugal effects make the "surface gravity" (and hence the leakage of radiation) much larger than over the rest of the surface. If accretion powers such a torus, then $\dot{m} \times$ (efficiency) $\gtrsim 10$.

If the outer parts are sufficiently slowly rotating that (29.), or a still steeper law, approximately holds, the characteristic Thomson optical depth must depend on radius r at least as steeply as

$$\tau_T(r) \propto r^{-2}. \qquad 31.$$

This in turn implies that the torus cannot remain optically thick (in the sense that $\tau_T > 1$) out to $r \gg r_g$ unless the viscosity parameter α at $r \simeq r_g$ is very low indeed. (This has been thought by some to be an implausible feature of such models. However, one could argue contrariwise that these objects resemble stars, in which the persistence of differential rotation certainly implies an exceedingly low effective α. Pursuing this analogy further suggests that large-scale circulation effects may play as big a role in energy transport as radiative diffusion does.)

If LTE prevails in such a torus, then the temperature at radius r, at locations well away from the rotation axis, is

$$T(r) \simeq [\tau_T(r_g)]^{1/4} T_E (r/r_g)^{-1} \qquad 32.$$

(cf. Equation 5). The condition for LTE [i.e. that photons can be thermalized within their diffusion time scale $\tau_T(r)(r/c)$] is more stringent than $\tau_T > 1$. Indeed, even at the pressure maximum ($r \simeq r_g$), the requirement is

$$\dot{m}\alpha^{-1} \simeq \tau_T(r_g) > 2 \times 10^3 M_8^{1/17}, \qquad 33.$$

and radiation pressure dominates gas pressure by a factor of $\sim 10^6 [\tau_T(r_g)]^{-1/4} M_8^{1/4}$—much larger than ever occurs in stellar structure. If $\tau_T(r_g)$ is even larger than (33.), so that LTE prevails out to $r \gg r_g$, the hole may be sufficiently well smothered that all the radiation effectively emerges from a photosphere, in appearance rather like an O or B star (24).

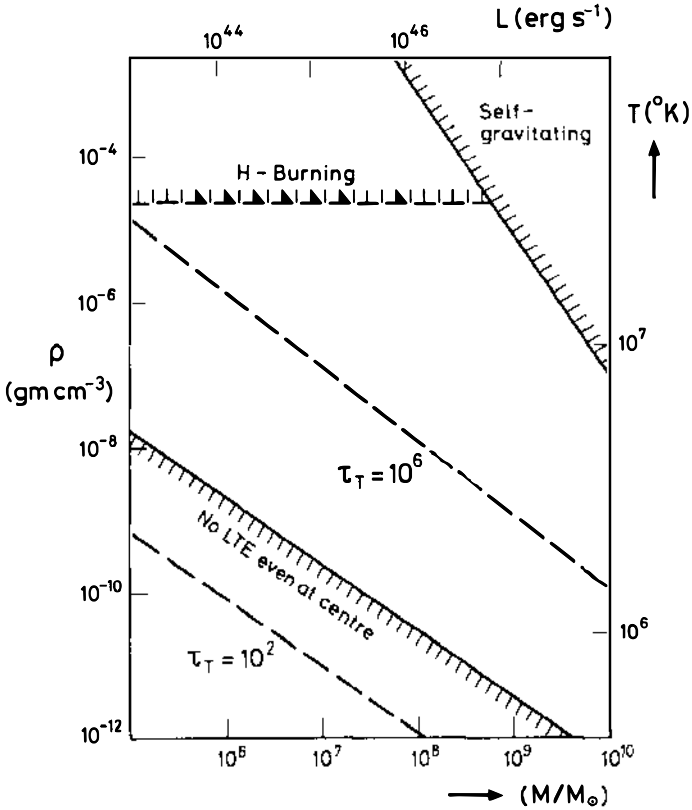


Figure 4 This diagram shows physical conditions near the pressure maximum of an optically thick radiation-supported torus around a hole of mass M. For a given M, the free parameter is the density (or Thomson optical depth τ_T), which scales as $\dot{m}\alpha^{-1}$. Rather high values of this parameter are needed in order to achieve LTE, even in the inner parts of the torus where the pressure is maximal (and where the temperature would then have the value given on the right-hand scale); if LTE is to extend out nearer the surface, then the torus may be so dense and massive near the center that nuclear burning and self-gravitation become significant. (In the approximate treatment given here, the pressure maximum is taken to occur at $r \simeq r_g$; in fact it is always at a larger radius than this, by an amount related inversely to the hole's angular momentum parameter.)

