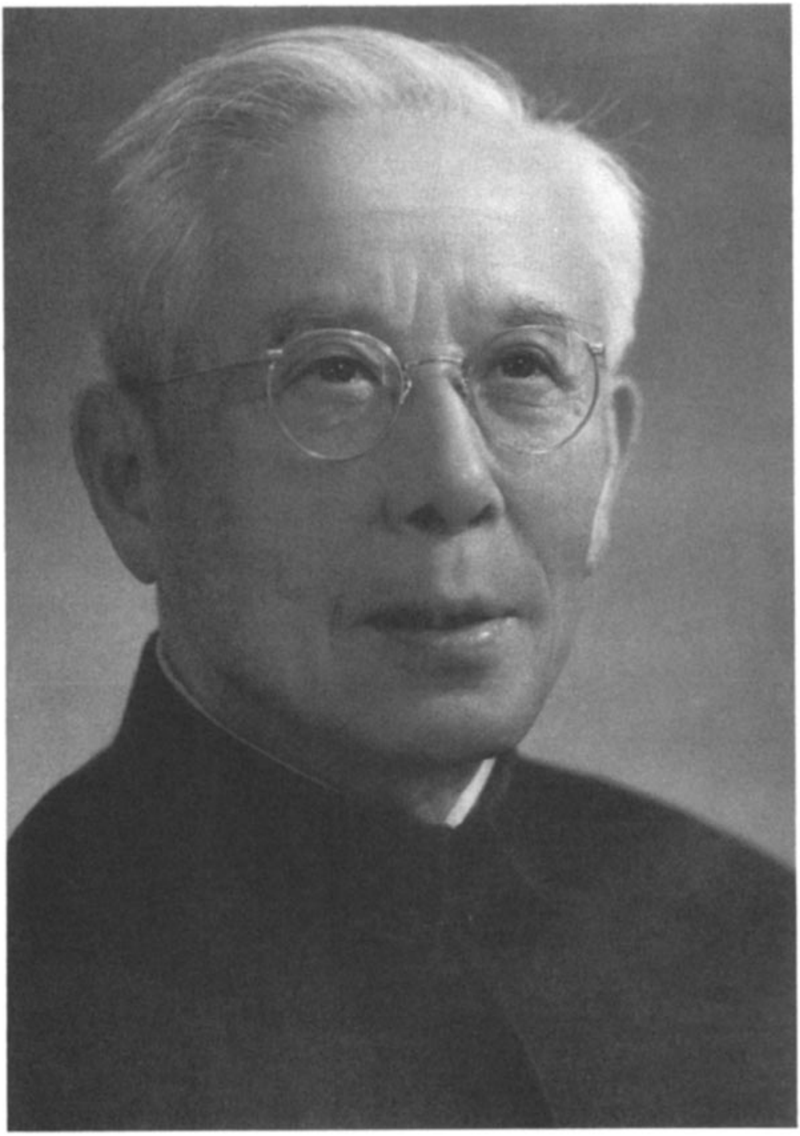

Annu. Rev. Fluid Mech. 1995. 27: 1–15

50 YEARS OF TURBULENCE RESEARCH IN CHINA

Pei-Yuan Chou

Department of Mechanics, Peking University, Beijing, China

Ru-Ling Chou

Lamont-Doherty Earth Observatory, Columbia University, Palisades, New York 10964

INTRODUCTION by J. L. Lumley

When the Editorial Board of the Annual Review of Fluid Mechanics invited Pei-Yuan Chou to prepare a historical article, we hoped that he would deal with the personalities and socio-cultural history of the development of fluid mechanics in China during his lifetime. We expressed this hope in our initial letter. We were encouraged to approach Professor Chou through intermediaries, rather than directly—Shan-Fu Shen of Cornell University was helpful in this regard, and later Professor Chou's daughter, Ru-Ling Chou, then of the NASA Goddard Institute for Space Studies (presently with Columbia Medical School). As our correspondence (entirely through third parties) developed, we began to infer that Professor Chou did not find our request appropriate, and did not feel that he should publicly discuss such things or his role in them. Rather, he preferred the sort of personal scientific retrospective paper that follows this brief introduction.

During the summer of Professor Chou's 90th birthday (August 28, 1992), I was pleased to attend an international conference held in his honor at Beijing under the auspices of Peking University, the Chinese Physical Society, and the Chinese Society of Theoretical and Applied Mechanics. The proceedings of the conference have been published by the University Press under the title *Some New*

0066-4189/95/0115-0001$05.00

Trends on Fluid Mechanics and Theoretical Physics, reflecting Professor Chou's two interests. During the conference a birthday reception and dinner were held for Professor Chou, and several speeches were presented, highlighting his career. In this introduction I have supplemented what I knew of Professor Chou with material from these speeches. In addition, I have drawn freely on the preface of the conference proceedings, prepared by Jia-Er Chen, the chairman of the program committee, as well as on a biography which Pei-Yuan Chou himself edited. Bill Sears, Ru-Ling Chou, and Akiva Yaglom have read the manuscript and made helpful suggestions.

At the conference in Beijing, Professor Chou was an impressive figure, clearly in full command of his faculties. His family and friends were happy to have had an opportunity to honor him because, sadly, Professor Chou passed away without warning or suffering on November 24, 1993.

Chou made very considerable scientific contributions to fluid mechanics and theoretical physics. In the latter area, he worked on the general theory of relativity and gravitation, about which I am not qualified to comment. When the Second World War broke out, however, he wanted to employ his scientific knowledge directly to serve China and the free world. He temporarily gave up the field of theoretical physics and embarked on the field of fluid dynamics. In the area of turbulence, he is regarded as the father of computational modeling. In an absolutely original paper published in the *Chinese Journal of Physics* 4,1 (1940), pp. 1–33, and later elaborated in the international literature in three papers in 1945 and 1947, he introduced equations for the second and third moments of the turbulent fluctuations, which he closed by a model equation relating the fourth moment to the second moment, giving an expression somewhat different from that proposed a bit later by Millionshchikov. Chou carried an equation for the enstrophy, which provided a time scale. Kolmogorov had also suggested the parameterization of turbulence by the local scales, but did not follow up on the suggestion. Unfortunately, Chou's suggestions came before computers were generally available; the amount that could be done by hand computation was quite limited. Nevertheless, there is no question that the hundreds of modelers all over the world at the present time, busily developing turbulence models which can be used in CFD codes for design and regulation, can trace their heritage directly to this work of Chou in 1940.

Chou received his AB and AM from the University of Chicago in

1926, and his PhD from Cal Tech in 1928 under the guidance of E. T. Bell, with whom he studied general relativity. He was the first Chinese PhD student to be graduated from Cal Tech. In the fall of 1928 he was a postdoctoral fellow at Leipzig under Werner Heisenberg. The next spring, when Heisenberg left Germany for the U.S., Wolfgang Pauli invited Chou to join him at the Eidgenössiche Technische Hochschule in Zurich, to continue his postdoctoral work in quantum mechanics. Returning to China in the fall of 1929, he became the youngest professor at Tsing Hua University, at the age of 27, where he was the only theoretician, and therefore responsible for teaching all the theoretical physics courses. During the war years, 1938–1947, he was a physics professor at the National Southwest Associate University as well. Chou spent his first sabbatical year, 1936–37, at the Institute for Advanced Study at Princeton, where he took part in Albert Einstein's seminar. During the period 1943–1947, Chou spent his second sabbatical year at Cal Tech. He then worked for the U.S. Office of Scientific Research and Development of the National Defense Research Committee, and later for the U.S. Naval Ordnance Test Station, on the problem of launching torpedoes from aircraft. When the war ended, Chou took his family back to Tsing Hua University. The beginning of Chou's career coincided with the introduction of modern physics into China, a process to which Chou (in collaboration with three or four physicists of the older generation) made major contributions. He was solely responsible for introducing relativity theory into China. Chou is also regarded as the founder of Mechanics as a discipline in China. In 1952, at the age of 50, he moved to Peking University, where he was successively Dean of Studies, Vice President, and finally in 1976, President. He was responsible for setting up the field of Mechanics there, which was the earliest section of Mechanics established in China, and which has trained large numbers of distinguished experts for the country. In fact, Chou personally supervised enormous numbers of students, and continued to do so until the age of 90; he has academic offspring everywhere. He was also at various times Vice President of the Chinese Academy of Science, and Chairman of the China Association for Science and Technology (CAST). He was President of the Chinese Physical Society, and Vice President of the Chinese Society of Theoretical and Applied Mechanics.

Chou was very active in international collaborations. He was the sole Chinese delegate to the first Pugwash Conference, held in

Canada in 1957, and at his death was a member of the Council of the Pugwash Meeting of Science and World Affairs. He was elected to both of the councils of the International Congress of Theoretical and Applied Mechanics (ICTAM) and the newly organized International Union of Theoretical and Applied Mechanics (IUTAM) in 1946 in Paris, and at the time of his death was a member of the General Assembly of IUTAM. In 1978 he led a Chinese education delegation to visit the United States at the invitation of the U.S. Government through the Committee of Scholarly Communications with the People's Republic of China (CSCPRC), before the normalization of diplomatic relations between the PRC and the U. S. The purpose of the visit was to negotiate a scholarly exchange program between the two countries. Chou invented the title "visiting scholar" during the negotiation, and this turned out to be a key factor leading to the successful establishment of the exchange. The scholarly exchange relation between China and the U.S. established then continues to the present time. Chou was one of the initiators of the Asian Congress of Fluid Mechanics, which is at the present time a useful and effective forum to bring together scholars in this field from the Pacific rim. He received an Honorary LLD from Princeton University in 1980. In 1982 he won China's National Natural Science Award, and in 1980 and again in 1985 he received from Cal Tech the Distinguished Alumni Award. He is the only Cal Tech alumnus to receive this award twice.