5.5 *Ion-Supported Tori*

We have seen that for spherically symmetric inflow, the cooling time scale—and even the electron-ion coupling time—can be longer than the free-fall time; the same conditions can prevail even for inflow with angular momentum, provided that $\dot{m}$ is low enough. As compared with Figure 2, all that is changed is that the inflow time is $\alpha^{-1}t_{\text{free-fall}}$ and the characteristic density for a given $\dot{m}$ is higher by α^{-1}. The condition for electron-ion coupling to be ineffective in the inner parts of a torus (cf. Figure 2) is

$$\dot{m}\alpha^{-2} < 50. \tag{34.}$$

When (34.) holds, the ions can remain at the virial temperature even if synchrotron and Compton processes permit the electrons to cool, and the disk swells up into a torus. The dominant viscosity is likely to be magnetic. Estimates of magnetic viscosity are very uncertain; Eardley & Lightman (49) suggest that α falls in the range 0.01–1.0. However, there is no reason why the magnetic α should fall as $\dot{m}$ is reduced, so (34.) should definitely be fulfilled for sufficiently low accretion rates.

An accretion flow where $\dot{m}$ is small, and where (furthermore) the radiative efficiency is low, may seem a doubly unpromising model for any powerful galactic nucleus. However, such a torus around a spinning black hole offers an environment where the Blandford-Znajek (29) process could operate (108). Even though it may not radiate much directly, the torus can then serve as a catalyst for tapping the hole's latent spin energy. Three conditions are necessary:

1. *Magnetic fields threading the hole must be maintained by an external current system.* The requisite flux could have been advected in by slow accretion; even if the field within the torus were tangled, it would nevertheless be well ordered in the magnetosphere. The torus would be a good enough conductor to maintain surface currents in the funnel walls, which could confine such a field within the hole's magnetosphere. The only obvious upper limit to the field is set by the requirement that its total energy should not exceed the gravitational binding energy of the torus. (An equivalent statement is that B should not exceed $\dot{m}^{1/2}\alpha^{-1/2}B_E$.)

2. *There must be a current flowing into the hole.* Although an ion-supported torus radiates very little, it emits some bremsstrahlung gamma rays. Some of these will interact in the funnel to produce a cascade (31) of electron-positron pairs (99, 108), yielding more than enough charge density to "complete the circuit" and carry the necessary current—enough, indeed, to make the magnetosphere essentially charge-neutral, in the sense that $(n^+ + n^-) \gg |(n^+ - n^-)|$, so that relativistic MHD can be applied.

3. *The proper "impedance match" must be achieved between the hole and the external resistance.* Phinney (99) has explored the physics of the relativistic wind, whose source is the pair plasma created in the magnetosphere and that flows both outward along the funnel and into the hole. By considering the location of the critical points, he finds consistent wind solutions where Ω^F is as large as $0.2\,\Omega^H$. This corresponds (cf. Equation 25) to 60% of the maximum power extraction (for a given B-field). Although

some energy is dissipated in the hole, this would still permit a few percent of the hole's rest mass energy to be transformed into a mixture of Poynting flux and a relativistic electron-positron outflow.

The Blandford-Znajek process could operate even if the field threading the hole were anchored to a thin disk, but a thick ion-supported torus provides an attractive model for strong radio galaxies because it could initiate collimated outflow (see the discussion in Section 6). The possibility of such tori depends, however, on the assumption that Coulomb scattering alone couples electrons to ions. This raises the question of whether some collective process might, realistically, be more efficient—if so, the electrons could drain energy from the ions and the torus would deflate. There are bound to be shearing motions, owing to differential rotation, which generate local pressure anisotropies in the plasma. There are certainly instabilities that isotropize the ion plasma, as well as instabilities that isotropize the electron plasma. The key question—which still seems open—is whether these two isotropization processes act almost independently, or whether they can transfer energy from ions to electrons.

[Although electromagnetic extraction of energy is especially important for ion-supported tori (objects where the accretion process is inevitably inefficient), this process could also augment the power generated within a radiation-supported torus. There is in principle no limit to the power that could be extracted from a spinning hole embedded in a dense and strongly magnetized cloud, provided that this power can escape preferentially along the rotation axis without disrupting the cloud. These optically thick radiation-driven jets (21), discussed primarily in the different context of SS 433, could occur in quasars. If the cloud were not sufficiently flattened to permit the excess energy to escape in preferential directions, material would be blown from the cloud, reducing its central pressure: this condition would persist until the total (accretion plus electromagnetic) power fell to L_E, but only a fraction came from accretion.]

6. JET FORMATION

Directed outflow is a ubiquitous feature of active galactic nuclei, and it is also seen in some small-scale prototypes of AGNs in our own Galaxy (e.g. SS 433). This is in itself evidence that a spherically symmetric model cannot be entirely realistic. For a full review of theories of jet propagation, with special relevance to radio galaxies, the reader is referred to Begelman et al. (17). The direct evidence for jets pertains exclusively to scales much larger than the primary power source. The scales probed by VLBI are typically a few parsecs ($\gtrsim 10^4\ r_g$ for plausible central masses); the only evidence for smaller-scale beaming comes from indirect arguments about the physics of optically violent variables (OVVs), or "blazars" (6, 87, 88). There are theoretical reasons for postulating that the relativistic outflow is initiated on scales of order r_g, but there are really no grounds for believing that a *narrow* collimation angle is established until the jets get out to VLBI scales or beyond: indeed, conditions in the medium $\lesssim 1$ pc from the central source cannot readily provide the kind of pressure-confined "nozzles" (27) that could best collimate them (107).

The radiation from the jets—the emission detected by VLBI and other radio techniques, as well as the emission in other wave bands from (for instance) the M87 jet—is presumably synchrotron radiation from electrons accelerated in situ. Plainly, any high-γ *random* motions produced at $r \simeq r_g$ would have been eliminated by radiative and adiabatic losses before the jet got out to 1 pc. In the superluminal sources, there is direct evidence for bulk relativistic outflow ($\gamma_b \gtrsim 5$). We do not know whether this outflow involves ordinary matter, electron-positron plasma, or even Poynting flux, and various authors have suggested schemes involving each of these options.

Any disk structure near a black hole provides a pair of preferred directions along the rotation axis; moreover, within the Lense-Thirring effect's domain of influence, this axis is maintained steady by the hole's gyroscopic effect. Magnetically driven winds from tori or from thin disks (23, 26) could generate outflowing jets with the attractive attribute of a self-confining toroidal field.

The evacuated vortices along the axes of thick accretion tori, which can be very narrow for an angular momentum distribution close to $\mathscr{L}$ = constant, suggest themselves as possible preexisting channels for directed outflow. The most widely discussed version of this idea, first proposed by Lynden-Bell (74), utilizes radiation pressure. A simple order-of-magnitude argument shows that a test particle (electron plus ion) released from rest outside a source with $r \simeq r_g$ and $(L-L_E)/L_E \gtrsim 1$ would attain a relativistic speed; a radiation-supported torus whose vortex has cone angle θ emits within this cone a greatly enhanced luminosity $\sim \theta^{-2} L_E$ per unit solid angle, which suggests that this photon beam might impart high Lorentz factors to any matter in its path.

Detailed study reveals flaws in this superficially attractive idea (4, 5, 90, 119). The main problem is that the radiation field within a long, narrow funnel is almost isotropic: there may indeed be a super-Eddington outward flux along it, but the radiation density far exceeds (flux/c) because of scattering, or absorption and reemission, by the walls. Consequently, a test electron travels *sub*relativistically along the funnel, at a speed such that the radiation appears nearly isotropic in its moving frame. The radiation flux only becomes well collimated by the time the particle escapes from the funnel, at $r = r_0$. Even for the (probably unstable) $\mathscr{L}$ = constant tori, r_0 is at least $\theta^{-2}r_g$; and out there the dilution (because r is now $\gg r_g$) cancels out the θ factor gained from the beaming. The net result is that γ-values of only ~ 2 can be reached for an electron-ion plasma, and maybe up to ~ 5 for electron-positron plasma. A second difficulty is that the Thomson depth along the funnel would become > 1, vitiating the test-particle approach adopted in the calculations, if the particles were numerous enough to carry a substantial fraction of L. [However, in the limit of very large optical depths, where radiation and matter can be treated as a single fluid, radiation pressure around a supercritical central source—a "cauldron" (21)—could efficiently generate a jet of ordinary matter with high γ_b.]