Older readers will not need to be reminded that Chou's professional career coincided with dramatic social upheavals in China: the Japanese invasion, the Second World War, the war between the Communists and Nationalists, the rise and fall of the Gang of Four, and the Cultural Revolution, just to touch the high points. It is a tribute to the delicacy of his political touch, and the respect in which he was held, that he managed to come through this turmoil unscathed, despite his having always been an activist. When he was a high school student in Shanghai in 1919, he was expelled for his active participation in the May Fourth Movement. During the years of the Cultural Revolution, Chou opposed the Gang of Four, boycotted their anti-scientific maneuvers, and in a strongly-worded letter to Premier Zhou En-Lai in 1972 indicated the grave danger of neglecting the importance of fundamental science. It was after the overthrow of the Gang of Four that Chou assumed the Presidency of Peking University, and exercised energetic leadership in restructuring and reforming the teaching and research functions of the university.

At the beginning of Chou's career, Chinese science was hamstrung by the complexity of the Chinese language and the problems it created for typing, printing, and ultimately for computation and electronic communication. It was Chou who saw the tremendous significance of the Computerized Editing and Laser Typesetting System for Chinese Characters, and gave from the very beginning the strongest possible support to the research team led by Professor Xuan Wang at Peking University, paving the way for the success of this research project—a project that has finally brought about a revolution in today's printing industry in China.

This short introduction gives just a hint of the many phases of Chou's long and productive career. We do not have space here to do more than mention that he was also responsible for research and critical evaluation of resource exploitation and ecological protection in southwestern China. He hosted vast numbers of foreign guests, and on many occasions represented China at international conferences and activities for world peace.

In this generation there were at least four giants in fluid mechanics from four countries; men who had, each in his own way, tremendous influence inside and outside his country, certainly by contributing to the development of areas of fluid mechanics, but also by providing intellectual and personal leadership; men of such stature that substantial numbers of distinguished workers in fluid mechanics in each country could trace their academic lineage to that great man. I have in mind von Karman in the United States, Kolmogorov in the (then) Soviet Union, G. I. Taylor in the United Kingdom, and Pei-Yuan Chou of China.

50 YEARS OF TURBULENCE RESEARCH IN CHINA

The basic dynamical equations for viscous fluids are the Navier-Stokes equations which are nonlinear partial differential equations of second order:

$$(\partial u_i/\partial t)+u^j u_{i,j} = -(1/\rho)p_{,i}+\nu\nabla^2 u_i \tag{1}$$

$$u^j_{,j} = 0 \tag{2}$$

where u is velocity, p is pressure, ρ is density, and ν is viscosity. However, due to the complexity of turbulent motion and to the nonlinearity of the equations themselves, it is difficult to solve turbulence problems by utilizing the Navier-Stokes equations directly.

For fully developed turbulent motions of incompressible fluids, Osborne Reynolds discovered that the motion can be divided into two parts: 1. the mean motion (which varies slowly in space and time) and 2. the motion of fluctuations (which is a rapidly varying function of space and time). All of the dynamical quantities, such as velocity and pressure, can be divided in this way, i.e.

$$u_i = U_i + w_i \quad p = \bar{p} + \tilde{\omega} \tag{3}$$

with

$$\bar{w}_i = 0 \quad \bar{\tilde{\omega}} = 0 \tag{4}$$

where U_i is the mean velocity, p is the mean pressure, and the fluctuations in velocity and pressure are denoted by w_i and $\tilde{\omega}$ respectively. Substituting these two parts of dynamical quantities into the Navier-Stokes equations and taking the time or space ensemble average, Reynolds obtained the equation for mean motion, which he published in his famous 1895 paper:

$$\partial U_i/\partial t + U^j U_{i,j} = -(1/\rho)\bar{p}_{,i} + (1/\rho)\tau^j_{i,j} + \nu\nabla^2 U_i \tag{5}$$

$$U^j_{,j} = 0. \tag{6}$$

The Reynolds stress,

$$\tau^j_i = -\rho\overline{w_i w^j} \tag{7}$$

is proportional to the double velocity fluctuation correlation, which obviously is unknown. The equations for mean motion thus derived are not closed. In order to tackle this unclosed problem, various forms for the Reynolds Stresses were proposed. Most scientists assumed that the Reynolds stress was linearly related to the gradients of the mean motion with a turbulent viscosity coefficient. Mixing-length theories by Prandtl (using momentum transport), Taylor (using vorticity transport), and von Karman (using similarity theory) were all proposed.

In 1940, P.-Y. Chou presented another approach, which was published in the *Chinese Journal of Physics*. He pointed out that because the Navier-Stokes equations are the basic dynamical equations of fluid motion, it is insufficient to consider only the mean turbulent motion. The turbulent fluctuations are as important as the mean motion and the equations for turbulent fluctuations also need to be considered.

Subtracting the mean motions (5, 6) from the Navier-Stokes equations (1, 2), Chou obtained the equations for the turbulent fluctuations,

$$(\partial w_i/\partial t) + U^j w_{i,j} + w^j w_{i,j} + w^j U_{i,j} = -(1/\rho)\tilde{\omega}_{,i} - (1/\rho)\tau^j_{i,j} + \nu\nabla^2 w_i \tag{8}$$

$$w^j_{,j} = 0 \tag{9}$$

from which he derived the dynamical equations of the double (Reynolds stress), triple, and quadruple correlations:

$$\frac{\partial}{\partial t}\overline{w_i w_k} + U_{i,j}\overline{w^j w_k} + U_{k,j}\overline{w^j w_i} + U^j(\overline{w_i w_k})_{,j} + (\overline{w^j w_i w_k})_{,j}$$

$$= -\frac{1}{\rho}(\overline{\tilde{\omega}_{,i} w_k} + \overline{\tilde{\omega}_{,k} w_i}) + \nu\nabla^2\overline{w_i w_k} - 2\nu g^{mn}\overline{w_{i,m} w_{k,n}} \tag{10}$$

$$\begin{aligned}
&\frac{\partial}{\partial t}\overline{w_i w_k w_l} + U_{i,j}\overline{w^j w_k w_l} + U_{k,j}\overline{w^j w_l w_i} + U_{l,j}\overline{w^j w_i w_k} \\
&\quad + U^j(\overline{w_i w_k w_l})_{,j} + (\overline{w^j w_i w_k w_l})_{,j} \\
&\quad = -\frac{1}{\rho}(\overline{\tilde{\omega}_{,i} w_k w_l} + \overline{\tilde{\omega}_{,k} w_l w_i} + \overline{\tilde{\omega}_{,l} w_i w_k}) + (\overline{w^j w_i})_{,j}\overline{w_k w_l} \\
&\quad + (\overline{w^j w_k})_{,j}\overline{w_l w_i} + (\overline{w^j w_l})_{,j}\overline{w_i w_k} + \nu g^{mn}(\overline{w_i w_k w_l})_{,mn} \\
&\quad - 2\nu g^{mn}(\overline{w_{i,m} w_{k,n} w_l} + \overline{w_{k,m} w_{l,n} w_i} + \overline{w_{l,m} w_{i,n} w_k})
\end{aligned} \tag{11}$$

$$\begin{aligned}
&\frac{\partial}{\partial t}\overline{w_i w_k w_l w_p} + U_{i,j}\overline{w^j w_k w_l w_p} + U_{k,j}\overline{w^j w_l w_p w_i} + U_{l,j}\overline{w^j w_p w_i w_k} \\
&\quad + U_{p,j}\overline{w^j w_i w_k w_l} + U^j(\overline{w_i w_k w_l w_p})_{,j} + (\overline{w^j w_i w_k w_l w_p})_{,j}, \\
&\quad = -\frac{1}{\rho}(\overline{\tilde{\omega}_{,i} w_k w_l w_p} + \overline{\tilde{\omega}_{,k} w_l w_p w_i} + \overline{\tilde{\omega}_{,l} w_p w_i w_k} + \overline{\tilde{\omega}_{,p} w_i w_k w_l}) \\
&\quad + (\overline{w^j w_i})_{,j}\overline{w_k w_l w_p} + (\overline{w^j w_k})_{,j}\overline{w_l w_p w_i} + (\overline{w^j w_l})_{,j}\overline{w_p w_i w_k} \\
&\quad + (\overline{w^j w_p})_{,j}\overline{w_i w_k w_l} + \nu g^{mn}(\overline{w_i w_k w_l w_p})_{,mn} \\
&\quad - 2\nu g^{mn}(\overline{w_{i,m} w_{k,n} w_l w_p} + \overline{w_{k,m} w_{l,n} w_i w_p} + \overline{w_{l,m} w_{p,n} w_i w_k} \\
&\quad + \overline{w_{i,m} w_{l,n} w_k w_p} + \overline{w_{k,m} w_{p,n} w_i w_l} + \overline{w_{i,m} w_{p,n} w_k w_l}).
\end{aligned} \tag{12}$$

By so doing, a hierarchy of equations for velocity fluctuation correlations is formed. However, any velocity correlation equation of a given order always contains an unknown velocity correlation of one higher order, i.e. this hierarchy is also not closed.

One way to treat the turbulence problem is to assume some physical restrictions on the velocity correlations at a certain order to close the system at that level. In his 1940 paper, Chou stopped at the equations of the triple velocity correlation by assuming the quadruple correlation to be proportional to the sum of the three products of two double velocity correlations:

$$\overline{w_i w_j w_k w_l} = c(\overline{w_i w_j}\,\overline{w_k w_l} + \overline{w_i w_k}\,\overline{w_j w_l} + \overline{w_i w_l}\,\overline{w_j w_k}). \tag{13}$$

In addition, further hypotheses on the correlations of the turbulent pressure gradient and velocity fluctuations were needed, and terms of the turbulent energy decay were assumed. This type of approach, subsequently developed first by Rotta (1951a,b) and then by a number of investigators in the 1960s and 1970s—especially after computers were widely employed (Launder et al 1975), has been referred to as "modeling theory."