Quite apart from these theoretical difficulties, models involving radiation-supported tori cannot be relevant to the objects where the most spectacular jets are seen (radio galaxies, M87, etc.). We have *upper* limits to the thermal luminosity from these AGNs; we also have *lower* limits to the energies involved in producing large-scale radio structure and, hence, to the masses involved. Combining these limits precludes there being any object emitting a thermal luminosity L_E (the level of isotropic emission that would be an inevitable concomitant of a radiation-supported torus with a narrow funnel).

An ion-supported torus maintained by accretion with low $\dot{M}$ can provide funnels along the rotation axis, just as a radiation-supported torus can. The expelled material would then be an electromagnetically driven wind of electron-positron plasma (99, 108). The rest mass energy of the pairs could be $\ll L/c^2$—indeed, most of the outflow could be in Poynting flux rather than being carried by the pairs themselves—making high beam Lorentz factors γ_b no problem. An energy flux of this kind could readily be converted into relativistic particles at large distances from its point of origin and is thus an attractive model for radio sources.

Two factors constrain the content and the Lorentz factor of jets emerging from scales of $\sim r_g$ (99, 107). First, an e^+-e^- jet that started off with too high a particle density would suffer annihilation before moving one scale height: this means that an energy flux L_E in pair kinetic energy, rather than in Poynting flux, is impossible unless γ_b is high. [The particle flux is then less for a given L; furthermore, the time scale available for annihilation, measured in the moving frame, is only $\gamma_b^{-1}(r/c)$.] But radiation drag effects give a second countervailing constraint that precludes particle jets with very high values of γ_b. Radiation pressure provides an acceleration only if it comes from the backward direction after transforming into the moving frame (97). If radiation comes from a source of finite size r_s, then the acceleration at a distance r would always saturate for $\gamma_b \simeq (r/r_s)$, no matter how high the luminosity of the source. Moreover, in a realistic model for a galactic nucleus, some fraction of the luminosity is scattered or reemitted on scales out to ~ 1 pc. This quasi-isotropic flux exerts a Compton drag force on any beam, and it is particularly serious for e^+-e^- beams, which have the least inertia relative to their scattering cross section.

The interaction of jets with the material at ~ 1 pc in AGNs is an interesting topic that has only recently been seriously discussed (86). Possibly, the beams generally deposit their energy in the emission-line region, and only in especially favorable cases does the jet material get collimated sufficiently to penetrate beyond $r \simeq 1$ pc.

7. SOME COMMENTS ON PHENOMENOLOGY

7.1 *The Continuum Spectrum*

The only direct clue to physical conditions in the central region (i.e. within a radius of, say, $100r_g$) is the rather featureless continuum luminosity; spectral lines originate farther out. The models we have discussed can radiate either thermally or nonthermally: indeed, one of the hardest things to estimate is what fraction of the power dissipated via viscous friction in a realistic flow pattern would go directly into ultrarelativistic particles (via shocks, magnetic reconnection, etc.) rather than being shared among all the particles. Unfortunately, observations are little help in discriminating between various continuum radiation mechanisms: a smooth spectrum could be produced equally well by several alternative mechanisms. For instance (99), there are at least four ways of getting a spectrum with $L(\nu) \propto \nu^{-1/2}$:

1. Thermal processes can mimic a power law if the spatial properties of the emitting medium vary in a suitable way (84). This particular slope arises if we consider bremsstrahlung from a spherical distribution of gas with density $n \propto r^{-3/2}$ (corresponding to free-fall) and $T \propto T_{\text{virial}} \propto r^{-1}$.
2. Relativistic particles may be steadily injected with a high Lorentz factor and then lose energy by synchrotron or inverse Compton emission beforc escaping, yielding nonthermal radiation with $L(\nu) \propto \nu^{-1/2}$.
3. Relativistic particles could be accelerated with an E^{-2} differential spectrum. An (over)simple theory of shock acceleration (9, 48) actually yields this law for a compression factor of 4 (the value expected for a strong nonrelativistic shock).
4. Comptonization of injected soft photons yields a power law, which would have the particular value $-\frac{1}{2}$ for a value of the parameter $y = (kT/m_e c^2)\tau_T^2$ that is slightly geometry dependent but typically close to unity.

It is true that theoretical arguments can rule out some of these emission processes in some particular instances: for example, bremsstrahlung can never generate a high luminosity ($L \simeq L_E$) without τ_T being so large that Comptonization reshapes the spectrum (71). These examples of mechanisms, any or all of which could be occurring within a single source, nevertheless highlight the necessity of other indicators (such as polarization or spectral breaks) for discriminating between them.

Obviously the values of M and $\dot{M}$ are crucial in determining the properties of an accreting hole; the angular momentum parameter $(J/J_{\max})$ is also important. We conclude further, and somewhat less trivially, that it is the value of $\dot{m} = \dot{M}/\dot{M}_E$ that determines the nature of the inflow. The value of M itself only enters explicitly (and with weak fractional powers) when reabsorption effects are important. This means that there is a genuine physical similarity, not merely a crude resemblance, between active galactic nuclei and the stellar-scale phenomena (X-ray binaries, etc.) observed within our own Galaxy.

While it is perhaps foolhardy to put forward any fully comprehensive unified scheme for the various kinds of AGNs, there have been several proposals to relate particular categories of objects, or particular features in their spectra, to specific mechanisms.

7.2 *Ordinary QSOs*

Most QSOs are radio-quiet and are neither violently variable nor highly polarized. The main bolometric luminosity, in the near-ultraviolet, could come from the photosphere of a radiation-supported torus around a $(10^7$–$10^8)$ $M_\odot$ hole. Blandford (24) has suggested that the characteristic surface temperature is determined by He recombination, which changes the mean molecular weight. An isentropic torus of the type discussed in Section 5.3 would need to have a very high central density (and a correspondingly low value of α) in order to be sufficiently optically thick to thermalize radiation out at the putative photosphere—indeed, its central pressure and temperature might have to be so high that nuclear energy released via hydrogen-burning (16) dominates accretion-generated power (see Figure 4).

Even if one accepts that there is something special about a photospheric temperature $T = 20{,}000$ K, the configuration need not resemble a stable torus. A more tentative and less controversial conjecture would simply be that typical QSOs are objects where the central hole is smothered by plasma clouds at distances $(10^2$–$10^3)$ r_g, which are dense enough to be close to LTE [but which are not necessarily supported quasi-statically by an $n \propto r^{-3}$ density distribution (cf. Equation 29) at smaller r]. Such a hypothesis would suffice to explain the "UV bump" in quasar spectra (78, 79). The filaments emitting the broad spectral lines would lie outside this photosphere. Realistically, one expects an additional nonthermal component due to shocks and/or magnetic flaring (by analogy with O star photospheres, except that in AGNs the escape velocity, and probably also the characteristic Alfvén speed, would be very much higher). The X rays could be attributed to this component, since in such a model no radiation would escape directly from $r \simeq r_g$.

7.3 *Radio Galaxies*

In radio galaxies, the direct radiative output from the nucleus is typically $\sim 10^{42}$ erg s^{-1}, less than the inferred output of the beams that fuel the extended radio components. The energy carried by the beams in Cygnus A exceeds the central luminosity by a factor of ~ 10. These objects must therefore channel *most* of their power output into directed kinetic energy. Moreover, the mass involved in producing the large-scale radio structure must be large—certainly $> 10^7\ M_\odot$. The thermal output from these AGNs is therefore $\lesssim 10^{-3}\ L_E$, implying that they cannot involve radiation-supported tori; nor can radiation pressure be important for accelerating the jet material. Such considerations suggest that strong radio sources may involve massive spinning black holes onto which matter is accreting very slowly (maybe $10^{-3}\ M_\odot$ yr^{-1}) to maintain an ion-supported torus, so that the holes' energy is now being tapped electromagnetically and being transformed into directed relativistic outflow (108).