Chou (1945) then derived the pressure fluctuation gradient from the Poisson equation:

$$\frac{1}{\rho}\nabla^2\tilde{\omega}_{,k} = -2(U^m_{,n} w^n_{,m})_{,k} + (\overline{w^m w^n} - w^m w^n)_{,mnk}. \tag{14}$$

The general solution to (14) can be written in the form

$$\frac{1}{\rho}\tilde{\omega}_{,k} = \frac{1}{2\pi}\iiint (U'^m_{,n} w'^n_{,m})'_{,k}\frac{1}{r}\,dV' - \frac{1}{4\pi}\iiint (\overline{w'^m w'^n} - w'^m w'^n)'_{,mnk}\frac{1}{r}\,dV'$$
$$+ \frac{1}{4\pi\rho}\iint\left[\frac{1}{r}\frac{\partial\tilde{\omega}'_{,k}}{\partial n'} - \tilde{\omega}'_{,k}\frac{\partial}{\partial n'}\left(\frac{1}{r}\right)\right]dS'.$$

The correlations between the pressure fluctuation and the velocity fluctuations were obtained:

$$\frac{1}{\rho}\overline{\tilde{\omega} w_i} = \frac{1}{2\pi}\iiint U'^m_{,n}(\overline{w'^n w_i})'_{,m}\frac{1}{r}\,dV' + \frac{1}{4\pi}\iiint (\overline{w'^m w'^n w_i})'_{,mn}\frac{1}{r}\,dV'. \tag{16}$$

The equations for vorticity decay were also found and the terms of energy decay were improved:

$$\frac{1}{\rho}(\overline{\tilde{\omega}_{,i} w_k} + \overline{\tilde{\omega}_{,k} w_i}) = a^n_{mik} U^m_{,n} + b_{ik} \tag{17}$$

where

$$a^n_{mik} = \frac{1}{2\pi}\iiint [(\overline{w'^n w_i})_{,mk} + (\overline{w'^n w_k})_{,mi}]\frac{1}{r}\,dV' \tag{18}$$

$$b_{ik} = \frac{1}{4\pi}\iiint [(\overline{w'^m w'^n w_i})_{,mnk} + (\overline{w'^m w'^n w_k})_{,mni}]\frac{1}{r}\,dV' \tag{19}$$

$$2\nu g^{mn}\overline{\frac{\partial w_i}{\partial x^m}\frac{\partial w_k}{\partial x^n}} = -\frac{3\nu}{3\lambda^2}(k-5)q^2 g_{ik} + \frac{2\nu k}{\lambda^2}\overline{w_i w_k}. \tag{20}$$

Thus, by building up the differential equations of the velocity correlations for each successive order from the equations of turbulent fluctuations, a method of solving the turbulent problem by successive approximations is achieved. In principle, one is able to obtain the velocity correlation up to any higher order, as long as the process of successive approximation is carried out at least one step further.

Chou (1945) also pointed out that a more rigorous way of treating the problem involves simultaneously solving the Reynolds equations of mean motion and the equations of turbulent fluctuation. However, this method is very difficult to carry out. There were three major obstacles:

1. The equations of mean motion and fluctuation form a set of nonlinear integro-differential equations; ascertaining their solutions for general shear turbulent motion of fluids is very difficult.
2. The mean velocity, mean pressure, and the turbulent velocity correlation functions of different orders are all slowly varying functions of space and time, while the turbulent velocity and pressure fluctuations are all rapidly varying functions of them.
3. To solve the set of nonlinear integro-partial differential equations of turbulent motion, an extra physical condition is needed (like Prandtl's similarity or self-preserving condition for the boundary layer problem in laminar fluid motion).

In addition to the difficulties mentioned above, the computer had not yet been invented; the simultaneous solutions of the equations of mean motion and velocity fluctuation were thus essentially impossible.

In the early 1950s, Chou and his students started to investigate the nature of turbulence; namely, they wanted to know: What is the "element" of turbulence? They started with the simplest type of turbulence—homogeneous isotropic turbulence. In this case, the equations of turbulent fluctuation are the same as the Navier-Stokes equations. Chou & Cai (1957) considered homogeneous isotropic turbulence in the final period of decay, wherein the Navier-Stokes equations can be linearized. As the element of turbulence, they chose an axially symmetric vortex which satisfies a condition of similarity:

$$\frac{\lambda}{\nu}\frac{d\lambda}{dt} = 2 \tag{21}$$

where λ is the micro-scale. The double velocity correlation between two distinct points was computed over all space and for all orientations of the vortex axis. The double velocity correlation thus obtained is already well known. Both the correlation and decay of the turbulence energy agree with experiment. The paper was presented at the 1956 International Congress of Applied Mechanics and published the following year (Chou & Cai 1957).

Using the above axially symmetrical vortex solution, Y.-N. Huang, an undergraduate student at Peking University at that time, computed the triple velocity correlation between two distinct points (Huang 1965). Later, Bennett (1976) performed an experiment under the guidance of S. Corrsin to measure this triple velocity correlation for small Reynolds number flows. His results verified Huang's (1977) theoretical predictions very well (see Figures 1 and 2).

In the initial period of decay for homogeneous isotropic turbulence, for instance, not far behind the grid in a wind tunnel where the Reynolds number of the flow is high, both the terms involving the viscosity and the temporal partial derivative in the Navier-Stokes equations can be ignored. Chou, Shi & Li (1965) used another similarity condition,

$$\frac{1}{U^3\lambda^2}\frac{d(U^2\lambda^3)}{dt} = \text{constant} \tag{22}$$

and obtained a different axially symmetrical vortex solution to the equations. By using this vortex as an element of turbulence, they computed the double velocity correlation between two different points in the fluid via the same averaging process as was used in the case of the final period of decay. By employing this double velocity correlation, one can compute the triple correlation from the Kármán-Howarth equation for the double