7.4 *Radio Quasars and Optically Violent Variables (OVVs)*

Data on OVVs (also known as "blazars") have been reviewed by Angel & Stockman (6; see also 87, 88). For the extreme members of the class, such as OJ 287 and AO 0235 + 164, the case for beaming seems compelling. The less luminous objects might also be beamed, but they could alternatively involve unbeamed synchrotron emission from $r \lesssim 10\ r_g$. More evidence on the hard X-ray spectrum of such objects would help to decide between these options. If gamma rays were emitted and (9.) were fulfilled, the resultant "false photosphere" of electron-positron pairs would scatter the optical photons and destroy any intrinsic high polarization (58). One would then be disposed to invoke relativistic beaming, which would increase the intrinsic source sizes compatible with the observed variability and reduce the luminosity in the moving frame; this would mean that (9.) was no longer fulfilled, and gamma rays could escape without being transformed into pairs.

7.5 *Hard X-Ray and Gamma-Ray Sources*

Boldt & Leiter (30, 68) have proposed a scheme whereby the output in gamma rays relative to X rays increases as $\dot{M}$ decreases. Low-redshift objects are postulated to have low $\dot{M}$ and to emit gamma rays; their high-z counterparts, however, are fueled at a higher rate, and they yield most of the

X-ray background without contributing proportionately to the gamma-ray background.

According to White et al. (128), the characteristic X-ray spectrum of active nuclei depends on whether their primary luminosity in hard photons is $\gtrless 10^{-2} L_E$. For a source size of $\sim 10 r_g$, this determines whether or not a pair photosphere is produced (9.). In sources with high L/r, where a pair photosphere *is* produced, the emergent Comptonized spectrum is softer. A small-scale analogue of this phenomenon may be the galactic compact source Cygnus X-1, which undergoes transitions between "high" and "low" states, the spectrum being softer for the former. The fact that many AGNs emit variable X rays with a flat spectrum (energy index 0.6; 89, 131) suggests that e^+-e^- production is inevitable, and that the effects of pairs on dynamics (99) and radiative transfer (130) need further attention.

8. DEMOGRAPHY OF AGNs

Even if AGNs are precursors on the route toward black hole formation (cf. Figure 1) rather than structures associated with black holes that have already formed, it seems hard to escape the conclusion that massive black holes must exist in profusion as remnants of past activity; they would be inconspicuous unless infall onto them recommenced and generated a renewed phase of accretion-powered output or catalyzed the extraction of latent spin energy.

Estimates of the masses and numbers of "dead" AGNs are bedeviled by uncertainty about how long individual active objects live and the evolutionary properties (i.e. the z-dependence) of the AGN population. As regards the latter, see, for instance, (81, 114, 127) for recent reviews of optical data, and (95, 126) for radio studies. It has long been known that the evolution is *strong*, amounting to a factor of up to 1000 in comoving density for the strongest sources; the evolution is *differential*, being less steep for lower-luminosity objects of all kinds. It is now feasible to refine these statements, though it is still premature to be extremely precise about the redshift dependence of the multivariate function $f(L_{rad}, L_{opt}, L_X)$. And we are still a long way from having much astrophysical understanding of *why* the luminosity function evolves in this way. Anyway, at the epoch $z = 2$, the population of strong sources declined on a time scale $t_{Ev} \simeq 2 \times 10^9 h_{100}$ yr; this is of course an *upper limit* to the "half-life" of a particular source, since there may be many generations of objects within the period t_{Ev}.

Soltan (120) has given an argument that bypasses the uncertainty in AGN lifetimes but nevertheless yields useful constraints on the masses involved in such phenomena and the kinds of galaxies in which they can reside. The overall energy budget for AGNs is dominated by QSOs (most of which are "radio quiet"); they contribute an integrated background luminosity amounting to

$$\sim 3000 M_{\odot} c^2 \text{ Mpc}^{-3}. \qquad 35.$$

This estimate involves an uncertain bolometric correction; the main measured contribution to $\nu L(\nu)$ is typically at $\sim 10^{15}$ Hz (i.e. in the ultraviolet). Although some individual quasars could emit more power in (for instance) the far-ultraviolet, hard X-rays, or gamma rays, we know enough about the isotropic background in these bands to be sure that such emission cannot permit a huge bolometric correction for the typical quasar. The main contribution to (35.) comes from quasars with $19 < B < 21$ (corresponding to bolometric luminosities of 10^{45}–$10^{46} h_{100}^{-2}$ erg s^{-1} if they are typically at $z \simeq 2$); the counts flatten off at fainter magnitudes, so (35.) is unlikely to be a severe underestimate. The energy output from radio galaxies and other manifestations of active nuclei is much smaller than that from optically selected quasars; we are therefore probably justified in using (35.) in the discussion that follows.

The individual remnant masses can be estimated as

$$8 \times 10^7 h_{100}^{-3} \varepsilon^{-1} (t_Q/t_{Ev}) M_{\odot}, \qquad 36.$$

where ε is the overall efficiency with which rest mass is converted into electromagnetic radiation over a typical quasar's active lifetime t_Q (defined as the time for which the magnitude is $M < -24$). If quasars were associated with all "bright" galaxies ($M < -21.3$), whose space density is known, the mean hole mass would be $\sim 2 \times 10^6 h_{100}^{-3} \varepsilon^{-1} M_{\odot}$. If only a small fraction of galaxies had ever harbored active nuclei, the masses (and lifetimes) of each would be correspondingly increased.

The above discussion is important if one wishes to relate nuclear activity to galactic morphology. However, it is also relevant as a discriminant between different models (99). If individual quasars are as long lived as is compatible with the cosmological evolution of the quasar population [i.e. $t_Q = t_{Ev}$ in (36.)], their remnant masses must be as large as $\sim 10^9 h_{100}^{-3} (\varepsilon/0.1) M_{\odot}$ (the present space density of remnants being only about that of radio sources with $P_{178} > 10^{23} h_{100}^{-2}$ W Hz^{-1} sr^{-1}). But the luminosity of a "typical" quasar ($M \simeq -25.5$ for $h_{100} = \frac{1}{2}$) corresponds to the Eddington limit for a mass of "only" $\sim 10^8 M_{\odot}$; so if quasars resemble radiation-supported tori, then they must have lifetimes such that $(t_Q/t_{Ev}) \ll 1$, and individual quasars must be "switched off" by something internal to the particular system, rather than being influenced by any change in the overall cosmic environment (which could occur only on time scales $\sim t_{Ev}$) (38, 77).

Because of space limitations, I do not speculate here about how different forms of AGNs might be interrelated. Another contentious issue is the role of *beaming* (25, 28, 32, 113), which has been advocated to explain compact radio sources (and perhaps also extreme optical outbursts in OVVs). There is no reason to invoke it, however, for the typical radio-quiet quasar, and in any case it cannot affect the estimation of (35.). Suffice it to say, as emphasized by Phinney (99), that tentative "demographic" studies of AGNs imply the following:

1. If strong radio sources involve ion-supported tori around holes of mass $\gtrsim 10^9\ M_\odot$, they could be the "reactivated" remnants of long-lived quasars (with $t_Q = t_{Ev}$).
2. If quasars radiate at the Eddington limit, then their lifetimes would be $\sim 4 \times 10^7 h(\varepsilon/0.1)$ yr (cf. Equation 4). Even if their efficiency ε is high, they must be short lived compared with t_{Ev}. The space density of remnants cannot exceed $10^{-3} h_{100}^2 (\varepsilon/0.1)^{-1}$ Mpc^{-3} unless their luminosities are highly "super-Eddington."
3. If Seyfert galaxies are quasar remnants, they cannot be modeled by radiation-supported tori.

9. SOME CONCLUDING COMMENTS

The foregoing sections have discussed some physical processes and some idealized models that are relevant to the general phenomenon of active galactic nuclei. It is, however, depressingly evident how tenuous are the links between these models and the actual observations. Partly, this is because the subject is just beginning; but it is also partly because the observations relate only very indirectly to the primary energy source—they may instead tell us about secondary reprocessing that has occurred on much larger scales.

Exhortations and hopes for the future can be summarized in three categories:

1. On the purely theoretical level, even the simple "toy" models discussed here need further investigation—they involve effects in Kerr geometry, collective processes and radiative transfer in pair-dominated plasma, and acceleration of high gamma particles, none of which are yet well explored or understood. We need to clarify the stability of the various axisymmetric configurations: this should narrow down the embarrassing freedom we now have in specifying the angular momentum and the enthalpy distribution in tori. Large-scale computer simulations could be crucial here.