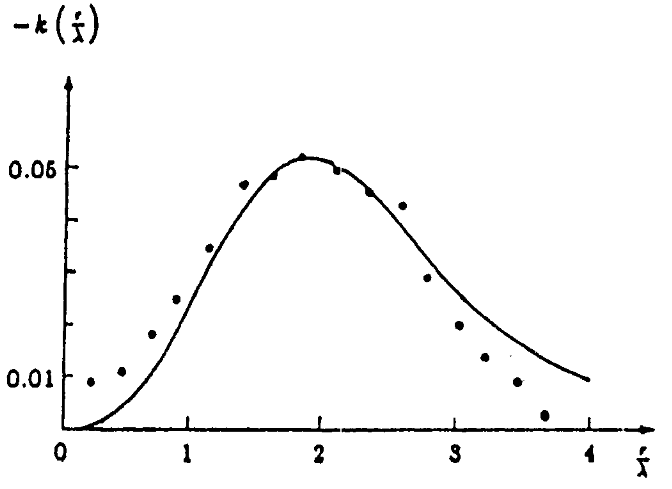

Figure 1 A comparison of Huang's theoretical prediction (curve) with experimental data. Grid Reynolds number $R_M = 1800$, $R_\lambda = 10.9$, $x/M = 480$.

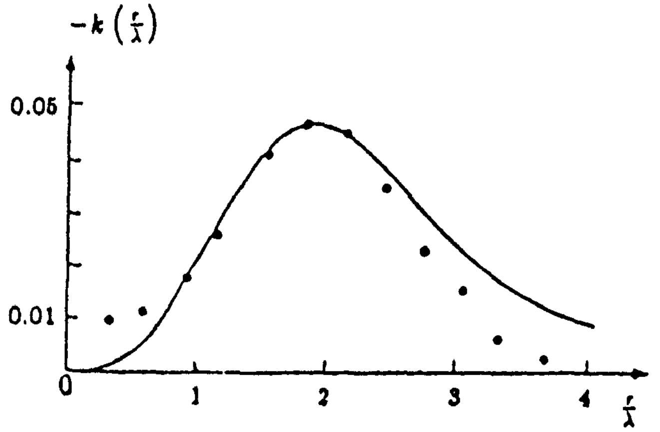

Figure 2 A comparison of Huang's theoretical prediction (curve) with experimental data. Grid Reynolds number $R_{\mathrm{M}} = 1300$, $R_\lambda = 5.1$, $x/M = 640$.

velocity correlation. Both correlations thus obtained agree qualitatively well with experimental measurements (Chou et al 1965).

Since the two conditions of similarity for solving the problem of homogeneous isotropic turbulence in its initial and final periods of decay are different, there should exist a single condition to embrace the two. Chou & Huang (1975) introduced the pseudo-similarity condition,

$$\frac{\lambda}{v}\frac{d\lambda}{dt} = \frac{1}{R_0}R_\lambda^2 + 2 \tag{23}$$

which covers the similarity condition in the initial period for large Reynolds number R_λ (Chou et al 1965) and that in the final period of decay for very small Reynolds number (Chou et al 1957). Using this newly introduced pseudo-similarity condition, Huang & Chou solved the nonlinear velocity fluctuation equations—which are the same as the Navier-Stokes equations—to first approximation for homogeneous isotropic turbulence, and obtained a solution that unified the initial to final periods of turbulence decay. Satisfactory agreement was obtained between computations and experimental results for the double and triple velocity correlations, and for the energy spectrum function from the initial to the final period of decay. These results were presented at the 1980 International Congress of Theoretical and Applied Mechanics, and at the First Asian Congress of Fluid Mechanics, and were published in 1981.

The introduction of the pseudo-similarity condition into the theory of homogeneous isotropic turbulence was crucial to its success. This condition was verified experimentally by Wei and his colleagues in the lowlevel turbulence wind tunnel in Peking University; the results were published in 1981 (Wei et al 1988; see Figures 3 and 4).

In 1985, Chou extended the results for homogeneous isotropic turbulence to the solution for general turbulent shear flow by generalizing the pseudo-similarity condition:

$$\frac{\lambda}{\nu}\left(\frac{\partial\lambda}{\partial t}+U^j\lambda_{,j}\right)=\frac{1}{R_0}\left\{R_\lambda^2-\left[R_l+\frac{k_1}{\nu}\lambda^2(g^{kl}U^j_{,k}U_{j,l})^{1/2}+\frac{k_2}{\nu}\lambda^3(g^{ij}g^{kl}g^{mn}\Omega_{ik,m}\Omega_{jl,n})^{1/2}\right]^2+2R_0\right\} \quad (24)$$

where $\Omega_{ij} = U_{i,k} - U_{k,i}$ and k_1, k_2, R_0 are constants. At the same time, the method of successive approximation mentioned before (Chou 1945) had also been further developed.