Computer simulations should permit us also to relax the assumption of stationarity (59), which has been implicit in most work on accretion flows. It may be more realistic to envisage that the feeding proccss and the subsequent viscous redistribution of angular momentum and drainage into the hole are sporadic. There is, after all, observational evidence for variability on all time scales. Three-dimensional gas-dynamical codes could also check whether the Lense-Thirring effect does indeed align the flow in the way simple arguments suggest. A further valuable computational development will be the advent of MHD codes able to treat electromagnetic processes around black holes, as well as the initiation (and possible magnetic confinement) of relativistic jets.

Detailed computations would also be worthwhile on other classes of relativistic systems relevant to the evolutionary tracks in Figure 1 (20). In particular, supermassive stars with realistic differential rotation should be investigated. For a suitable angular momentum distribution, these could acquire a high gravitational binding energy [cf. the massive disks (15, 75) that have been treated analytically]. Redistribution of angular momentum within such objects would be likely to cause their inner regions to collapse, leaving a massive torus around the resultant black hole. If too massive, this would be subject to gravitational instability and could fragment. Otherwise, it would evolve on a Kelvin or viscous time scale (whichever was shorter). Such models also remind us that evolution need not be restricted to the slow time scales of order t_E (Equation 4), but that rare "hypernovae" may occur.

2. The "peripheral fuzz" at $r \gg r_g$ in the emission-line region and the radio structures involves physics that is less extreme and more familiar than that in the central relativistic domain. However, it is here that one is perhaps more pessimistic about theoretical progress. This is because in the central region, even though the physics may be exotic, we have a relatively "clean" problem : axisymmetric flow in a calculable gravitational field. On the other hand, in the large-scale sources, environmental effects are plainly crucial: progress will be slow, for the same reasons that weather prediction is difficult.

The subject has proceeded in a highly compartmentalized way: the central engine, the emission-line region, the radio jets, etc., are modeled somewhat disjointly. To a certain extent this is inevitable—after all, the relevant scales may differ by many powers of ten. As data proliferate on source morphology, it no longer seems premature to develop more comprehensive models, nor to understand the relation of AGNs to their parent galaxies: If we compare spirals and ellipticals, are the central masses different? Is the fueling different? What other environmental influences determine the kind of AGN that is observed? And do stellar-mass compact objects within our own Galaxy offer many clues to the mechanisms of AGNs?

3. It is perhaps salutary (especially for relativists) to remain aware that

Einstein's theory is empirically validated only in the weak-field limit. An extra motive for studying the central region is therefore to seek a diagnostic (by refining our models for galactic nuclei) that could test strong-field general relativity and check whether the space-time around a rotating black hole is indeed described by the Kerr metric.

Ginzburg (53) has recently remarked on how surprisingly *slowly* most sciences develop. Concentrated activity over a short time-base may give the illusion that progress is fast, but the advance of science—particularly where data are sparse—displays a slowly rising trend, with large-amplitude "sawtooth" fluctuations superposed on it as fashions come and go. There has been progress toward a consensus, in that some bizarre ideas that could be seriously discussed a decade ago have been generally discarded. But if we compare present ideas with the most insightful proposals advanced when quasars were first discovered 20 years ago (such proposals being selected, of course, with benefit of hindsight), progress indeed seems meager. It is especially instructive to read Zeldovich & Novikov's (1964) paper entitled "The Mass of Quasi-Stellar Objects" (133). In this paper, on the basis of early data on 3C 273, they conjectured the following: (*a*) Radiation pressure perhaps balances gravity, so the central mass is $\sim 10^8\ M_\odot$. (*b*) For a likely efficiency of 10%, the accretion rate would be $3\ M_\odot\ \mathrm{yr}^{-1}$. (*c*) The radiation would come from an effective "photosphere" at a radius $\sim 2 \times 10^{15}$ cm (i.e. $\gg r_g$), outside of which line opacity would cause radiation to drive a wind. (*d*) The accretion may be self-regulatory, with a characteristic time scale of ~ 3 yr. These suggestions accord with the ideas that remain popular today, and we cannot yet make many firmly based statements that are more specific.

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