Based upon various experiments, it is found that the velocity correlations of the odd orders are usually much smaller than those of the even ones. During the process of approximation we can then truncate the velocity correlations of the odd orders to terminate the hierarchy of dynamical equations of velocity correlations. The method of successive approxi-

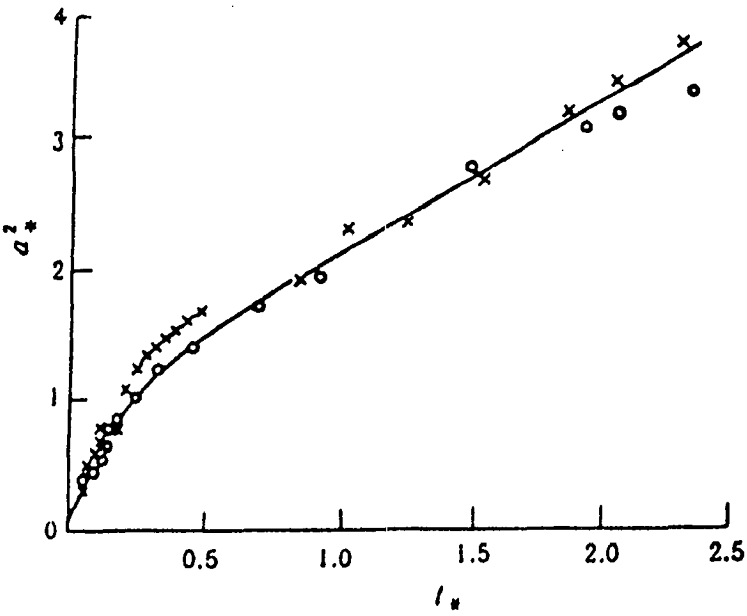

Figure 3 Decay of the normalized turbulent energy during the whole period from the initial to the final stage. The curve is from the theory of vorticity structure. From Wei et al (1988).

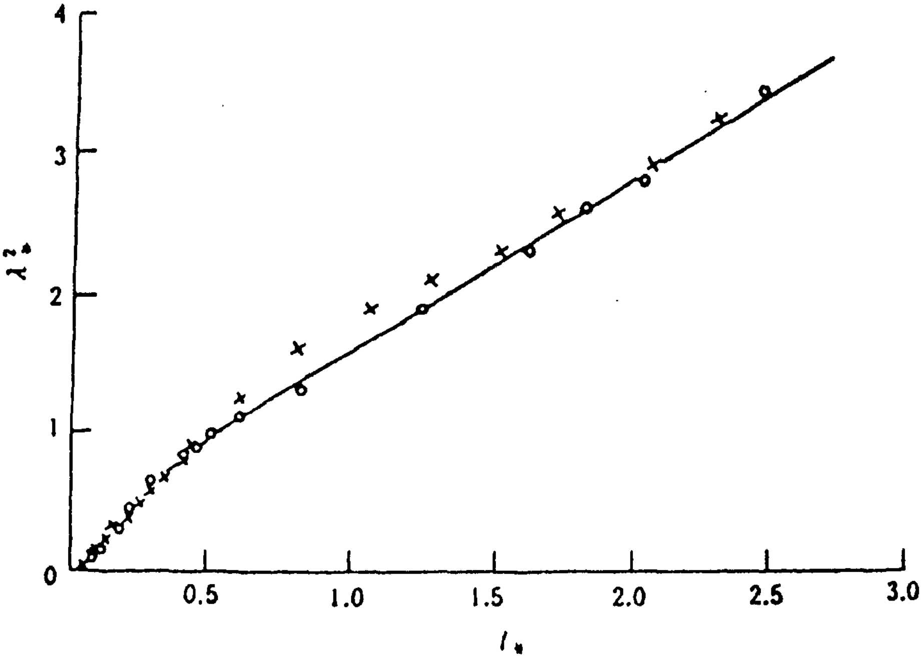

Figure 4 Decay of the normalized turbulence microscale during the whole period from the initial to the final stage. The curve is from the theory of vorticity structure. From Wei et al (1988).

mation is carried out as follows: For the first approximation we solve the equations of mean motion (5, 6) and of the double velocity correlation (10) in which the terms involving the triple correlations are dropped; the second approximation is to solve the equations of the triple (11) and quadruple correlations (12) together with the equations in the first approximation, while the quintuple correlations are ignored. Apparently, this process of approximation can be continued.

This method of successive approximation using only its first approximation has been applied to solve the channel flow and plane wake problems by Chou (1985). Calculations including the second approximation for plane wakes and channel flows were carried out by Chou & Chen (1987). Their computed triple correlation of the plane wake was compared with the results calculated using the closure conditions on the triple correlation put forth by Launder et al (1975). There exist distinct differences between the two theoretical predictions of the triple correlations with their experimental measurements (see Figure 5).

From the preceding presentations we can see that turbulence problems can be solved by the method of successive approximation—namely, by

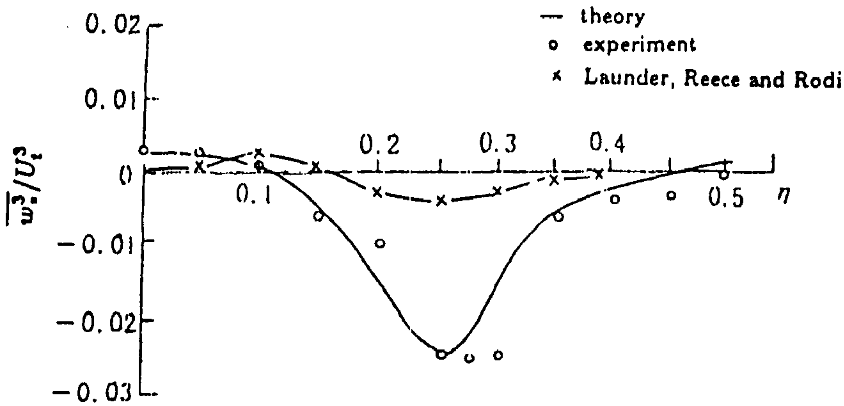

Figure 5 A comparison between theory and experiment for the triple correlation for a plane wake.

building a hierarchy of velocity correlations of various orders and solving the differential equations satisfied by them. However, performing all these integrations is cumbersome, and certain assumptions always have to be imposed to terminate the hierarchy.

Since 1987, we have been developing a new method, a method of successive substitution, to solve the equations of mean motion and turbulent velocity fluctuations simultaneously. The basic idea of this method is as follows. First, we start with the equations of mean motion (5, 6) and double velocity correlation (10), and neglect the triple velocity correlation involved. The solutions obtained for the Reynolds stress and mean velocity are considered a zeroth-order approximation. We then put these zeroth-order solutions into the equations of turbulent fluctuation (8, 9) and solve for the velocity fluctuation w_i. After obtaining this velocity fluctuation, we can compute the Reynolds stress (7). By substituting this new Reynolds stress back into the equation of mean motion, we can solve for the new mean velocity. Both the new Reynolds stress and mean velocity are considered to be the first-order approximation. By repeating this cycle, higher-order approximation of the mean velocity and velocity correlations can be computed.

In short, after obtaining the turbulent velocity fluctuation, we can compute the Reynolds stress and the velocity correlations of higher orders. Furthermore, by using the above mentioned method of successive substitution, the different higher approximations of the mean velocity and the hierarchy of turbulent velocity correlations can be established. By solving simultaneously the equations of mean motion and of turbulent fluctuation and then building up the velocity correlations of different orders, the computations are very much simplified when compared with those required to build and solve the hierarchy of partial differential equations satisfied by the velocity correlations of different orders.

After fifty years of continuous effort, the three major difficulties in solving simultaneously the equations of mean motion and of turbulent fluctuation have finally been overcome. The task of finding the solution to fully developed turbulent flows first put forward in 1945 can now be accomplished.

Literature Cited

Bennett JC. 1976. *Measurements of grid-generated, nearly isotropic turbulence at small Reynolds' numbers*. PhD thesis. Dept. Mech. and Material Sci., John Hopkins Univ. Baltimore

Chou P-Y. 1940. On an extension of Reynolds' method of finding apparent stress and the nature of turbulence. *Chin. J. Phys*. 1–53

Chou P-Y. 1945. On velocity correlations and the solutions of the equations of turbulent fluctuation. *Q. Appl. Math*. 111(l): 38–54

Chou P-Y. 1985. On the condition of pseudo-similarity and the theory of turbulence. *Sci. Sinica* XXVIII(4): 405–81

Chou, P-Y, Cai S-T. 1957. The vorticity structure of homogeneous isotropic turbulence in its final period of decay. *Acta Mech. Sinica* 1(1): 3–14

Chou, P-Y, Chen S-Y. 1987. On the theory of turbulence for incompressible fluid. *Sci. Sinica* (4): 369–80; *A Symposium to Honor CC Lin*, 1988, ed. DJ Benney, FH Shu, C Yuan, pp. 59–80. Singapore: World Sci.; Sadhana, *India Acad. Proc. Eng. Sci*., Aug. 1987, IO(Parts 3 & 4): 403–17; Des Sciences techniques, *Ser. Mec. Appl*., 1989, 34: 195–211

Chou, P-Y, Huang Y-N. 1975. On the statistical vorticity structure theory of homogeneous isotropic turbulence. *Sci. Sinica* XVIII(2): 199–222

Chou, P-Y, Shi X-G, Li S-N. 1965. The homogeneous isotropic turbulent motion at high Reynolds numbers. *Peking Univ. J*. 1: 39–52

Huang, Y-N. 1965. The triple velocity correlation of homogeneous isotropic turbulence in the final period of decay. *Acta Mech. Sinica* XIII(2): 122–32

Huang, Y-N. 1977. *Kexue Tongbao* 22: 345–47

Huang, Y-N, Zhou P-Y. 1981. On the solutions of Navier-Stokes equations and the theory of homogeneous isotropic turbulence. *Sci. Sinica* XXIV(9): 1207–30; Chou PY. 1981. *Proc. Indian Acad., Section Eng. Sci*. IV(Part 2): 177–97

Launder BE, Reece GJ, Rodi W. 1975. *J. Fluid Mech*. 68: 537–666

Rotta JC. 1951a. Statistiche Theorie Nichthomogenes Turbulenz 1. *Z. Phys*. 129: 547–72

Rotta JC. 1951b. Statistiche Theorie Nichthomogenes Turbulenz 2. *Z. Phys*. 132: 51–77

Wei, Z-L, Zhu Q-K, Niu Z-N, Yu D-C. 1988. The experimental results of micro-structure in grid-produced turbulence. *Acta Mech. Sinica* 4(l): 7–14